Norton Eq. 10.8 - zero axial load

$$d^3 = \frac{32\, N_{GOODMAN}}{\pi}\left(\frac{\sqrt{\left(K_{f\,b}M_a\right)^2 + \frac{3}{4}\left(K_{f\,t}T_a\right)^2}}{S_N} + \frac{\sqrt{\left(K_{t\,b}M_m\right)^2 + \frac{3}{4}\left(K_{t\,t}T_m\right)^2}}{S_{ut}}\right)$$

$N_{GOODMAN}$ Goodman factor of safety

M_a	alternating bending moment	$K_{f\,b}$	fatigue bending stress concentration
T_a	alternating torque	$K_{f\,t}$	fatigue torsion stress concentration
M_m	mean bending moment	$K_{t\,b}$	static bending stress concentration
T_m	mean torque	$K_{t\,t}$	static bending stress concentration

$K_f = 1 + q\,(K_t - 1)$ $\quad q = \frac{1}{1+\frac{\sqrt{a}}{\sqrt{r}}}$

S_{UT} ultimate tensile strength

S_N fatigue strength interpolated for N cycles using S_e and S_m

$S_e = C_{LOAD}\, C_{SIZE}\, C_{SURF}\, C_{TEMP}\, C_{RELI}\, S_e'$

ASME standard B106 (obsolete)
fully reversed bending, zero mean bending, constant torsion, Gough failure

$$d^3 = \frac{32\, N_{GOUGH}}{\pi}\sqrt{\left(K_{f\,b}\frac{M_a}{S_N}\right)^2 + \frac{3}{4}\left(\frac{T_m}{S_Y}\right)^2}$$

key - yield failure in direct shear (guillotine shear)

$T = F_T\,(d/2)$ $F_T = 2\,T / d$ $A_S = w\,L$ $\tau = F_T / A_S$ $\sigma' = \sqrt{3}\,\tau$

$$N_Y = \frac{S_Y}{\sigma'} = \frac{S_Y}{\sqrt{3}\tau} = \frac{S_Y\ A_S}{\sqrt{3}\ F_T} = \frac{S_Y\ w\ L\ d}{2\sqrt{3}\ T}$$

$N_Y = 2$ $T_{MAX} = 1500$ in.lbf $S_Y = 40$ ksi $d = 1.5$ in

$w = 0.375$ in Table 10-2 Norton

$$L = \frac{2\sqrt{3}\ T\ N_Y}{S_Y\ w\ d} = \frac{2\sqrt{3}\ (1500\ \text{in.lbf})(2)}{(0.375\ \text{in})(1.5\ \text{in})}\left(\frac{\text{in}^2}{40{,}000\ \text{lbf}}\right) = 0.462\ \text{inch}$$

use L = 0.5 inch

key - yield failure in direct bearing (crushing)

$A_B = L\,(H/2) = L\,(w/2)$ $\sigma' = F_T / A_B$

$$N_Y = \frac{S_Y}{\sigma'} = \frac{S_Y}{\sqrt{3}\tau} = \frac{S_Y\ A_B}{F_T} = \frac{S_Y\ w\ L\ d}{4\ T}$$

same data

$$L = \frac{4\ T\ N_Y}{S_Y\ w\ d} = \frac{4\ (1500\ \text{in.lbf})(2)}{(0.375\ \text{in})(1.5\ \text{in})}\left(\frac{\text{in}^2}{40{,}000\ \text{lbf}}\right) = 0.533\ \text{inch}$$

use L = 0.625 inch

must also check shaft material for direct bearing if weaker than key

key - fatigue failure in direct shear

w = 0.375 inch L = 0.625 inch use larger value for L

$N_{GOODMAN} = 2$ $T_{MAX} = 1500$ in.lbf $T_{MIN} = 0$ in.lbf $S_{UT} = 60$ ksi 90% reliability

$T_{MEAN} = 750$ in.lbf $T_{ALT} = 750$ in.lbf

$S_{e'} = S_{UT} / 2 = 30$ ksi Eq. 6.5a Norton

$C_{LOAD} = 1$ page 362 Norton

$A_{95} = w\ L$ because entire area has stress between 95% to 100% of maximum - page 363 Norton

Fig 6-25 Norton is only for bending

$A_{95} = w\ L = 0.2344\ in^2$ $d_{equiv} = \sqrt{\dfrac{A_{95}}{0.0766}} = 1.7492$ inch Eq. 6.7d Norton

$C_{SIZE} = 0.869\ d^{-0.097} = 0.8231$ Eq. 6.7b Norton

$C_{SURF} = 2.7\ S_{UT}^{-0.265} = 0.9123$ machined, Eq. 6.7e and Table 6-3 Norton

$C_{TEMP} = 1$ normal temperature, Eq. 6.7f Norton

$C_{RELIAB} = 0.897$ 90% reliability, Table 6-4 Norton

$S_e = C_{LOAD}\ C_{SIZE}\ C_{SURF}\ C_{TEMP}\ C_{RELIAB}\ S_{e'}$ Eq. 6.6 Norton

$S_e = (1)\ (0.8231)\ (0.9123)\ (1)\ (0.897)\ (30\ ksi) = 20.207$ ksi

$S_f = S_e = 20.207$ ksi infinite life

$$N_{ALT} = \frac{S_f\ w\ L\ d}{2\sqrt{3}\ T_{ALT}} = \left(\frac{20{,}207\ lbf}{in^2}\right)\frac{(0.375\ in)(0.625\ in)(1.5\ in)}{2\sqrt{3}(750\ in.lbf)} = 2.734$$

$$N_{MEAN} = \frac{S_{UT}\ w\ L\ d}{2\sqrt{3}\ T_{MEAN}} = \left(\frac{60{,}000\ lbf}{in^2}\right)\frac{(0.375\ in)(0.625\ in)(1.5\ in)}{2\sqrt{3}(750\ in.lbf)} = 8.119$$

$$N_{GOODMAN} = \frac{N_{MEAN}\ N_{ALT}}{N_{MEAN} + N_{ALT}} = 2.045 \quad \text{OK}$$

key - yield failure in direct shear (guillotine shear)

½ inch DIA shaft delivers ½ HP at 400 rpm

select 1020 HR square key with $N_{YS} = 2$ for static yield

1020 HR $S_Y = 30$ ksi Norton Table A-9

d = 0.5 inch w = 0.125 inch Norton Table 10-2

$$P = T\,\omega \qquad T = \frac{P}{\omega} = \left(\frac{1/2\ \text{HP}}{}\right)\left(\frac{\text{min}}{400\ \text{rev}}\right)\left(\frac{\text{rev}}{2\pi\ \text{rad}}\right)\left(\frac{33{,}000\ \text{ft.lbf}}{\text{HP min}}\right)\left(\frac{12\ \text{in}}{\text{ft}}\right) = 78.78\ \text{in.lbf}$$

$$L = \frac{2\sqrt{3}\ T\ N_{YS}}{S_Y\ w\ d} = \frac{2\sqrt{3}\,(78.78\ \text{in.lbf})(2)}{(0.125\ \text{in})(0.5\ \text{in})}\left(\frac{\text{in}^2}{30{,}000\ \text{lbf}}\right) = 0.291\ \text{inch}$$

use L = 0.375 inch