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ORIGINAL PAPER

Self-organization of sensor networks for detection of pervasive faults

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Abstract Resource aware operation of sensor networks requires adaptive re-organization to dynamically adapt to the operational environment. A complex dynamical system of interacting components (e.g., computer network and social network) is represented as a graph, component states as spins, and interactions as ferromagnetic couplings. Using an Ising-like model, the sensor network is shown to adaptively self-organize based on partial observation, and real-time monitoring and detection is enabled by adaptive redistribution of limited resources. The algorithm is validated on a test-bed that simulates the operations of a sensor network for detection of percolating faults (e.g. computer viruses, infectious disease, chemical weapons, and pollution) in an interacting multi-component complex system.

Keywords Sensor network · Graph theory · Ising model · Pervasive faults

1 Introduction

Real-time situational awareness is necessary for both—military and civil applications for detection (of known and unknown events) and for control to maintain desired performance. Sensor networks are often used to monitor large and distributed systems composed of interdependent components, examples include Structural Health Monitoring, Military Operations in Urban Terrain (MOUT), weather, habitat, and pollution monitoring [1–4]. Usage of distributed sensor networks for pattern recognition and detection of percolating faults becomes a challenging task because of the need to process a large volume of generated data in real time [5]. Moreover, sensor nodes are often severely constrained for the use of available resources—such as processing power, energy, communication bandwidth, etc. Since events such as the growth of anomalies are usually rare and localized events, data from *all* sensor nodes need not be processed simultaneously at *all* times. Thus, the network can often be operated at a reduced capacity by using only a fraction of available resources while extracting the necessary information in real time.

Complex systems of interacting dynamic components found in computer networks, social networks, chemistry, and biology have recently been studied using the concepts of statistical mechanics and graph theory [6]. The tools of statistical mechanics have been applied to investigate the ensemble behavior of a large number of interacting units. For instance, representing nodes in a graph as energy levels and its edges as particles occupying it, complex networks have been shown to follow Bose statistics, where certain known characteristics of the network appear as thermodynamically distinct phases of a Bose gas [7]. A detailed review of statistical mechanics of complex networks is reported by Albért and Barabási [6] and, in the context of statistical physics, Strogatz [8] has explored the behavior of interacting dynamical systems in various disciplines.

Ising's ferromagnetic spin model [9,10] has been traditionally used to study critical phenomena (e.g., spontaneous magnetization) in various systems. Essentially it allows

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modeling the collective behavior of an ensemble of interacting systems. Its ability to model local and global influences on constituting units that make a binary choice (e.g. ± 1) has been shown to characterize the behavior of systems in diverse disciplines other than statistical mechanics, e.g., finance [11], biology [12], and sociophysics [13].

This paper extends the application of Ising model to the field of sensor networks to enable resource-aware real-time monitoring of a dynamic environment. Tools of statistical mechanics and graph theory have been applied to formulate an algorithm for self-organization and adaptive redistribution of available resources in a sensor network. From these perspectives, contributions of the work reported in this paper are delineated below.

- Construction of an adaptive model for scheduling sensor node activity for self-learning and adaptation.
- Construction of a state-dependent Hamiltonian to model neighborhood interactions and time-dependent external influences.
- Synergistic usage of statistical mechanical concepts (e.g., Boltzman distribution and thermodynamic equilibrium).
- Construction of an importance sampling function for probabilistic activation of sensor nodes.

The paper is organized in five sections including the present one. Section 2 presents Ising model formulation that forms the backbone of the sensor network algorithm. Section 3 formulates the algorithm for self organization of sensor networks based on the principles of statistical mechanics and graph theory. Section 4 implements the algorithm on a simulation test-bed and presents the results of algorithm validation on a test problem. Section 5 summarizes and concludes the paper with recommendations for future research.

2 Ising model formulation

A weighted graph has been used to represent interacting components in a large system and the choice of this framework naturally leads to an Ising-like formulation. Let $\mathbb{G} = (V, E, W)$ be a weighted graph, where $V = \{v_1, v_2, \ldots, v_N\}$ is the set of individual components of the system under consideration; an edge $(v_i, v_j) \in E$ is a two-element subset of V representing interdependence between a pair of components v_i and v_j; and the function $W : E \to \mathbb{R}$ yields $W((v_i, v_j)) = w_{ij}$ as the strength of interaction between the components v_i and v_j. Every interaction in $\mathbb{G}$, identified as an edge and its weight, is represented in a nearest neighbor model in the sense of Ising. However, unlike a conventional Ising model, $\mathbb{G}$ can potentially be a directed graph where elements of E are 2-tuples and interactions between components may not be symmetric. Self loops are assumed to be absent in $\mathbb{G}$ as they are not meaningful in this context.

Given a weighted graph $\mathbb{G}$, its weights are used to construct a time-dependent Hamiltonian $\mathscr{H}^\tau$.

$$\mathscr{H}^\tau = -\sum_{\langle i,j\rangle} w_{ij}\sigma_i\sigma_j - \mathscr{B}^\tau \sum_i \sigma_i \tag{1}$$

where $\langle i, j\rangle$ denotes a pair of nearest neighbor spins; w_{ij} and $\mathscr{B}^\tau$ are neighborhood interactions and time-dependent external field, respectively. Each node is assigned a spin σ_i Eq. (1) to represent its current state. The spin σ_i of node v_i is ± 1 to represent its state as functional $(+1)$ or failed (-1). In general, the interactions w_{ij} in Eq. (1) can be time-dependent but they are assumed to be constants here. Every w_{ij} is taken to be strictly positive; thus, being in the same spin state as its neighbors is energetically favorable for a node. This assumption is representative of a typical multi-component system where malfunctioning neighbors make a node more likely to change its state from $+1$ to -1 under similar external influences, which is analogous to ferromagnetic influences in an Ising model. In a more general setting, a larger set (i.e., cardinality > 2) of states as opposed to binary $\{+1, -1\}$ may be required to represent nodes states e.g. *Potts* model. Although (possibly) required to model more complex dynamics of interacting systems, this is not considered in this paper and is suggested as an avenue for future research.

Let $\mathcal{S} = \{\mathcal{S}_1, \mathcal{S}_2, \ldots, \mathcal{S}_N\}$ be a sensor network, where every component $v_i \in V$ is being monitored by a sensor node $\mathcal{S}_i$ which is represented as a function $\mathcal{S}_i : \mathcal{P} \to \mathbb{R}^n$ that maps the physical space $\mathcal{P}$ of observable parameters into the measurement space $\mathbb{R}^n$. The pattern of a sensor data sequence, generated from a component $v_i \in V$, could be statistically characterized as the state probability vector $\mathbf{p}^i$ of a finite state machine, and a (non-negative scalar) distance measure $\mu_i \triangleq d(\mathbf{p}^i, \mathbf{q}^i)$ of $\mathbf{p}^i$ from a given reference pattern $\mathbf{q}^i$ can be computed [5]. Instead of directly incorporating sensor data sequences, the scalar measures μ_i are used for construction of a Hamiltonian in the Ising model to make the computation independent of specific sensor modalities as described below.

The field $\mathscr{B}^\tau$ in Eq. (1) is representative of external influences and points in the negative direction. Thus enforcing the components of the system to flip to their respective failed (-1) state. The value of $\mathscr{B}^\tau$ at node v_k and time τ is estimated as the sum:

$$\mathscr{B}^\tau(k) = -\sum_{i=1}^{N} B_i\left(\mu_i^\tau, \{\mu_{i_j}^\tau\}\right)\delta_i(k) \tag{2}$$

where $B_i\left(\mu_i^\tau, \{\mu_{i_j}^\tau\}\right)$ is magnitude of the local field at node v_i, which is a function of the measure μ_i^τ at node v_i and the set of measures $\{\mu_{i_j}^\tau\}$ of its nearest neighbors v_{i_j}. The functional form of B_i is taken to be identical for all nodes v_i, and

$\delta_i(k)$ is the unit impulse function, i.e., $\delta_i(k) = 1$ if $k = i$ and $\delta_i(k) = 0$ if $k \neq i$.

Given the spin states and anomaly measures at a given time instant, it follows from Eq. (2) that self-organization of a sensor network for redistribution of resources reduces to estimation of the probabilities of the possible subsequent states of the underlying system. A statistical mechanical approach that is used to compute the probability of subsequent states of the observed system is given in the next section.

3 Self-organization of sensor networks

A statistical mechanical representation of the sensor network is formulated as follows. The thermodynamic state I of the system represented by the graph $\mathbb{G}$ can be given by the spin sequence $(\sigma_1, \sigma_2, \ldots, \sigma_N)$; the probability P_I that the system is in this state is given by the *Gibbs Distribution* [9]:

$$P_I(\sigma_1, \sigma_2, \ldots, \sigma_N) = \frac{1}{Z_N} \exp(-\beta E_I) \tag{3}$$

where the energy E_I of the thermodynamic state I is derived from the Hamiltonian in Eq. (1); the parameter β is proportional to the inverse temperature and Z_N is the associated *partition function* defined as:

$$Z_N = \sum_I \exp(-\beta E_I) \tag{4}$$

where the sum runs over all possible spin sequences or thermodynamic states I. The partition function may not be computationally tractable, especially, for systems with an irregular lattice and a large number of interacting nodes. The situation is simplified by the following assumptions [14]:

- *Markov dynamics*: the future state depends only on the present state.
- *Quasi-static equilibria at all time instants*: the probability of state transitions corresponding to large changes in energy is assumed to be zero and the system follows the single-flip dynamics.
- *Detailed balance*: let P_I be the probability of being in thermodynamic state I and p_{IJ} be the probability of transition from the thermodynamic state I to state J. Then, detailed balance [14] implies that

$$\left(P_I p_{IJ} = P_J p_{JI}\right) \Rightarrow \left(\frac{P_J}{P_I} = \frac{p_{IJ}}{p_{JI}}\right) \tag{5}$$

  and $p_{IJ} = p_{JI} \exp\left(-\beta(E_J - E_I)\right)$ follows from Eq. (4).

Equation (5) eliminates the partition function Z_N and tells how the ratios of transition probabilities p_{IJ} and p_{JI} should behave but it does not provide the solution for p_{IJ}. For this purpose, p_{IJ} is derived from the so-called heat-bath dynamics [14] as:

$$p_{IJ} = \frac{\exp\left(-\beta(E_J - E_I)\right)}{1 + \exp\left(-\beta(E_J - E_I)\right)} > 0 \tag{6}$$

Irreducibility of the state transition matrix $\left[p_{IJ}\right]$ is ensured because of strict positivity of each p_{IJ}.

The change in energy ΔE_{flip} due to a single-spin-flip (from $+1$ to -1 or vice versa) at a node v_i is given by:

$$\Delta E_i^{\text{flip}} = 2 \sum_{\langle i,j \rangle} w_{ij} \sigma_i \sigma_j + 2\mathscr{B}^\tau \sigma_i \tag{7}$$

where $\langle i, j \rangle$ denotes the nearest neighbor j of the node i. The flip probability p_i^{flip} of node v_i is obtained by using Eq. (6) as:

$$p_i^{\text{flip}} = \exp\left(-\beta \Delta E_i^{\text{flip}}\right) \Big/ \left[1 + \exp\left(-\beta \Delta E_i^{\text{flip}}\right)\right] \tag{8}$$

Thus for a node v_i, p_i^{flip} is the likelihood that it would change its state from $\sigma_i \to -\sigma_i$ given the states of its neighbors and magnitude of external influence. The expected probability $\hat{p}^{\text{flip}}$ that a randomly chosen node would flip (and generate information) in the next sampling instant is obtained as:

$$\hat{p}^{\text{flip}} = \frac{1}{N} \sum_{i=1}^{N} p_i^{\text{flip}} \tag{9}$$

A high (low) value of $\hat{p}^{\text{flip}}$ indicates, irrespective of the chosen node, a higher (lower) likelihood of new information begin available when a randomly chosen sensor node is set to be active. The number N_s of sensors that should be activated in the next sampling instant, out of a total of N sensors, is therefore a monotonically increasing function of $\hat{p}^{\text{flip}}$. The choice of a piecewise linear function for N_s yields:

$$N_s = \min\left(M, \max\left((1+\epsilon) N \hat{p}^{\text{flip}}, m\right)\right) \tag{10}$$

where $0 < \epsilon \ll 1$ is a detection threshold; m is the minimum number of sensors that must remain active at all time instants; and $M \leq N$ is the maximum capacity of the sensor network.

Given the flip probabilities p_i^{flip} for each node and $N_s \leq N$ computed by Eq. (10), the optimal solution for choosing the nodes that should be activated would yield N_s sensor nodes with the highest probability to be active at any instant. This deterministic method where the most probable N_s sensor nodes are activated would be blind to any activities at the nodes not chosen by the method. Thus, a sub-optimal approach is followed where the sensor nodes are not chosen deterministically but N_s nodes are sampled from an importance sampling function. This ensures that there is a finite probability of activating *every* sensor node at *all* time instants, although the sampling probability is higher for nodes with high expected activity and vice versa.

The importance sampling function is defined as:

$$p_i^s = \frac{p_i^{\text{flip}}}{N\,\hat{p}^{\text{flip}}} \tag{11}$$

to provide larger sampling weights to nodes with higher expected activity in the next instant. In essence, N_s nodes are drawn from this distribution at each instant and are scheduled to be activated in the next sampling instant. The collected data are used to modify the flip probability p_i^{flip} of the node v_i and the importance sampling function p_i^s is re-calculated. The state of a node is changed from +1 to $\sigma_i = -1$ when it is detected to have reached its respective failed state. The failed state of a node is signalled when the normalized measure μ_i computed for that node saturates and reaches a value of unity [15]. Unless updated by the sensor network, μ_i and spin σ_i are maintained at their last recorded values. Thus at all times the network works with partial information gathered by a fraction of sensor nodes. Following an iterative procedure, collected information is used to predict the likelihood of new activities. This estimate is then used to re-organize the network to adapt to the changing environment and collect new data, which is used to modify previously made predictions. This approach is similar to recursive filters (e.g., Kalman filter) where predictions are made using a model and the measurement history, which are then corrected based on new data.

It is noteworthy that the model predicts a non-zero probability of a $-1 \rightarrow +1$ flip, which corresponds to a less likely event of healing or on-line repair. Due to the field pointing in the negative direction, the probability of a $-1 \rightarrow +1$ flip is very small. Thus, the sensor network has a non-zero probability of activating a sensor for a failed component, but it may do so with a very small probability.

The parameter β serves the role of *bias control* for p_i^s (see Eqs. (8) and (11)). Low (high) values of β cause p_i^s to move towards uniform distribution (δ-distribution) independent of ΔE_{flip}.

It must be noted that irrespective of the state of sensors (i.e., active or inactive), the underlying system evolves in time, whereas a limited number, $N_s \leq N$, of sensor nodes are activated at each sampling instant. From this perspective, the efficiency η is defined as:

$$\eta = \left(\sum_{i=1}^{N} \mu_i\right)_m \Bigg/ \left(\sum_{i=1}^{N} \mu_i\right)_a \tag{12}$$

where the subscripts m and a imply *measured* and *actual* values, respectively.

Unlike a typical computational statistical mechanical problem, the goal here is not to compute macroscopic parameters but to estimate the probabilities of future thermodynamic states of the system when a particular state is sensed at time τ. In this respect the following two issues are addressed for the sensor network at all sampling instants: (1) the number of nodes to activate and (2) their distribution in the sensor network.

4 Simulation results and discussion

A simulation test bed has been constructed to validate the proposed methodology. The test bed consists of a two-dimensional (25 × 25) array of sensor nodes (i.e., a total of 625 nodes) with four nearest neighbors in the orthogonal directions. All nodes begin with a functional state (i.e., $\sigma_i = +1$) and a small number (= 5) of randomly chosen nodes are injected (seeded) with faults that slowly continue to grow until the node reaches the failed state. Infectious nodes infect their neighbors with a probability, called *transmission probability*, equal to the fraction of the total number of failed nodes present in the system at that instant of time τ. Thus, the transmission probability increases as the fault percolates through the system. The simulation scenario resembles the *Susceptible Exposed Infectious Removed* (SEIR) model of epidemic spread [16] (e.g. infectious diseases and computer viruses).

The fault process in this example problem was taken to be linear i.e. represented by a linear growth of anomaly measure once infected. Following the Susceptible Exposed Infectious Removed (SEIR) model of epidemic spread, nodes with a critical value of normalized anomaly measure (≥0.8 in this case) are termed infectious and transmit the fault process to their neighbors with a transmission probability as described above. Nodes that are infected with the fault process but not yet infectious fall into the exposed category. Failed nodes (with normalized anomaly measure = 1) are considered removed i.e. they cannot be infected again with the fault process. Nodes neighboring the exposed or infectious nodes are taken to be susceptible.

Each node v_i in the simulation test bed is monitored by a sensor whose τ-dependent data are compressed into a scalar measure μ_i^τ, and the local field B_i in Eq. (2) for each node v_i is identically modeled in the following form.

$$B\left(\mu_i^\tau, \{\mu_{i_j}^\tau\}\right) = B_0 \left(\mu_i^\tau + \sum_{i_j} \mu_{i_j}^\tau \exp(-\alpha|i - i_j|)\right) \tag{13}$$

where $|i - i_j| = 1$ for nearest neighbor interactions.

Self-organization of the sensor network is implemented in the simulation test-bed based on the following algorithm.

The interaction coefficients w_{ij} in Eq. (1) are chosen to be 0.8 for each neighbor pair (v_i, v_j); inverse temperature β is set to 0.333; parameters N, M, m and ϵ in Eq. (10) are set to 625, 320, 20 and 0.001, respectively; and parameters B_0 and α in Eq. (13) are set to 5 and 1, respectively, in the simulation.

Algorithm 1: Self-Organization of Sensor Networks

while *(1)* **do**
  Step 1: Collect data from active senor nodes.
  Step 2: Compute local measure μ_i^τ for active nodes at current time τ and update their spin states.
  Step 3: Compute the local field B^l (Eqs. 2 and 13)
  Step 4: Compute change in energy ΔE (Eq. 7)and calculate new p_i (Eq. 6) for each node i.
  Step 5: Update importance function p^s (Eq. 11) and calculate N_s (Eq. 10)
  Step 6: Draw N_s samples from p^s and to activate sensors nodes in the next time step.
endw

As stated earlier the parameter N is the total number of nodes in the system being monitored, while M and m are the maximum and the minimum limit on the sensor nodes that the sensor network can activate at a particular instant of time. The maximum limit M would be dictated by the capacity of the sensor network to handle data traffic and processing resources available. The lower limit m, on the other hand, is a design parameter chosen to maintain a desired level of detection probability. The parameter $0 < \epsilon \ll 1$ allows the algorithm to be robust against errors such as in modeling. A higher (lower) value can be chosen to achieve higher (lower) degree of robustness.

The probability p_i^{flip} that a node would change its state from $\sigma_i \rightarrow -\sigma_i$ is dependent on the current state of its neighbors through the interaction coefficient w_{ij} and the value of their anomaly measure via parameter α. Interaction coefficient $w_{ij} > 0$ corresponds to ferromagnetic interaction, modeling a case where failed neighbors increase the likelihood on inducing fault process in a node. Parameter $\alpha > 0$ models a case where the effect of fault growth in a faraway neighbor is smaller as compared to a node in the immediate vicinity.

The plots of algorithm efficiency η for three typical simulation runs are shown in Fig. 1a. At the beginning, η has a relatively low trend when the sensor network learns the fault pattern; then η approaches unity as the network dynamically adapts itself to the fault pattern. Figure 1b shows the fraction of active sensor nodes at any given time. The number of active sensor nodes dynamically changes and is always a small fraction (e.g., <25%) of the total number of sensor nodes in these tests. It must be noted that when all nodes are functional their anomaly measure is a constant which starts to change only when fault process is injected. Thus the efficiency starts with a value of unity and drops (representing the injection of fault) only to converge back up again as the algorithm learns the patterns and dynamically re-allocates resources. Figure 1a and b show the ability of the sensor network to self-organize to adapt to the dynamic environment. As shown by this example problem, the proposed algorithm potentially enables real-time monitoring and detection by working with a small fraction of sensor nodes to conserve valuable resources.

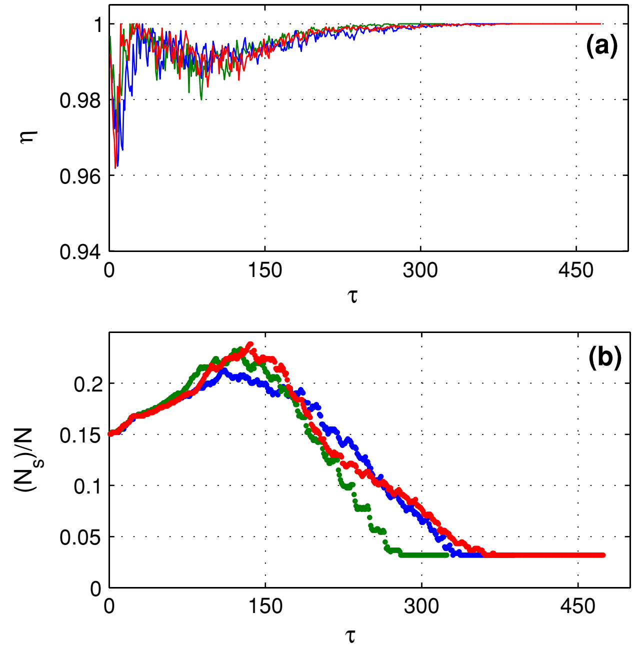

Fig. 1 **a** Algorithm efficiency and **b** fraction of active sensors

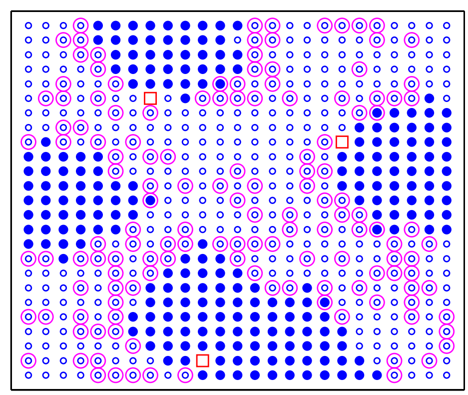

Fig. 2 A snap shot of the sensor grid. Solid dots (*blue*): $\sigma_i = -1$. Small circles (*blue*): $\sigma_i = +1$. Squares (*red*): failed but yet undetected by the sensor network. Big circles (*magenta*): sensors active at this instant

Figure 2 displays a snapshot of the operational environment where the adaptive redistribution of resources can be seen. A larger number of sensors, neighboring nodes detected to have failed i.e. $\sigma_i = -1$, were activated. While nodes already detected to have failed by the sensor network were activated with a small probability. Thus network resources were directed more towards functional nodes to activate their sensors than for nodes detected as failed. As the fault pattern evolves in time, the sensor network dynamically adapts to the changes by making corrections to its predicted estimates while using partial information at all times.

The choice of the parameters in the proposed methodology is dictated by the underlying system being monitored and level of desired performance. Parameters such as the interaction coefficient w_{ij} and α, maybe determined empirically from the underlying system using methods such as *mutual information* [17] and *transfer entropy* [18]. While design parameters such as M, m and ϵ are determined by factors such as the capacity of the sensor network.

5 Summary, conclusions and future work

This paper introduces a concept of adaptive self-organization of sensor networks by using weighted graphs and an Ising-like model based on the principles of Statistical Mechanics. Given past measurements, probabilities of future states are computed to construct an importance sampling function to probabilistically activate a small fraction of sensor nodes. Numerical simulation has been conducted on a test-bed of interacting multi-component systems to demonstrate the adaptive self-organizing capability of the proposed methodology. Simulation results show that the algorithm is able to adaptively monitor and detect pervasive faults.

While there are many research areas in sensor network, the following topics are recommended for future research in the context of the work presented in this paper.

- Higher-order neighborhood systems and more than binary states for component nodes, e.g. using Potts model.
- Investigation of information-theoretic concepts such as *mutual information* [17] to determine the strength of interactions w_{ij}, and *transfer entropy* [18] to ascertain direction of information flow.
- Construction of the Hamiltonian in Eq. (1) directly from the state probability vectors [5] rather than compressing the sensed information into a scalar measure μ.
- Analysis for robustness to modeling uncertainties and parametric uncertainties, and its effect on convergence properties of the proposed algorithm.
- Application of methods such as Artificial Neural Networks (ANN) for self organization of sensor network for redistribution of limited resources and evaluation of the proposed algorithm relative to such methods.

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