ductile materials (steel, stainless steel, aluminum, titanium, bronze)

distortion energy failure theory using maximum octahedral shear

von Mises equivalent stress σ'

$$\sigma' = \sqrt{\sigma_X^{\;2} + \sigma_Y^{\;2} - \sigma_X \sigma_Y + 3\,\tau_{XY}^{\;2}}$$ Eq 5.7d Norton

normal stress and shear stress $\sigma_Y = 0$

$$\sigma' = \sqrt{\sigma_{NORMAL}^{\;2} + 3\,\tau_{SHEAR}^{\;2}}$$

normal stress and shear stress (round shaft with bending moment M and torsional moment T)

$$\sigma' = \sqrt{\sigma_{BENDING}^{\;2} + 3\,\tau_{TORSION}^{\;2}} \qquad \sigma = \frac{M\,c}{I} \qquad \tau = \frac{T\,c}{J} \qquad c = \frac{d}{2} \qquad I = \frac{\pi d^4}{64} \qquad J = \frac{\pi d^4}{32}$$

$$\sigma' = \sqrt{\left(\frac{M\,c}{I}\right)^2 + 3\left(\frac{T\,c}{J}\right)^2} = \sqrt{\left(\frac{64M\,(d/2)}{\pi d^4}\right)^2 + 3\left(\frac{32T\,(d/2)}{\pi d^4}\right)^2} = \frac{32}{\pi d^3}\sqrt{M^2 + \frac{3}{4}T^2}$$

$$\sigma' = \frac{32}{\pi d^3}\sqrt{M^2 + \frac{3}{4}T^2}$$

biaxial normal stress (thin wall pressure vessel) $\tau_{XY} = 0$

$$\sigma' = \sqrt{\sigma_{TANGENTIAL}^{\;2} + \sigma_{AXIAL}^{\;2} - \sigma_{TANGENTIAL}\sigma_{AXIAL}}$$

pure shear (direct shear, key shear, shaft with no bending) $\sigma_X = \sigma_Y = 0$

$$\sigma' = \sqrt{3}\;\tau_{SHEAR}$$

brittle materials (cast iron, ceramics)

maximum tensile failure theory

Dowling equivalent stress $\tilde{\sigma}$

1) find principal normal stresses using Mohr's circle $\sigma_1 \quad \sigma_2 \quad \sigma_3$

2) calculate Dowling constants $C_1 \quad C_2 \quad C_3$ using Eq. 5.12c Norton $S_{UT} > 0 \qquad S_{UC} < 0$

$$C_1 = \frac{1}{2}\left[\left|\sigma_1 - \sigma_2\right| + \frac{2\, S_{UT} - \left|S_{UC}\right|}{-\left|S_{UC}\right|}\left(\sigma_1 + \sigma_2\right) \right]$$

$$C_2 = \frac{1}{2}\left[\left|\sigma_2 - \sigma_3\right| + \frac{2\, S_{UT} - \left|S_{UC}\right|}{-\left|S_{UC}\right|}\left(\sigma_2 + \sigma_3\right) \right]$$

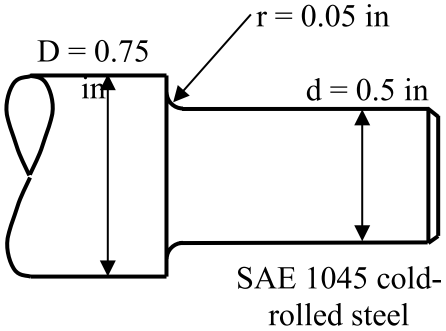
$$C_3 = \frac{1}{2}\left[\left|\sigma_3 - \sigma_1\right| + \frac{2\, S_{UT} - \left|S_{UC}\right|}{-\left|S_{UC}\right|}\left(\sigma_3 + \sigma_1\right) \right]$$

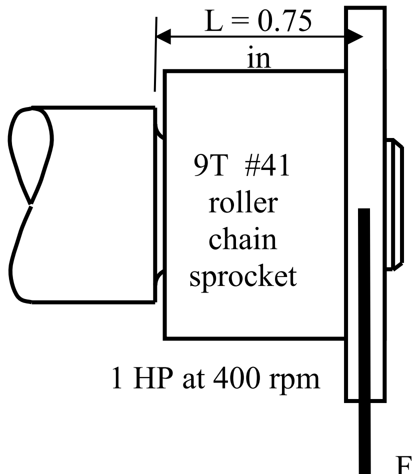
3) find maximum

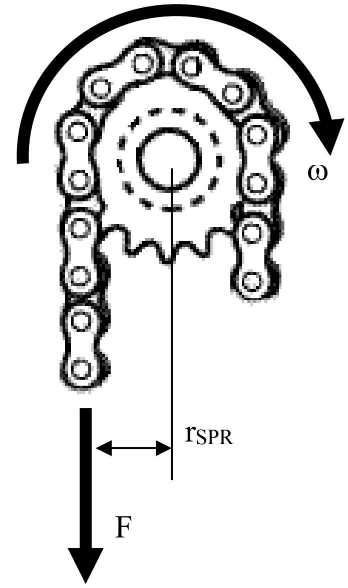
$$\tilde{\sigma} = \mathrm{MAX}\left(C_1\ C_2\ C_3\ \sigma_1\ \sigma_2\ \sigma_3 \right) \qquad \tilde{\sigma} = 0 \quad \text{if} \quad \mathrm{MAX} < 0$$

4) compare Dowling equivalent stress to ultimate tensile strength

$$N = S_{UT} / \tilde{\sigma}$$



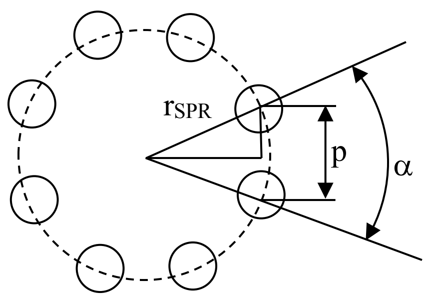




$P = T\,\omega$ ω must be rad/time NOT rev/time

$$T = P / \omega = 1\ \text{HP}\left(\frac{33000\ \text{ft.lbf}}{\text{HP.min}}\right)\left(\frac{\text{min}}{400\ \text{rev}}\right)\left(\frac{\text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{12\ \text{in}}{\text{ft}}\right) = 157.6\ \text{in.lbf}$$

#41 ANSI chain, pitch p = 0.5 inch, maximum working load = 314 lbf, McMaster-Carr



pitch circumference = $2\,\pi\, r_{SPR} \approx (9T)\,p$
$r_{SPR} \approx (9T)\,p / 2\,\pi = 0.7162$ in

$\alpha = 360° /$ number of teeth $= 40°$
p = 0.5 in
$\sin \alpha/2 = (p/2) / r_{SPR}$
EXACT $r_{SPR} = (p/2) / (\sin \alpha/2) = 0.731$ in

$T = F\, r_{SPR}$ $F = T / r_{SPR} = 220$ lbf

bending moment $M = F\,L = 165$ in.lbf

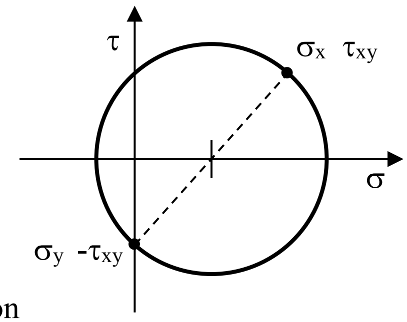
torsion moment $T = 157.6$ in.lbf

$I = \pi\, d^4 / 64 = 3.068 \times 10^{-3}\ \text{in}^4$ $J = \pi\, d^4 / 32 = 6.136 \times 10^{-3}\ \text{in}^4$

$c = d / 2 = 0.25$ in

$$\sigma_{NOM} = M\,c / I = \frac{(165\ \text{in.lbf})(0.25\ \text{in})}{(3.068\text{x}10^{-3}\ \text{in}^4)} = 13.45\ \text{ksi}$$

$$\tau_{NOM} = T\,c / J = \frac{(157.6\ \text{in.lbf})(0.25\ \text{in})}{(6.136\text{x}10^{-3}\ \text{in}^4)} = 6.421\ \text{ksi}$$



maximum shear failure criterion (OLD)

$\sigma_x = 13.45$ ksi $\quad \sigma_y = 0 \quad \tau_{xy} = 6.421$ ksi

$\tau_{MAX} = \sqrt{(\sigma_x / 2)^2 + \tau_{xy}^2} = 9.30$ ksi

1045 cold-rolled steel $\quad S_y = 77$ ksi $\quad$ Table A-9 Norton

$N = \dfrac{0.5\, S_y}{\tau_{MAX}} = 4.14$ $\quad$ Eq 5.10 and 5.11 Norton

von Mises failure criterion

$\sigma_x = 13.45$ ksi $\quad \sigma_y = 0 \quad \tau_{xy} = 6.421$ ksi

$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x \sigma_y + 3\tau_{xy}^2} = 17.45$ ksi $\quad$ Eq 5.7d Norton

1045 cold-rolled steel $\quad S_y = 77$ ksi $\quad$ Table A-9 Norton

$N = \dfrac{S_y}{\sigma'} = 4.41$ $\quad$ Eq 5.8a Norton

iterative solution for new diameter

$\sigma_{NOM} = M\, c / I = \dfrac{M \frac{d}{2}\, 64}{\pi\, d^4} = \dfrac{32\, M}{\pi\, d^3}$ $\qquad$ $\tau_{NOM} = T\, c / J = \dfrac{T \frac{d}{2}\, 32}{\pi\, d^4} = \dfrac{16\, T}{\pi\, d^3}$

try NEW d = 0.625 in

NEW σ' = OLD $\sigma' \left(\dfrac{0.5}{0.625}\right)^3 = 0.512$ OLD σ'

static stress concentration factors

shaft bending $\quad D/d = 1.5 \quad A = 0.93836 \quad b = -0.25759$ $\quad$ Figure C-2 Norton

$r/d = 0.1 \quad K_t = A(r/d)^b = 1.698$

$\sigma = K_t\, \sigma_{NOM} = 22.84$ ksi

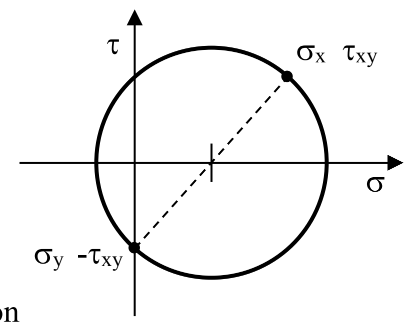
shaft torsion D/d = 1.5 interpolated A = 0.8526 b = -0.2334 Figure C-3 Norton

r/d = 0.1 $K_t = A(r/d)^b = 1.459$

$\tau = K_t\, \tau_{NOM} = 9.368$ ksi

on-line calculator
https://www.amesweb.info/StressConcentrationFactor/SteppedShaftWithShoulderFillet.aspx

bending $K_t = 1.74$ torsion $K_t = 1.41$



<u>maximum shear failure criterion (OLD)</u>

$\sigma_x = 22.84$ ksi $\sigma_y = 0$ $\tau_{xy} = 9.368$ ksi

$$\tau_{MAX} = \sqrt{(\sigma_x/2)^2 + \tau_{xy}^2} = 14.77 \text{ ksi}$$

1045 cold-rolled steel $S_y = 77$ ksi Table A-9 Norton

$$N = \frac{0.5\, S_y}{\tau_{MAX}} = 2.61$$ Eq 5.10 and 5.11 Norton

<u>von Mises failure criterion</u>

$\sigma_x = 22.84$ ksi $\sigma_y = 0$ $\tau_{xy} = 9.368$ ksi

$$\sigma' = \sqrt{\sigma_x^2 + \sigma_y^2 - \sigma_x\sigma_y + 3\tau_{xy}^2} = 28.02 \text{ ksi}$$ Eq 5.7d Norton

1045 cold-rolled steel $S_y = 77$ ksi Table A-9 Norton

$$N = \frac{S_y}{\sigma'} = 2.75$$ Eq 5.8a Norton

<u>key length</u>

assume 1020 hot-rolled $S_y = 30$ ksi Table A-9 Norton

d = 0.5 inch DIA shaft w = 0.125 inch square key Table 10-2 Norton assume $L_{KEY} = 0.625$ inch < L overhang

$T = 157.6$ in.lbf $r_{SHAFT} = 0.25$ inch $F_{KEY} = T / r_{SHAFT} = 630.4$ lbf

$A_{SHEAR} = w\,L = 0.078125\ in^2$ $\tau = F_{KEY} / A_{SHEAR} = 8069.1$ psi

$\sigma' = \sqrt{3}\ \tau = 13.98$ ksi

$N_{YIELD} = S_Y / \sigma' = 2.15$

fatigue stress concentration (notch sensitivity) factors

full reversed bending $\sigma_{NOM} = 13.45$ ksi $K_t = 1.698$

1045 cold-rolled steel $S_{ut} = 91$ ksi Table A-9 Norton

Neuber $\sqrt{a} = 0.0692\ \sqrt{in}$ interpolated Table 6-6 Norton

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.0692\sqrt{in}}{\sqrt{0.05\ in}}} = 0.7637$$ Eq 6.13 Norton

$K_f = 1 + q\,(Kt - 1) = 1.533$ Eq 6.11b Norton

$\sigma = K_f\,\sigma_{NOM} = 20.62$ ksi

$\sigma_{alt}' = \sigma = 20.62$ ksi

constant torsion $\tau_{NOM} = 6.421$ ksi $K_t = 1.459$

no fatigue stress concentration (notch sensitivity) factor $\tau = K_t\,\tau_{NOM} = 9.368$ ksi

$\sigma_{mean}' = \sqrt{3}\ \tau = 16.23$ ksi Eq 5.7d Norton

modified endurance limit

1045 cold-rolled steel $S_{ut} = 91$ ksi Table A-9 Norton

$S_e' = 0.5\ S_{ut} = 45.5$ ksi Eq 6.5a Norton

$C_{LOAD} = 1.0$ bending Eq 6.7a Norton

$C_{SIZE} = 0.869\ d^{-0.097} = 0.9294$ circular section Eq 6.7b Norton

$C_{SURF} = 0.76$ machined Figure 6-26 Norton

$C_{SURF} = A(S_{ut})^b = 0.817$ $A = 2.7$ $b = -0.265$ machined Eq 6.7e and Table 6-3 Norton

use $C_{SURF} = 0.76$

$C_{TEMP} = 1.0$ normal environmental temperature range Eq 6.7f Norton

$C_{RELI} = 0.814$ $R = 99\%$ Table 6-4 Norton

$S_e = C_{LOAD}\ C_{SIZE}\ C_{SURF}\ C_{TEMP}\ C_{RELI}\ S_e' = (1)\ (0.9294)\ (0.76)\ (1)\ (0.814)\ 45.5\ \text{ksi} = 26.16\ \text{ksi}$

fatigue factor of safety

$$\left(\frac{400\ \text{rev}}{\text{min}}\right)\left(\frac{60\ \text{min}}{\text{hr}}\right)\left(\frac{24\ \text{hr}}{\text{day}}\right) = 576{,}000\ \text{rev/day}$$

use infinite life $N>10^6$ $S_N = S_e = 26.16$ ksi $S_{ut} = 91$ ksi

$\sigma_{alt}' = 20.62$ ksi $\sigma_{mean}' = 16.23$ ksi

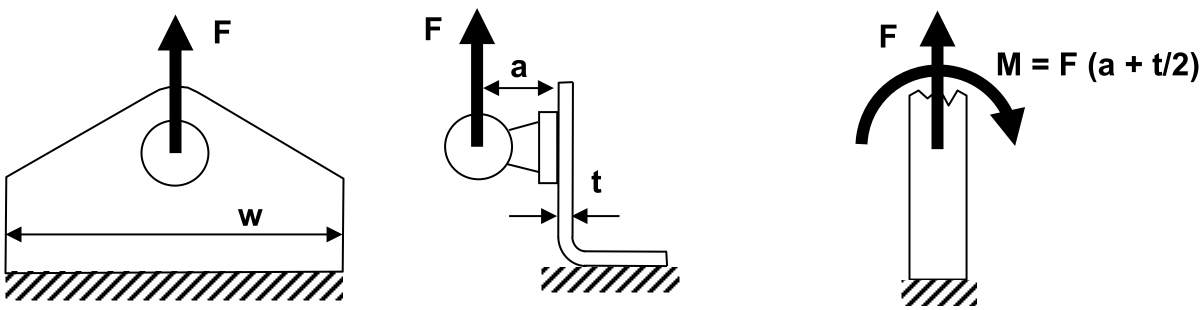
$N_{alt} = S_N / \sigma_{alt}' = 1.269$ $N_{mean} = S_{ut} / \sigma_{mean}' = 5.607$

$N_{GOODMAN} = N_{alt}\ N_{mean} / (N_{alt} + N_{mean}) = 1.03$

1045 cold-rolled steel $S_y = 77$ ksi Table A-9 Norton

$N_{FIRST} = S_y / (\sigma_{alt}' + \sigma_{mean}') = 2.09$

A ball mount bracket for gas springs is made from AISI 1010 hot rolled steel as shown below. Determine vertical load F on the ball that would cause static yielding.



w = 50.8 mm t = 2 mm a = 12.7 mm

AISI 1010 hot-rolled steel S_y = 179 MPa Table A-9 Norton

$$\sigma = \frac{Mc}{I} + \frac{F}{A}$$ at front face

c = t /2 = 1 mm $I = w\,t^3 / 12 = 33.87\ mm^4$ $A = w\,t = 101.6\ mm^2$ (a + t/2) = 13.7 mm

use $\sigma = S_y$ for yield

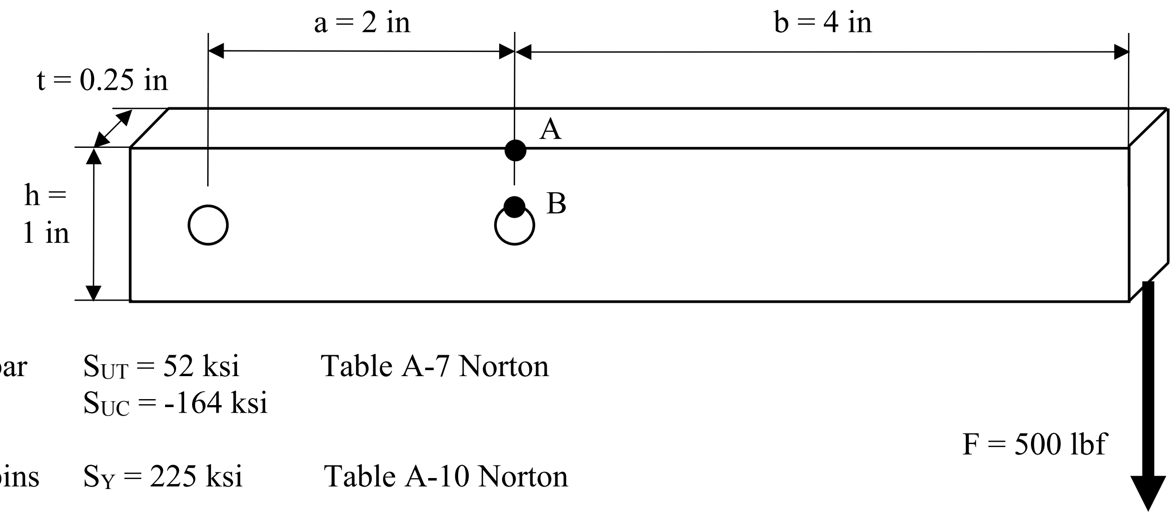
$$S_y = F\left(\frac{(a + t/2)c}{I} + \frac{1}{A}\right)$$

$$179\frac{N}{mm^2} = F\left(\frac{(13.7\ mm)(1\ mm)}{33.87\ mm^4} + \frac{1}{101.6\ mm^2}\right) = F(0.4045 + 0.009843)\frac{1}{mm^2}$$

F = 432.0 N = 97.12 lbf

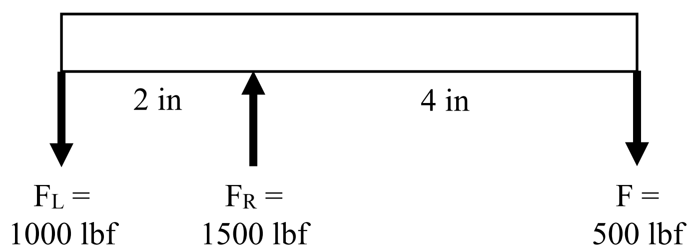
Class 50 cast iron bar mounted with two 0.25 inch DIA dowel pins (A8 steel hardness HRC52)

values shown in red are for 0.5 inch DIA dowel pins



bar S_{UT} = 52 ksi Table A-7 Norton
S_{UC} = -164 ksi

pins S_Y = 225 ksi Table A-10 Norton



direct shear on right pin

$A = \pi\, d^2 / 4 = 0.0491 \text{ in}^2$ 0.1963 in^2

$\tau = F_R / A = 30.55$ ksi 7.639 ksi

$\sigma' = \sqrt{3}\ \tau = 52.91$ ksi 13.23 ksi

$N = S_Y / \sigma' = 4.25$ 17.00

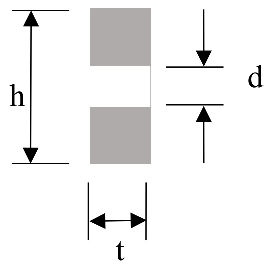
bending of bar at top A

maximum M at right pin M = (500 lbf) (4 in) = 2000 in.lbf

$I = t\,(h^3 - d^3) / 12 = 0.0205\ in^4$ $0.01823\ in^4$

$c = h / 2 = 0.5\ in$

$\sigma = M\,c / I = 48.78\ ksi$ $54.85\ ksi$



$\tilde{\sigma} = \sigma$

$N = S_{UT} / \tilde{\sigma} = 1.07$ 0.95

direct crushing of bar above right pin B

assume tight fit $A_{PROJ} = d\,t = 0.0625\ in^2$ Section 4.8 Norton $0.125\ in^2$

$\sigma_{CRUSH} = F_R / A_{PROJ} = -24.00\ ksi$ compressive $-12.00\ ksi$

bending of bar above right pin B

$c = d / 2 = 0.125\ in$ same M same I 0.25

$\sigma_{BEND} = M\,c / I = +12.20\ ksi$ tension $+27.43\ ksi$

Dowling equivalent stress Eq. 5.12c Norton

$\sigma_1 = +12.20\ ksi$ $\sigma_2 = 0$ $\sigma_3 = -24.00\ ksi$ $27.43\ ksi$ 0 $-12.00\ ksi$

$S_{UT} = 52\ ksi$ $S_{UC} = -164\ ksi$

$$\frac{2\,S_{UT} - |S_{UC}|}{-|S_{UC}|} = 0.3659$$

$$C_1 = \frac{1}{2}\left[\,|\sigma_1 - \sigma_2| + \frac{2\,S_{UT} - |S_{UC}|}{-|S_{UC}|}(\sigma_1 + \sigma_2)\,\right] = +8.33\ ksi$$ $18.73\ ksi$

$$C_2 = \frac{1}{2}\left[\,|\sigma_2 - \sigma_3| + \frac{2\,S_{UT} - |S_{UC}|}{-|S_{UC}|}(\sigma_2 + \sigma_3)\,\right] = +7.61\ ksi$$ $3.80\ ksi$

$$C_3 = \frac{1}{2}\left[\ \left|\sigma_3 - \sigma_1\right| + \frac{2\ S_{UT} - \left|S_{UC}\right|}{-\left|S_{UC}\right|}\left(\sigma_3 + \sigma_1\right)\ \right] = +15.94 \text{ ksi}$$ 22.54 ksi

$\tilde{\sigma} = \text{MAX}\left(\ C_1\ C_2\ C_3\ \sigma_1\ \sigma_2\ \sigma_3\right) = +15.94$ ksi Note: not MAX(abs( )) 27.43 ksi

$N = S_{UT} / \tilde{\sigma} = 3.26$ 1.90