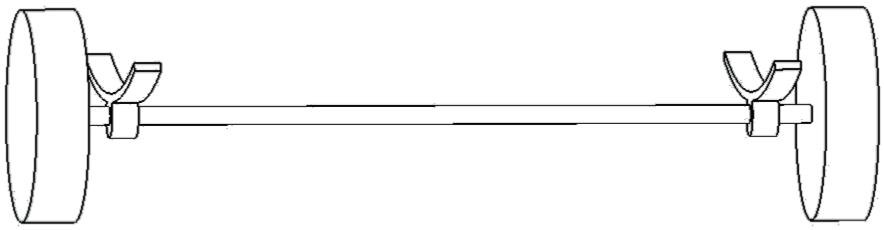
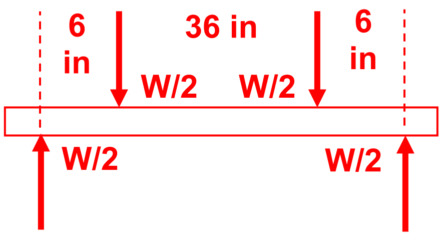
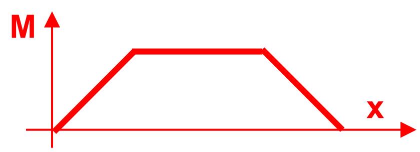
Fatigue Design Procedure

Alternating stress	**Mean stress**
1) select life N	
2) evaluate alternating loads	2) evaluate mean loads
3) choose tentative geometry	3) same
4) calculate stress concentration K_t	4) calculate stress concentration K_t (might be different loading)
5) choose tentative material and look up properties S_{ut} S_y S_m $S_{e'}$ $S_{f'}$ q	5) same
6) calculate fatigue stress concentration $K_f = 1 + q\,(K_t - 1)$	
7) calculate alternating stresses	7) calculate mean stresses
8) calculate von Mises stress σ_a'	8) calculate von Mises stress σ_m'
9) determine fatigue strength modification factors C_{XXXX}	
10) calculate corrected fatigue strength S_e S_f	
11) interpolate allowable S_N from S-N curve	
12) calculate factor of safety $N_a = S_N / \sigma_a'$	12) calculate factor of safety $N_m = S_{ut} / \sigma_m'$
13) use Goodman diagram (Case 3 Norton) $N_{GOODMAN} = N_a N_m / (N_a + N_m)$	
14) check for yield at first loading $N_{FIRST} = S_y / (\sigma_a' + \sigma_m')$	

Select a suitable axle diameter for a small boat trailer for infinite life and for life of 10^5 cycles. Assume the axle has uniform diameter. The boat and trailer weighs 1500 pounds. Ignore the tongue weight. Design for a safety factor of 3. The center-to-center distance between the wheels is 4 feet and center-to-center distance between the axle bearings is 3 feet. Outside diameter of the tires is 24 inches. Use 1040 cold-rolled steel and state all assumptions. Be conservative because this axle goes swimming every time you put the boat in the water.







$M_{MAX} = (W/2) (6 \text{ in}) = 4500$ in.lbf

1040 cold-rolled steel $S_y = 71$ ksi $S_{ut} = 85$ ksi Table A-9 Norton

$S_e' = 0.5\, S_{ut} = 42.5$ ksi Eq 6.5a Norton no stress concentrations

$\sigma = M c / I$ $c = d / 2$ $I = \pi d^4 / 64$ $N_{alt} = S_N / \sigma_{alt}$ $\sigma_{alt} = S_N / N$

$\sigma = 32 M / \pi d^3 = S_N / N$ $d^3 = 32 M N / \pi S_N$

preliminary design use $S_N = S_e'$

$$d^3 = \frac{32 M N}{\pi S_e'} = \frac{32(4500 \text{ in.lbf})(3)}{\pi}\left(\frac{\text{in}^2}{42.5\text{x}10^3 \text{ lbf}}\right) \qquad d = 1.48 \text{ in}$$

$C_{LOAD} = 1.0$ bending Eq 6.7a Norton

$C_{SIZE} = 0.869\, d^{-0.097} = 0.8366$ circular section using d = 1.48 in Eq 6.7b Norton

$C_{SURF} = 0.52$ corroded in tap water Figure 6-26 Norton

$C_{TEMP} = 1.0$ normal environmental temperature range Eq 6.7f Norton

$C_{RELI} = 0.814$ R = 99% Table 6-4 Norton

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e' = (1)\,(0.8366)\,(0.52)\,(1)\,(0.814)\,42.5\text{ ksi} = 15.05\text{ ksi}$

infinite life $S_N = S_e$ $$d^3 = \frac{32\,M\,N}{\pi\,S_e} = \frac{32(4500\text{ in.lbf})(3)}{\pi}\left(\frac{\text{in}^2}{15.05\text{x}10^3\text{ lbf}}\right)$$ d = 2.09 in

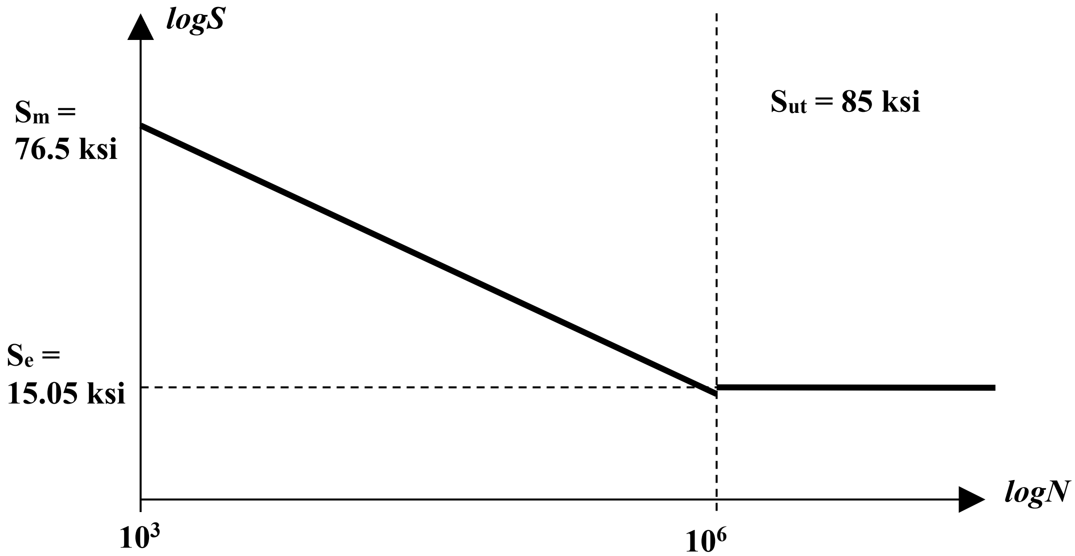
update $C_{SIZE} = 0.869\, d^{-0.097} = 0.8090$ circular section using d = 2.09 in Eq 6.7b Norton

update S_e = prior S_e (0.8090) / (0.8366) = 14.55 ksi

update d = prior d $\sqrt[1/3]{(0.8366)/(0.8090)} = 2.11$ in use d = 2.125 in for infinite life

$N = 10^5$ cycles

$S_m = 0.9\, S_{ut} = 76.5$ ksi bending Eq. 6.9 Norton (no C_{FAC} applied to S_M)



$$\frac{\log N - \log 10^3}{\log 10^6 - \log 10^3} = \frac{\log S_N - \log S_m}{\log S_e - \log S_m} \qquad 0.6667 = \frac{\log S_N - 1.8837}{-0.70612} \qquad S_N = 25.88\text{ ksi}$$

$N = 10^5$ $S_N = 25.88$ ksi $$d^3 = \frac{32\,M\,N}{\pi\,S_N} = \frac{32(4500\text{ in.lbf})(3)}{\pi}\left(\frac{\text{in}^2}{25.88\text{x}10^3\text{ lbf}}\right)$$ d = 1.745 in

update $C_{SIZE} = 0.869\, d^{-0.097} = 0.8233$ circular section using d = 1.745 in Eq 6.7b Norton

update S_e = prior S_e (0.8233) / (0.8090) = 14.81 ksi update using interpolation $S_N = 25.59$ ksi

update for $N = 10^5$ cycles $d = 1.751$ in

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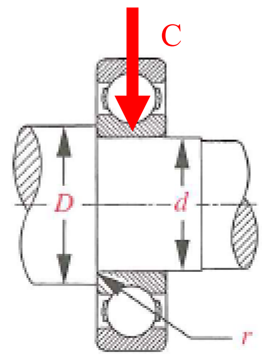
How far will the trailer travel for 10^5 cycles?

tire circumference = $\pi D_{TIRE} = 75.40$ in

$$\left(\frac{75.40 \text{ in}}{\text{rev}}\right)\left(10^5 \text{ rev}\right)\left(\frac{\text{ft}}{12 \text{ in}}\right)\left(\frac{\text{mile}}{5280 \text{ ft}}\right) = 119 \text{ miles}$$

better to use infinite life



Design the end of a 40 mm shaft using 1040 cold-rolled steel for a standard ball bearing in Figure 11-23 Norton based on infinite fatigue life. Assume no torque is transmitted. Determine factor of safety.

select 6307 bearing
bore ID = 35 mm
OD = 80 mm
width w = 21 mm
max fillet radius r_{MAX} = 0.060 in
dynamic load rating C = 7500 lbf

use D = 40 mm d = 35 mm = 1.378 in r = 1.5 mm = 0.05906 in < r_{MAX}

1040 cold-rolled S_{UT} = 586 MPa = 85 ksi Table A-9

assume maximum load C at center of bearing w/2

M = C (w/2) = 266,213 N.mm

c = d/2 = 17.5 mm

$I = \pi d^4 / 64 = 73{,}662 \text{ mm}^4$

$\sigma_{nom} = M c / I = 63.24$ MPa

D/d = 1.1426 interpolated A = 0.95967 b = -0.22922 Fig C-2 Norton

r/d = 0.042857 $K_t = A(r/d)^b = 1.9756$

S_{ut} = 85 ksi interpolate Table 6-6 Neuber $\sqrt{a} = 0.075 \text{ in}^{0.5}$

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = \frac{1}{1+\frac{0.075 \text{ in}^{0.5}}{\sqrt{0.05906 \text{ in}}}} = 0.7642 \qquad \text{Eq. 6.11a}$$

$K_f = 1 + q (K_t - 1) = 1.7455$ Eq. 6.11b

ignore Neuber $\mathbf{K_f = K_t}$

$\sigma'_{alt} = K_f \; \sigma_{nom} = 124.9$ MPa Eq. 6.12

$S_e' = 0.5 \; S_{ut} = 293$ MPa Eq. 6.5a

$C_{LOAD} = 1$ bending Eq. 6.7a

$C_{SIZE} = 0.869\,(d)^{-0.097} = 0.8424$ d = 1.378 in Eq. 6.7b

assume machined but not ground A = 4.51 b = -0.265 **for MPa** Table 6-3

$C_{SURF} = A\,(S_{UT})^{b} = 0.8331$ Eq. 6.7e

$C_{TEMP} = 1$ assume normal temperture Eq. 6.7f

$C_{RELI} = 0.814$ assume 99% reliability Table 6-4

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e' = 167.4$ MPa Eq. 6.6

$S_N = S_e$ infinite life

$N_{alt} = S_N / \sigma'_{alt} = 1.34$

++

try 1010 cold-rolled steel $S_{UT} = 365$ MPa = 53 ksi Table A-9

$S_e' = 0.5\, S_{ut} = 182.5$ MPa Eq. 6.5a

assume machined but not ground A = 4.51 b = -0.265 **for MPa** Table 6-3

$C_{SURF} = A\,(S_{UT})^{b} = 0.9444$ Eq. 6.7e

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e' = 118.2$ MPa Eq. 6.6

$S_N = S_e$ infinite life

$\sigma'_{alt} = K_f\, \sigma_{nom} = 124.9$ MPa Eq. 6.12

$N_{alt} = S_N / \sigma'_{alt} = 0.95$

Design an air tank for 150 psi using 4130 steel annealed with diameter 20 inches

S_{UT} = 81 ksi Table A-9

guess 20 gauge sheet t = 0.0359 inch thickness available on-line

thin wall pressure vessel r = 10 inch P = 150 psi

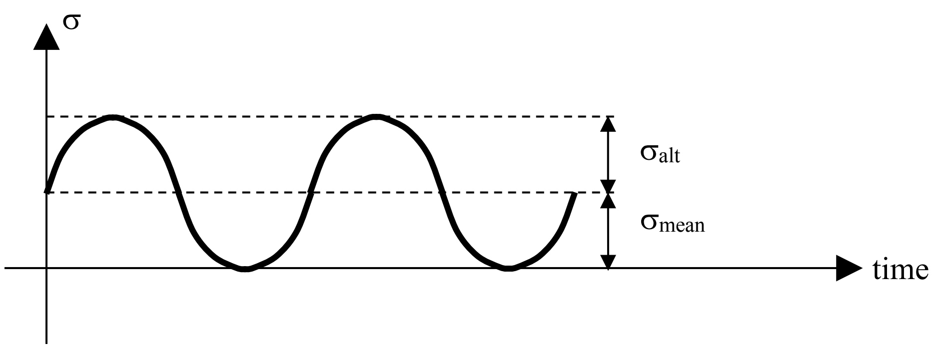
$\sigma_{TAN} = P\ r / t = 41.78$ ksi Eq. 4.49a

$\sigma_{RAD} = 0$ Eq. 4.49b

$\sigma_{AXIAL} = P\ r / 2\ t = 20.89$ ksi Eq. 4.49c

$\sigma_1 = \sigma_{TAN}$ $\sigma_2 = \sigma_{AXIAL}$ $\sigma_3 = 0$

$\sigma'_{MAX} = \sqrt{\sigma_1^{\ 2} - \sigma_1 \sigma_2 + \sigma_2^{\ 2}} = 36.18$ ksi Eq. 5.7c



full cycle $\sigma'_{mean} = 18.09$ ksi $\sigma'_{alt} = 18.09$ ksi

$S_e' = 0.5\ S_{ut} = 40.5$ ksi Eq. 6.5a

$C_{LOAD} = 0.7$ axial Eq. 6.7a

$C_{SIZE} = 1$ axial footnote page 363 5th edition

assume cold-rolled A = 2.78 b = -0.265 **for ksi** Table 6-3

$C_{SURF} = A\ (S_{UT})^b = 0.8425$ Eq. 6.7e

$C_{TEMP} = 1$ assume normal temperture Eq. 6.7f

$C_{RELI} = 0.753$ assume 99.9% reliability Table 6-4

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e' = 17.99$ ksi Eq. 6.6

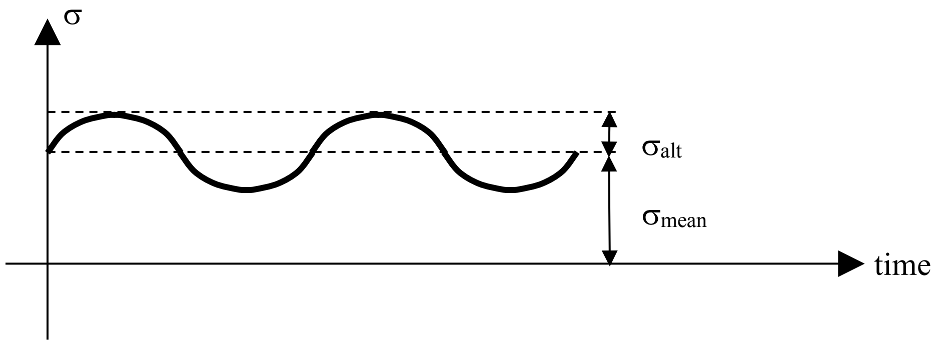
$S_N = S_e$ infinite life

$N_{mean} = S_{UT} / \sigma'_{mean} = 4.478$

$N_{alt} = S_N / \sigma'_{alt} = 0.9945$

$N_{GOODMAN} = N_{alt}\, N_{mean} / (N_{alt} + N_{mean}) = 0.814$

more realistic charge to 150 psi and keep above 75 psi



$P_{mean} = 112.5$ psi $P_{alt} = 37.5$ psi

$\sigma'_{mean} = 27.14$ ksi $\sigma'_{alt} = 9.05$ ksi

$N_{mean} = 2.98$ $N_{alt} = 1.99$ $N_{GOODMAN} = 1.19$

prefer $N_{GOODMAN} > 3$

$$t > \frac{3}{1.19}(0.0359 \text{ in}) = 0.0905 \text{ inch}$$

try 12 gauge t = 0.1046 inch

for 75 psi to 150 psi loading

$\sigma'_{mean} = 9.31$ ksi $\sigma'_{alt} = 3.10$ ksi

$N_{mean} = 8.70$ $N_{alt} = 5.80$ $N_{GOODMAN} = 3.48$

check 13 gauge t = 0.0897 inch $N_{GOODMAN} = 2.97$

Example 6-5 Norton

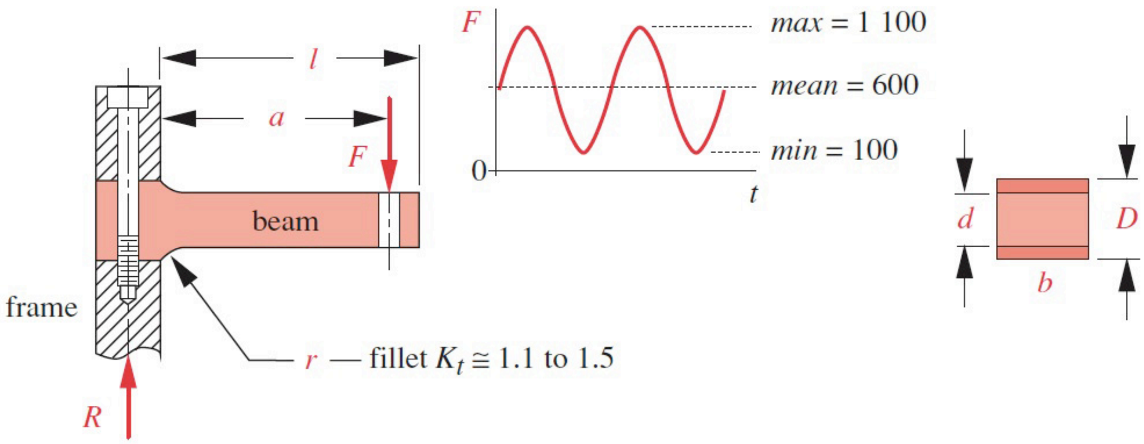


FIGURE 6-47

Design of a Cantilever Bracket for Fluctuating-Bending Loading

desire N = 2.5 for infinite life

ℓ = 6.0 in a = 5.0 in b = 2.0 in d = 1.0 in D = 1.125 in r = 0.5 in

1040 steel normalized machined 99.9% reliability room temperature

S_{UT} = 80 ksi Table A-9

F_{MEAN} = 600 lbf F_{ALT} = 500 lbf

$M_{MEAN} = a\, F_{MEAN} = 3000$ in.lbf $M_{ALT} = a\, F_{ALT} = 2500$ in.lbf

$I = b\, d^3 / 12 = 0.1667\ \text{in}^4$ $c = d / 2 = 0.5$ in

$\sigma_{MEAN\_NOM} = M_{MEAN}\, c / I = 9$ ksi $\sigma_{ALT\_NOM} = M_{ALT}\, c / I = 7.5$ ksi

D/d = 1.125 r/d = 0.5 A = 1.012 b = -0.221 interpolated Appendix C-10

$K_t = A\,(r/d)^b = 1.18$

S_{UT} = 80 ksi $\sqrt{a} = 0.8\sqrt{\text{in}}$ Table 6-6

$$q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}} = 0.898$$ Eq. 6.11a

$K_f = 1 + q\,(K_t - 1) = 1.16$ Eq. 6.11b

$\sigma' = \sigma_{BENDING}$

$\sigma'_{MEAN} = K_t\, \sigma_{MEAN\_NOM} = 10.62$ ksi $\sigma'_{ALT} = K_f\, \sigma_{ALT\_NOM} = 8.71$ ksi

$S_e' = 0.5\, S_{ut} = 40$ ksi Eq. 6.5a

$C_{LOAD} = 1.0$ bending Eq. 6.7a

$A_{95} = 0.05\, b\, d = 0.1\ in^2$ Fig. 6-25

$d_{EQUIV} = \sqrt{\dfrac{A_{95}}{0.0766}} = 1.143$ in Eq. 6.7d

$C_{SIZE} = 0.869\,(d)^{-0.097} = 0.8578$ Eq. 6.7b

machined A = 2.7 b = -0.265 **for ksi** Table 6-3

$C_{SURF} = A\,(S_{UT})^b = 0.8453$ Eq. 6.7e

$C_{TEMP} = 1$ assume normal temperture Eq. 6.7f

$C_{RELI} = 0.753$ assume 99.9% reliability Table 6-4

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e' = 21.84$ ksi Eq. 6.6

$S_N = S_e$ infinite life

$N_{MEAN} = S_{UT} / \sigma'_{MEAN} = 7.533$ $N_{ALT} = S_N / \sigma'_{ALT} = 2.507$

$N_{GOODMAN} = N_{alt}\, N_{mean}\ /\ (N_{alt} + N_{mean}) = 1.88$

modify geometry

$\ell = 6.0$ in a = 5.0 in b = 2.0 in d = 1.2 in D = 1.4 in r = 0.5 in

$N_{GOODMAN} = 2.6$ Case 3

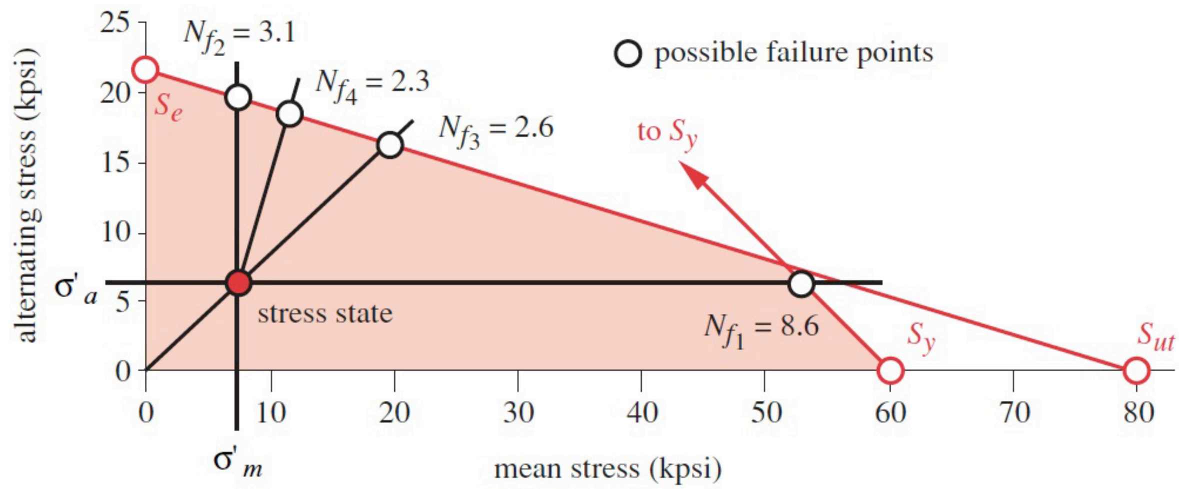


FIGURE 6-48

Modified-Goodman Diagram for Example 6-5 Showing Final Solution Data from Table 6-12