Brinell hardness tester

steel ball of diameter D pressed into flat plate with constant force F

measure size of circular indentation

steel F = 3000 kgf D = 10 mm

aluminum F = 500 kgf D = 10 mm

calculate subsurface stress for testing aluminum

steel ball on flat aluminum plate

F = 500 kgf = 4905 N 1 kgf = 9.81 N

steel ball $R_1 = 5$ mm $E_1 = 206.8$ GPa $\nu_1 = 0.28$ Norton Table A-1

aluminum plate $R_2 = \infty$ $E_2 = 71.7$ GPa $\nu_2 = 0.34$ Norton Table A-1

$$m_1 = \frac{1-\nu_1{}^2}{E_1} = \frac{1-(0.28)^2}{206.8\ \text{GPa}}\left(\frac{\text{GPa}}{10^3\,\text{MPa}}\right)\left(\frac{\text{MPa.mm}^2}{\text{N}}\right) = 4.4565 \times 10^{-6}\,\frac{\text{mm}^2}{\text{N}}$$ Norton Eq. 7.9a

$$m_2 = \frac{1-\nu_2{}^2}{E_2} = \frac{1-(0.34)^2}{71.7\ \text{GPa}}\left(\frac{\text{GPa}}{10^3\,\text{MPa}}\right)\left(\frac{\text{MPa.mm}^2}{\text{N}}\right) = 12.335 \times 10^{-6}\,\frac{\text{mm}^2}{\text{N}}$$ Norton Eq. 7.9a

$$B = \frac{1}{2}\left(\frac{1}{R_1}+\frac{1}{R_2}\right) = \frac{1}{2}\left(\frac{1}{5\ \text{mm}}+\frac{1}{\infty}\right) = 0.1\ \text{mm}^{-1}$$ Norton Eq. 7.9b

$$a^3 = 0.375\frac{m_1+m_2}{B}F = 0.375\frac{(4.4565+12.335)}{0.1\ \text{mm}^{-1}}\left(\frac{10^{-6}\,\text{mm}^2}{\text{N}}\right)4905\ \text{N} = 0.3089\ \text{mm}^3$$ Eq. 7.9d

a = 0.6760 mm

$$p_{MAX} = \frac{3}{2}\frac{F}{\pi a^2} = \frac{3}{2}\frac{(4905\ \text{N})}{\pi(0.6760\ \text{mm})^2} = 5126\ \text{MPa}$$ Norton Eq. 7.8b

$$\tau_{MAX} = \frac{p_{MAX}}{2}\left[\frac{(1-2\nu)}{2}+\frac{2}{9}(1+\nu)\sqrt{2(1+\nu)}\right]$$ Norton Eq. 7.12b

aluminum plate $\nu_2 = 0.34$ $\tau_{max} = 0.3237\, p_{max} = 1659$ MPa Eq. 7.12b von Mises $\sigma' = \sqrt{3}\,\tau_{MAX} = 2874$ MPa

strongest aluminum in Norton Table A-2 7075 heat treated $S_Y = 503$ MPa

Brinell tester will dent strongest aluminum $\sigma' > S_Y$

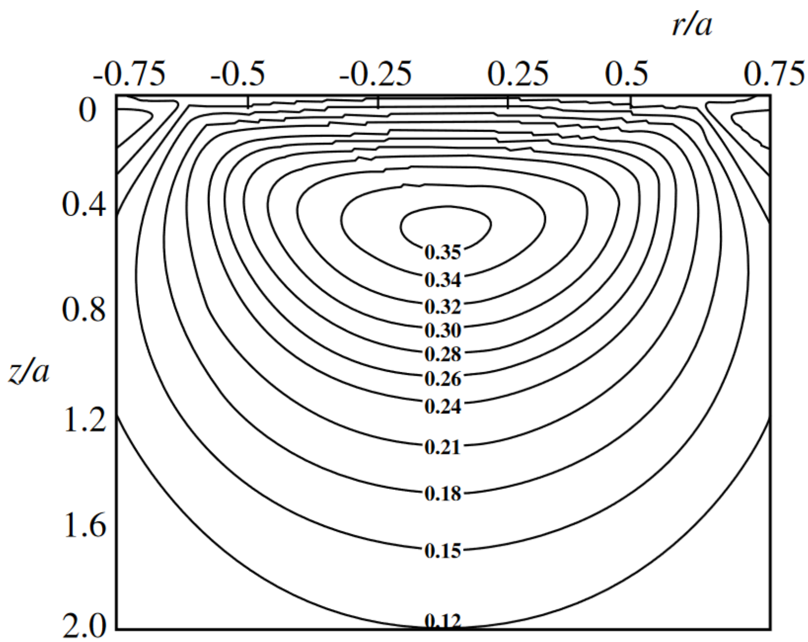


FIGURE 3.7 Contours of maximum shear stress normalized by Hertz stress p_0, beneath nominal circular point contact of radius a in material with $\nu = 0.3$.

A ball and socket joint uses a 10 mm DIA steel ball (4340 with HB 430) inside a 10.1 mm DIA phosphor bronze socket (CA510 with HRB 95). Determine factor of safety for the ball and for the socket in static yield at a load of 2000 N.

Table A-1 for E and ν Table A-10 for 4340 Table A-4 for phosphor bronze

steel ball (convex) $R_1 = +5$ mm $\nu_1 = 0.28$ $E_1 = 206.8$ GPa $S_Y = 1365$ MPa

phosphor bronze socket (concave) $R_2 = -5.05$ mm $\nu_2 = 0.33$ $E_2 = 110.3$ GPa $S_Y = 552$ MPa

$m_1 = (1-\nu_1^2) / E_1 = 0.004456\ \text{GPa}^{-1}$ Eq. 7.9a

$m_2 = (1-\nu_2^2) / E_2 = 0.008079\ \text{GPa}^{-1}$

$B = (1/R_1 + 1/R_2) / 2 = 0.0009901\ \text{mm}^{-1}$ Eq. 7.9b

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F \qquad \text{Eq. 7.9d}$$

$$a^3 = 0.375 \left(\frac{(0.004456 + 0.008079\ \text{m}^2}{10^9\ \text{N}} \right) \left(\frac{1000\ \text{mm}}{\text{m}} \right)^2 \left(\frac{\text{mm}}{0.0009901} \right) (2000\ \text{N}) \qquad \text{Eq. 7.9d}$$

$a = 2.118$ mm

$$p_{max} = \frac{3}{2} \frac{F}{\pi a^2} = \left(\frac{3}{2} \right) \frac{2000\ \text{N}}{\pi (2.118\ \text{mm})^2} = 212.9\ \text{MPa} \qquad \text{Eq. 7.8b}$$

ball

$$\nu_1 = 0.28 \qquad \tau_{max} = \frac{p_{max}}{2} \left(\frac{1 - 2\nu_1}{2} + \frac{2}{9} (1 + \nu_1) \sqrt{2(1 + \nu_1)} \right) = 0.3376\ p_{max} = 71.88\ \text{MPa} \qquad \text{Eq. 7.12b}$$

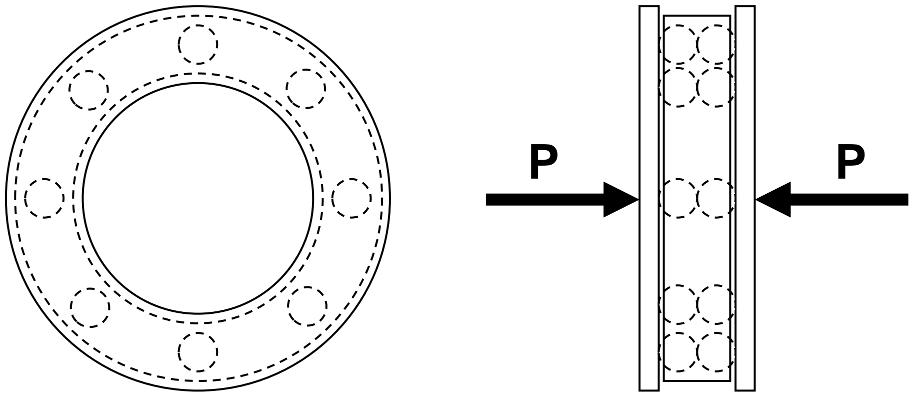
$\sigma' = \sqrt{3}\ \tau = 124.5$ MPa $N_{FS} = Sy / \sigma' = 10.96$

socket

$$\nu_2 = 0.33 \qquad \tau_{max} = \frac{p_{max}}{2} \left(\frac{1 - 2\nu_2}{2} + \frac{2}{9} (1 + \nu_2) \sqrt{2(1 + \nu_2)} \right) = 0.3260\ p_{max} = 69.4\ \text{MPa} \qquad \text{Eq. 7.12b}$$

$\sigma' = \sqrt{3}\ \tau = 120.2$ MPa $N_{FS} = Sy / \sigma' = 4.59$

Sixteen steel balls (0.156 inch DIA, S-5 steel with HRC 59) are used in torque adjusters for cordless electric drills. The balls are stacked in pairs inside eight axial holes within a 6/6 nylon ring spacer and compressed between two flat steel disks (4340 steel with HB 430) by a compression spring as shown below. The axial holes in the nylon ring do not constrain axial motion of the balls. Determine factor of safety in the balls and in the plates for compressive load P = 30 lbf. The eight pairs of balls share load P equally.



AXIAL VIEW **RADIAL VIEW**

contact between two steel spheres S-5 HRC 59 $S_Y = 280$ ksi Table A-10

$d = 0.156$ in $F = P / 8 = 3.75$ lbf

$R_1 = R_2 = d / 2 = 0.078$ in steel $\nu_1 = \nu_2 = 0.28$ $E_1 = E_2 = 30\text{x}10^6$ psi

$m_1 = m_2 = (1-\nu^2) / E = 3.072\text{x}10^{-8}$ in^2/lbf Eq. 7.9a

$B = (1/R_1 + 1/R_2) / 2 = 12.82$ in^{-1} Eq. 7.9b

$$a^3 = 0.375\frac{m_1 + m_2}{B}F = 0.375\left(\frac{6.144\text{x}10^{-8}\ \text{in}^2}{\text{lbf}}\right)\left(\frac{\text{in}}{12.82}\right)(3.75\ \text{lbf})$$ Eq. 7.9d

$a = 0.001889$ in

$$p_{max} = \frac{3}{2}\frac{F}{\pi a^2} = \left(\frac{3}{2}\right)\frac{3.75\ \text{lbf}}{\pi(0.001889\ \text{in})^2} = 501.8\ \text{ksi}$$ Eq. 7.8b

$$\tau_{max} = \frac{p_{max}}{2}\left(\frac{1-2\nu}{2} + \frac{2}{9}(1+\nu)\sqrt{2(1+\nu)}\right) = 0.3376\ p_{max} = 169.4\ \text{ksi}$$ Eq. 7.12b

$\sigma' = \sqrt{3}\ \tau = 293.4$ ksi

sphere on sphere $N_{FS} = S_Y / \sigma' = 0.95$

contact between steel sphere and steel plate

$R_1 = 0.078$ in $R_2 = \infty$ $\nu_1 = \nu_2 = 0.28$ $E_1 = E_2 = 30\text{x}10^6$ psi

$m_1 = m_2 = (1-\nu^2) / E = 3.072\text{x}10^{-8}$ in^2/lbf Eq. 7.9a

$B = (1/R_1 + 1/R_2) / 2 = 6.4103$ in^{-1} Eq. 7.9b

$$a^3 = 0.375 \frac{m_1 + m_2}{B} F$$ Eq. 7.9d

a = 0.002380 in

$$p_{max} = \frac{3}{2} \frac{F}{\pi a^2} = 316.1 \text{ ksi}$$ Eq. 7.8b

$$\tau_{max} = \frac{p_{max}}{2} \left(\frac{1-2\nu}{2} + \frac{2}{9}(1+\nu)\sqrt{2(1+\nu)} \right) = 0.3376\, p_{max} = 106.7 \text{ ksi}$$ Eq. 7.12b

$\sigma' = \sqrt{3}\, \tau = 184.8$ ksi

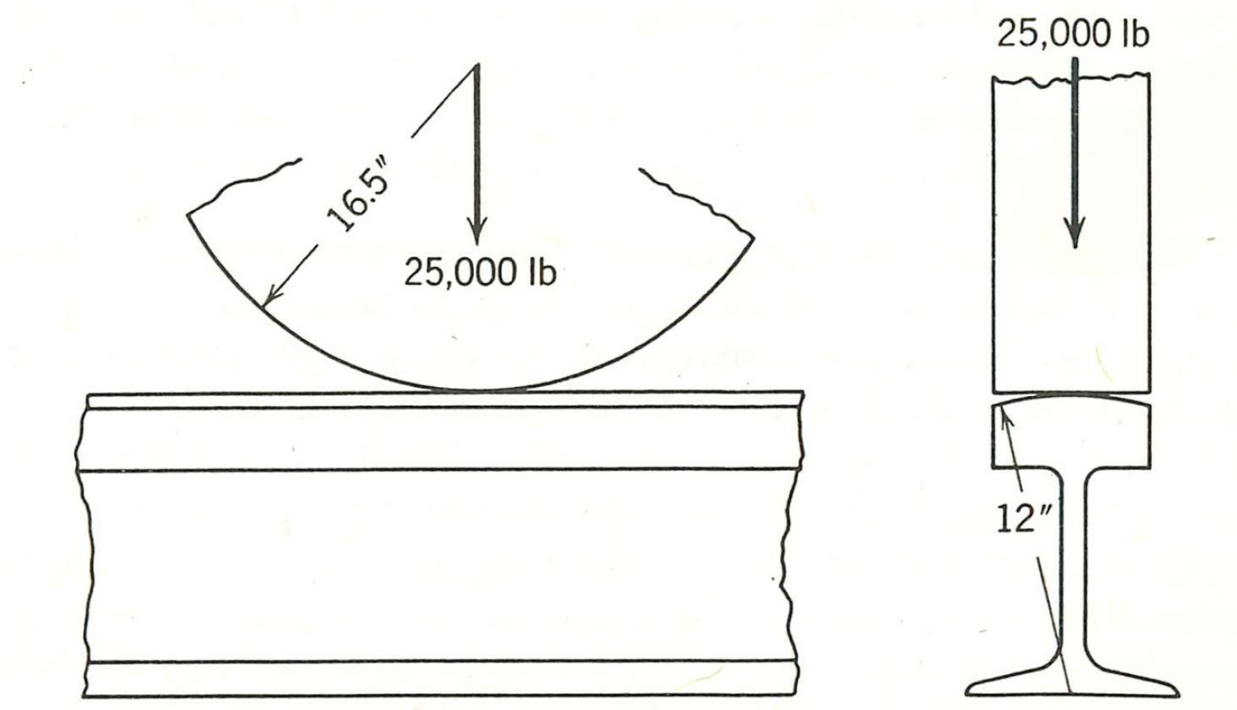
sphere on flat plate sphere S-5 HRC 59 $S_Y = 280$ ksi Table A-10

$N_{FS} = S_Y / \sigma' = 1.52$

sphere on flat plate plate 4340 with HB 486 $S_Y = 230$ ksi Table A-10

$N_{FS} = S_Y / \sigma' = 1.24$

A steel railway car wheel with 33 inch DIA rolls on a steel rail whose top surface has a cross section radius of 12 inches as shown below. The wheel load on the rail is 25,000 lbf. Assume width of the rail is 6 inches and **ignore the curvature of the rail cross section**. Determine width of the contact patch and maximum von Mises stress in the rail and wheel.



$R_1 = 16.5$ in $\quad R_2 = \infty \qquad$ steel $\quad \nu_1 = \nu_2 = 0.28 \qquad E_1 = E_2 = 30\text{x}10^6$ psi

$m_1 = m_2 = (1-\nu^2) / E = 3.072\text{x}10^{-8}$ in^2/lbf $\qquad$ Eq. 7.9a

$B = (1/R_1 + 1/R_2) / 2 = 0.0303$ in^{-1} $\qquad$ Eq. 7.9b

$$a = \sqrt{\frac{2}{\pi}\left(\frac{m_1 + m_2}{B}\right)\left(\frac{F}{L}\right)} = \sqrt{\frac{2}{\pi}\left(\frac{3.072\text{x}10^{-8}\ \text{in}^2 + 3.072\text{x}10^{-8}\ \text{in}^2}{\text{lbf}}\right)\left(\frac{\text{in}}{0.0303}\right)\left(\frac{25{,}000\ \text{lbf}}{6\ \text{in}}\right)}$$

$a = 0.0733$ in $\qquad$ Eq. 7.15b

width = 2a = 0.1467 in

rectangular contact area = 2 a L = 0.8802 in^2

$$p_{max} = \frac{2\,F}{\pi\,a\,L} = \frac{2(25{,}000\ \text{lbf})}{\pi(0.0733\ \text{in})(6\ \text{in})} = 36.19\ \text{ksi} \qquad \text{Eq. 7.14b}$$

$\tau_{max} = 0.304\, p_{max} = 11.0$ ksi $\qquad$ Eq. 7.17b $\quad$ steel cylinder on flat plate $\sigma' = \sqrt{3}\,\tau = 19.06$ ksi

include radius of rail cross section – General Contact Section 7.10

steel $\nu_1 = \nu_2 = 0.28$ $E_1 = E_2 = 30\text{x}10^6$ psi

$m_1 = m_2 = (1-\nu^2) / E = 3.072\text{x}10^{-8}$ in^2/lbf Eq. 7.9a

crossed cylinders

$R_1 = 16.5$ in $R_1' = \infty$ in $R_2 = 12$ in $R_2' = \infty$ in $\theta = 90°$

$$A = \frac{1}{2}\left(\frac{1}{R_1} + \frac{1}{R_1'} + \frac{1}{R_2} + \frac{1}{R_2'}\right) = 0.07197 \text{ in}^{-1}$$ Eq. 7.19a

$$B = \frac{1}{2}\sqrt{\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)^2 + \left(\frac{1}{R_2} - \frac{1}{R_2'}\right)^2 + 2\left(\frac{1}{R_1} - \frac{1}{R_1'}\right)\left(\frac{1}{R_2} - \frac{1}{R_2'}\right)\cos 2\theta} = 0.01136 \text{ in}^{-1}$$

Eq. 7.19b

$$\cos\phi = \frac{B}{A}$$ $\phi = 80.92°$ Eq. 7.19c

$$a = k_a \sqrt[3]{\frac{3\,F\left(m_1 + m_2\right)}{4\,A}} = 0.2812 \text{ in}$$ Eq. 7.19d $k_a = 1.1157$ interpolated Table 7-5

$$b = k_b \sqrt[3]{\frac{3\,F\left(m_1 + m_2\right)}{4\,A}} = 0.2274 \text{ in}$$ Eq. 7.19d $k_b = 0.9024$ interpolated Table 7-5

elliptic contact area = $\pi\,a\,b = 0.2009$ in^2

$$p_{MAX} = \frac{3}{2}\frac{F}{\pi\,a\,b} = \frac{3}{2}\frac{\left(25{,}000 \text{ lbf}\right)}{\pi\left(0.2812 \text{ in}\right)\left(0.2274 \text{ in}\right)} = 186.7 \text{ ksi}$$ Eq. 7.18b

$\tau_{max} \sim 0.34\,p_{max} = 63.48$ ksi page 482 Norton 5th edition

crossed cylinders $\sigma' = \sqrt{3}\,\tau = 110.0$ ksi