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Generalized Ising model for dynamic adaptation in autonomous systems

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Abstract – The paper presents a concept of *Statistical Mechanics* for observation-based adaptation in autonomous systems, which is typically exhibited by simple *biological systems*. Time-critical operations of autonomous systems (*e.g.*, unmanned undersea vehicles (UUVs)), require *in situ* adaptation in the original plan of action and rapid response to evolving contextual changes and situation awareness for enhanced autonomy. In this regard, a concept of dynamic plan adaptation (DPA) is formulated in the setting of a generalized Ising model (*e.g.*, the Potts model) over a discretized configuration space, where the targets (*e.g.*, undersea mines) are distributed. An exogenous time-dependent potential field is defined that controls the movements of the autonomous system in the configuration space, while the decision-theoretic tool for dynamic plan adaptation is built upon local neighborhood interactions. The efficacy of the DPA algorithm has been evaluated by simulation experiments that demonstrate early detection of localized neighborhood targets as compared to a conventional search method involving back and forth motions.

Introduction. – Tools of Statistical Mechanics [1] have been used to study stationary and quasi-stationary behavior of evolving complex systems. This area has emerged as a new discipline in the applied-physics literature, known as *thermodynamic formalism of complex systems* [2,3]. The concepts of Statistical Mechanics, which were originally developed to study the collective properties of physical systems (*e.g.*, solids, liquids, and gases), have been extensively used for a diverse range of applications including chemical and biological systems (*e.g.*, colloids, emulsions, liquid crystals, complex fluids, polymers, biopolymers, cellular structures, and bacterial chemotactic networks) [4–7], economical and sociological systems [8,9], ecological systems [10], mechanical systems [11], complex networks [12,13], and for time series data analysis [14–18].

Specifically, the simple structure of Ising model [19], that is considered to be one of the profound foundations of Statistical Mechanics, has an immense potential to model neighborhood dependencies between the interacting elements of a complex system. These interactions in turn produce the collective global behavior through mutual interdependence. Technical literature abounds with diverse applications of Ising model [7,15,20]; however, the domain of applications of such exclusive models from physical sciences has so far been limited from extension to many other disciplines including the science of autonomy and artificial intelligence. A critical issue in the science of autonomous systems (*e.g.*, unmanned undersea vehicles (UUVs)) is to enhance onboard autonomy that facilitates *in situ* adaptation to contextual changes, that refer to the observed phenomenon of the environment (*e.g.*, an event detection). Recent literature [21–23] in autonomous systems has addressed several critical issues of offline coverage planning (*e.g.*, area segmentation, obstacle avoidance, and optimization of path trajectory) to search for targets (*e.g.*, undersea mines) distributed in a region; however, the concept of *in situ* plan adaptation has not been addressed.

In this regard, this paper introduces a concept of observation-based *dynamic plan adaptation* (DPA) in autonomous systems for improvement in the local search performance. The concept of DPA is formulated in the setting of a lattice spin system, where the search region is partitioned into a grid to form a finite-dimensional lattice structure. A generalized Ising-model (*i.e.*, the Potts model [24,25]) is constructed over the lattice to model

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neighborhood influences that facilitate adaptation in the autonomous systems. Note: the generalized Ising model refers to any spin model (*e.g.*, the Potts model, the XY model, and the Heisenberg model) that is constructed from n-dimensional vector spins with an alphabet whose cardinality exceeds two. However, this paper utilizes the Potts model where the spin set is extended beyond the binary alphabet. To maintain generality this model is simply referred to as a generalized Ising model. The model involves i) an exogenous time-varying potential function term to control the movement of the autonomous system in the search space and ii) a local adaptation term that models the neighborhood influences on the nominal path trajectory.

Background information. – Several time critical operations of autonomous systems, such as mine counter measure (MCM) and anti-submarine warfare (ASW), involve search, detection, and tracking of targets (*e.g.*, undersea mines and submarines), where it is imperative to plan the (sub)optimum navigation path offline based on the *a priori* information. This *a priori* information may include contextual knowledge, such as the probability maps of target locations and enemy coarse of action (ECoA). Therefore, an offline plan (*e.g.*, a navigation path trajectory) is generated for such autonomous systems based on the *a priori* information using different multi-objective optimization methods, such as the *Genetic algorithms* [26]. The objective of the generated plan is to facilitate optimal execution of a mission task (*e.g.*, undersea operations to search for seabed mines using sonar technology and land operations to search for explosives) within limited time margins. The offline plan performs well under known scenarios with perfect *a priori* information. However, the unknown situational uncertainties including both anticipated events (*e.g.*, environmental changes) and unanticipated events (*e.g.*, an enemy attack) demand for *in situ* adaptation of the original plan based on local observations. Furthermore, the *a priori* information may be faulty, thereby, necessitating *in situ* adaptation of the autonomous systems. Specifically, onboard autonomy is designed to make local decisions with agile response to observed neighborhood events (*e.g.*, target detection). Recent literature in autonomous systems has addressed the critical issues of *coverage planning* such as area segmentation, obstacle avoidance, and optimization of path trajectory to achieve multiple objectives (*e.g.*, minimization of the total time and early detection of targets) [21–23]. Apparently, the concept of *in situ* plan adaptation has not yet been addressed.

Local adaptation in the navigation path for maneuvering around a target is typically observed in *biological systems*. A common example of a biological system that exhibits adaptation is that of an ant that drops a chemical called pheromone to act as a navigator to hunt for food. Therefore, the ant follows the areas that have high intensity of pheromone and also drops the pheromone in areas of interest where the food is present and that could be searched around to detect more food. Thus, searching for high-intensity pheromone regions enables localized improvement in performance. As such, learning from such biological systems, the problem of localized plan adaptation in autonomous systems is investigated based on detection of a target (*e.g.*, an undersea mine) for additional search in target neighborhood before reverting to the original path trajectory. Often, the targets are located closely in a clustered fashion or demonstrate a certain deployment pattern (*e.g.*, a straight line). Therefore, it is advantageous to search around the region of a detected target to hunt for (possible) locally clustered targets before indulging into search of the remaining large area. If no neighboring targets are found, the search resumes back to the nominal trajectory. Another example is that of an emergency situation, where while moving along the normal course of action (CoA), an autonomous system detects a shooting enemy, in this scenario, saving life and hunting for a possible enemy convoy hiding in the neighborhood becomes more critical necessity then to follow the CoA. As such, the performance of an adaptive autonomous system depends on *in situ* response to observation (*e.g.*, early detection of neighborhood targets); this in general improves search efficiency even if the operation is halted due to unexpected emergencies (*e.g.*, a severe storm) before complete coverage is achieved.

Space partitioning and lattice formation. – As presented earlier in the introduction section, the concept of dynamic plan adaptation (DPA) is formulated in the setting of a lattice spin system. The search region (*e.g.*, the area where the potential targets are believed to be distributed) of the autonomous system is partitioned into a grid to form a finite-dimensional lattice structure such that each grid element (*i.e.*, a cell) represents a lattice site. A generalized Ising model (*i.e.*, a Potts model [24,25]) is constructed over the lattice, which involves: i) an exogenous time-varying potential function term to control the movement of the autonomous system in the search space, and ii) a local adaptation term that models the neighborhood influences on the nominal path trajectory. The construction of an energy potential of this spin model is similar to the pheromone of *biological systems*, which acts as a navigator to the autonomous system to search for critical targets.

Let $\mathcal{S} \subset \mathbb{R}^2$ be the search region for the autonomous system. Let $\mathcal{P} = \{P_\xi : \xi = 1, \cdots, |\mathcal{P}|\}$ be a partition of $\mathcal{S}$ such that it is mutually exclusive and exhaustive, *i.e.*, $P_\xi \bigcap P_\nu = \phi$ for all $\xi \neq \nu$ and $\bigcup_{\xi=1}^{|\mathcal{P}|} P_\xi = \mathcal{S}$, respectively. Therefore, the partition $\mathcal{P}$ forms a grid of the search space $\mathcal{S}$. The partition is constructed such that the dimensions of each element (*i.e.*, cell) of the grid structure fall within the scanning radius of the sensors mounted on the autonomous system and the distance that it travels to complete one instance of measurement. For example, in typical unmanned undersea vehicles (UUVs), these sensors include sonar detectors that transmit sonar waves down towards the seabed, from where they are reflected back. A typical object in the path of the sonar wave, such as a mine, reflects back a signal of high amplitude, thus enabling detection. The scanning radius of such sonar sensors (that cover the space orthogonal to the direction of motion of UUVs) is typically of the order of 10 m on the seabed. Furthermore, these UUVs travel for approximately 10 m while taking sonar measurements, which defines one measurement instance. Therefore, the grid size for such UUVs is $\sim 10\,\mathrm{m}\times 10\,\mathrm{m}$. (Note: the details of sonar technology are beyond the scope of this paper and are not discussed further).

Once the spatial partitioning is done to construct a grid, a lattice is defined, where each site of the lattice is isomorphic to a grid element (*i.e.*, a cell) and represents a physical state of that element. Therefore, the terms "cell" and "site" are used interchangeably in this paper. Let $\Sigma=\{\sigma_j : j=1,\cdots,|\Sigma|\}$ be a finite set of symbols, called the alphabet, which defines all possible states for each partition cell $P_\xi \in \mathcal{P}$. For example, in the physical description of a polycrystalline alloy, Σ denotes the list of atom species that may occupy a lattice site. For a standard Ising model, such an alphabet denotes the up and down states of the spin orientations. In this paper, the physical description of the state of each partition cell is described by an alphabet Σ that is constructed with three possible symbols (*i.e.*, $|\Sigma|=3$). These symbols are defined as $\sigma_1=0$, $\sigma_2=-1$, and $\sigma_3=1$, that represent the following possible states of each partition cell: i) *target present*, ii) *target not present*, and iii) *unexplored*, respectively. While the the autonomous system explores the search area by taking sonar measurements at each cell, the states are updated by onboard analysis and interpretation of the measured sonar data via appropriate pattern analysis methods. Therefore, these states represent the three possible conditions of a partition cell. The objective of this paper is to facilitate adaptation based on the observed states of the neighborhood.

Let us now define a mapping $\Omega : \mathcal{P}\times\mathcal{T}\to\Sigma$ that assigns each partition cell a symbol from the alphabet Σ, such that $\gamma_\xi(t)\equiv\Omega(P_\xi,t)$ is the state of a lattice site ξ at time $t\in\mathcal{T}$, where $\mathcal{T}$ is the time span. Then, the configuration space of the autonomous system at time $t\in\mathcal{T}$ is constructed as the Cartesian product

$$\Gamma(t)=\bigotimes_{\xi=1}^{|\mathcal{P}|}\gamma_\xi(t), \tag{1}$$

where Γ is the collective state of the lattice that can have at most $|\Sigma|^{|\mathcal{P}|}$ possible state configurations. The lattice system is then defined as a set $\mathcal{L}=\{\mathcal{P},\Gamma\}$. Therefore, as the autonomous system continuously searches different cells on the lattice, while moving from one cell to another, Γ unfolds in space-time via exhibiting different configurations, that represents an evolution of the checkerboard pattern in the search space.

Generalized Ising model formulation. – A three-state generalized Ising model (*i.e.*, a Potts model) is now constructed over the lattice system $\mathcal{L}$. A local energy term at a lattice site ξ is defined as

$$E_\xi^{\mathcal{L}}(t)=\sum_{\langle\xi,\nu\rangle_\kappa} J_{\xi\nu}\Psi\left(\gamma_\xi(t),\gamma_\nu(t)\right)+\Phi\left(B_\xi(t),\gamma_\xi(t)\right), \tag{2}$$

where $\langle\xi,\nu\rangle_\kappa$ implies summation over a κ-neighborhood of ξ, for some $\kappa\in\mathbb{N}$. The κ-neighborhood of a lattice site ξ is defined as

$$\mathcal{N}_\kappa(\xi)=\{\nu : \max(|\xi_x-\nu_x|,|\xi_y-\nu_y|)\leq\kappa\}, \tag{3}$$

where $\xi_x,\xi_y\in\mathbb{N}$ and $\nu_x,\nu_y\in\mathbb{N}$ denote the x- and y-coordinates of lattice sites ξ and ν, respectively. Therefore, the first term in the right-hand side of eq. (2) defines the total interaction potential due to the sum of the effects of neighbors on the state at a lattice site ξ. This term is called as the *adaptation term* because the effects of the observed states in the neighborhood cause changes in the resultant energy potential at a lattice site ξ, that enables real-time adaptation in the navigation path trajectory. The coefficient $J_{\xi\nu}$ denotes the interaction strength between two distinct lattice sites ξ and ν. For $\eta=\max(|\xi_x-\nu_x|,|\xi_y-\nu_y|)$, *i.e.*, the distance between two neighborhood sites, $J_{\xi\nu}$ is given as

$$J_{\xi\nu}=\begin{cases}\eta^{-\alpha}, & \forall\xi\neq\nu \text{ and } \eta\in\{1,\cdots,\kappa\},\\ 0, & \text{otherwise,}\end{cases} \tag{4}$$

where $\alpha\in(0,\infty)$ is a control parameter. The interaction function Ψ is defined as

$$\Psi\left(\gamma_\xi(t),\gamma_\nu(t)\right)=\begin{cases}\psi_0, & \text{for } \gamma_\xi(t)=1, \quad \gamma_\nu(t)=0,\\ \psi_{-1}, & \text{for } \gamma_\xi(t)=1, \quad \gamma_\nu(t)=-1,\\ 0, & \text{otherwise,}\end{cases} \tag{5}$$

which implies the following conditions.

1. Any lattice site ξ whose state is either $\gamma_\xi(t)=0$ (*i.e., an explored site where a target is present*) or $\gamma_\xi(t)=-1$ (*i.e., an explored site where no target is present*) is not influenced by the state of the neighborhood. Therefore, the interaction potentials of sites that are in the above states are zero.

2. All neighborhood sites with a state $\gamma_\nu(t)=1$ (*i.e., unexplored site*) exert no influence on any lattice site because they transmit no information to the neighborhood.

3. ψ_0 defines the influence of a site ν in the neighborhood with $\gamma_\nu(t)=0$ (*i.e., an explored site where a target is present*) on a site ξ with $\gamma_\xi(t)=1$ (*i.e., unexplored site*).

4. ψ_{-1} defines the influence of a site ν in the neighborhood with $\gamma_\nu(t)=-1$ (*i.e., an explored site where no target is present*) on an unexplored site ξ.

The interaction function is based on the following principle. The effect of a target detection at a certain lattice site causes distortion in the space-time potential field in the local neighborhood of the target resulting in an increase in energy by ψ_0, scaled to the interaction strength, thereby creating a dome-like structure. Therefore, as the autonomous system scans the area, Γ unfolds in space-time, and any detected target's neighborhood with localized increase in energy becomes a high-priority area. In this neighborhood, the unexplored sites (*i.e.*, with a state $\gamma_\xi(t) = 1$) with a high interaction potential tend to settle down to low-energy states (*i.e.*, *explored site*). Thus, the autonomous system follows the high potential sites and by scanning them turns them to low-energy states by converting them to explored sites that have no neighborhood interaction. If another target is detected in the neighborhood, it generates its own high interaction potential. This leads to constructive interference with the potential of earlier target, and so on. Therefore, following high potential sites leads to an adaptation in the nominal trajectory of the autonomous system such that the high-priority areas are scanned earlier, thereby improving localized search performance. The interaction function shows that the explored sites, where no target is detected, also exert a small influence on the neighboring unexplored sites where the energy increases by a factor ψ_{-1}. Therefore, Γ tends to unfold in the neighborhood of explored sites with a priority, thereby generating a more uniform and orderly search. The construction of energy functional for adaptation of an autonomous system is conceptually similar to that of the chemical pheromone used for tracking by the *biological systems*, such as an ant.

The second term in the right-hand side of eq. (2) defines a navigation control function Φ that depends on an exogenous time-varying potential field $B_\xi(t)$ and the state $\gamma_\xi(t)$ at a lattice site ξ. The function Φ is defined as

$$\Phi\left(B_\xi(t), \gamma_\xi(t)\right) = \begin{cases} -\phi_0, & \text{for } \gamma_\xi(t) = 0, \\ -\phi_{-1}, & \text{for } \gamma_\xi(t) = -1, \\ B_\xi(t), & \text{for } \gamma_\xi(t) = 1, \end{cases} \tag{6}$$

where $\phi_0 > 0$ and $\phi_{-1} > 0$ correspond to low-energy states of the explored sites that have the presence (*i.e.*, $\gamma_\xi(t) = 0$) and absence (*i.e.*, $\gamma_\xi(t) = -1$) of a target, respectively. As described earlier, the explored sites have no neighborhood interaction potential and therefore, they settle down to these low-energy states. On the other hand, the exogenous potential field $B_\xi(t)$ defines the time-varying potential at unexplored sites (*i.e.*, $\gamma_\xi(t) = 1$) and is given as

$$B_\xi(t) = B_\xi^\star - C_{\xi,\mu}(t), \tag{7}$$

where $B_\xi^\star$ is the potential field that is constructed to navigate the autonomous system with no *in situ* adaptation. The relative cost potential function $C_{\xi,\mu}(t)$ defines the total decrease in potential at a site ξ due to travel and turn costs that are incurred to reach the site ξ from a current position $\mu(t)$ of the autonomous system at time $t \in \mathcal{T}$. This cost function is given as

$$C_{\xi,\mu}(t) \equiv \chi_{tr} T_{tr} + \chi_{tu} T_{tu}, \tag{8}$$

where T_{tr} is the cost of traveling one cell, T_{tu} is the cost of turning, and χ_{tr} and χ_{tu} define the total number of cells and the total number of turns, respectively, that are required to reach ξ from $\mu(t)$ along the shortest path. Equation (7) implies that the potential of a lattice site depends on the position and orientation of the autonomous system. For example, a far away site on the lattice with respect to the current position of the autonomous system will have less potential as compared to a nearby site, because of high traveling and turning costs, unless the far away site has a neighborhood target. Therefore, eq. (2) describes the total energy potential at any lattice site ξ, which is the sum of: i) neighborhood interaction potential due to nearby target locations, and ii) a time-varying field that depends on an externally applied potential and the traveling and turning costs.

Adaptive exploration example. – Following eq. (2), $E_\xi^{\mathcal{L}}(t)$ describes the net potential at time t on a certain lattice site ξ. By virtue of energy minimization principle, the tendency of the autonomous system coupled with the lattice system is to unfold the lattice site that has the highest potential at the earliest possible. This enables sequential unfolding of all unexplored high-potential sites, which turns the local energy potentials at different lattice sites to low-energy states via exploration. The effect is conceptually similar to a *sandpile*, where the high-energy particles fall one after the other to low-potential states. At the beginning of the search, when all sites are unexplored, the potential at every point is given by the exogenous field $B_\xi(0)$. Subsequently, the autonomous system unfolds the lattice by searching the sites that have the highest values of $E_\xi^{\mathcal{L}}(t)$, thereby minimizing the energy at these sites to that of the explored sites.

The concept of dynamic plan adaptation (DPA) has been evaluated on a dynamic plan execution simulator (DPES) that is capable of executing complex mission scenarios for autonomous systems. The six plots in fig. 1 exhibit the snapshots of the DPES program window, which demonstrate the exploration of a search area as time progresses. The search area is partitioned into a grid size of 60×60 to generate a lattice structure. For a typical mine counter measure (MCM) operation, each cell is approximately $10\,\mathrm{m} \times 10\,\mathrm{m}$ that is normally the scan range of sonar sensors on a UUV for one measurement instance. The starting point of the search is the top left corner while the end point is located at the bottom right corner. The typical back and forth motion of an autonomous system is optimal for searching an area in terms of minimum number of turns when no adaptation is needed. Therefore, for area coverage planning, the exogenous potential field $B_\xi^\star$ is designed for back and forth motion such that the potential field has an increasing

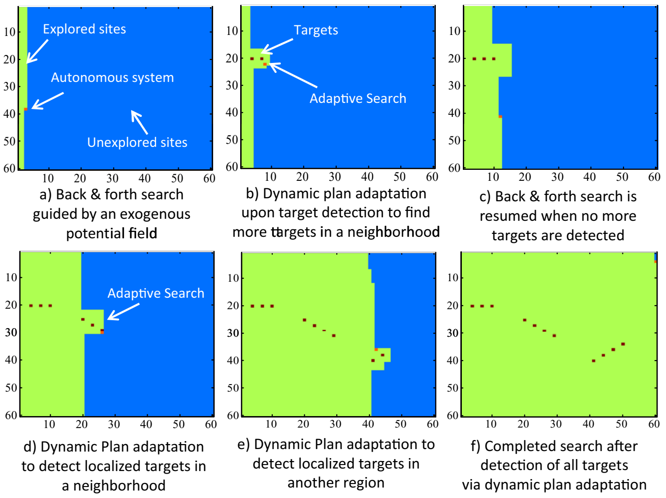

Fig. 1: (Colour on-line) Dynamic plan adaptation for detection of neighborhood targets.

magnitude from column to column, starting from a minimum value of magnitude 1 at the end point, while having equipotential sites on each column. The other parameters in eqs. (2) to (8) have been selected to be $\kappa = 5$, $\phi_0 = 250$, $\phi_{-1} = 500$, $\alpha = 0.5$, $\psi_0 = 1000$, $\psi_{-1} = 0.5$, $T_{tr} = 1$, and $T_{tu} = 2$; however, the specific values of these parameters do not have significant effect on the algorithm performance as long as they are of the same relative order of magnitude. Targets are randomly distributed in a clustered fashion, in this case straight lines, which is a commonly found deployment pattern of undersea mines using a naval ship. Figure 1 illustrates the motion of the autonomous system that finds these localized neighborhood targets via dynamic plan adaptation. Figure 1a) shows a starting stage of plan execution where the autonomous system is primarily guided by the exogenous potential field $B_\xi(t)$ and performs the typical back and forth motion till no target is detected. Figure 1b) shows plan adaptation of the autonomous system upon detection of a target to search for more targets in the neighborhood. This enables further detection of neighborhood targets, thereby improving localized search performance. Figure 1c) shows that the autonomous system resumes back and forth motion when no more targets are detected in the neighborhood. Figures 1d) and e) show dynamic plan adaptations in target neighborhoods at other locations; and fig. 1f) shows the completed search. The algorithm has been tested on several other scenarios including different target deployment patterns.

Conclusions and future work. – This paper has introduced a potential energy concept of dynamic plan adaptation (DPA) in autonomous systems based on *in situ* observation for early detection of neighborhood targets. The concept of DPA is formulated in the setting of a generalized Ising model of the configuration space that is partitioned into a lattice structure. Dynamic adaptation is achieved via local neighborhood interactions of the lattice sites, which generate a high potential in regions around detected targets, facilitating early search in unexplored neighborhood. The efficacy of the DPA algorithm has been evaluated on a dynamic plan execution simulator (DPES), which indicate improvement in the local performance of the autonomous system by early detection of neighborhood targets. This is better than the conventional search method involving back and forth motions.

The work reported in this paper is a step toward building a reliable autonomous system for adaptive search and tracking. Further research is necessary before its implementation in actual underwater environments (*e.g.*, MCM and ASW applications). While there are many research issues that need to be addressed, the following research topics are being currently pursued:

– Autonomous plan adaptation to accommodate environmental changes and disturbances (*e.g.*, obstacles and turbulence).

– Collaboration of a team (*e.g.*, swarm) of autonomous systems based on potential energy method that facilitates real-time local adaptation of each autonomous system for improvement of the collective global mission performance.

– Recursive learning of target deployment patterns based on the search history to facilitate more efficient adaptation.

– Extension of the Potts model to other forms of spin models (*e.g.*, Heisenberg) for diverse applications of autonomous systems.

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