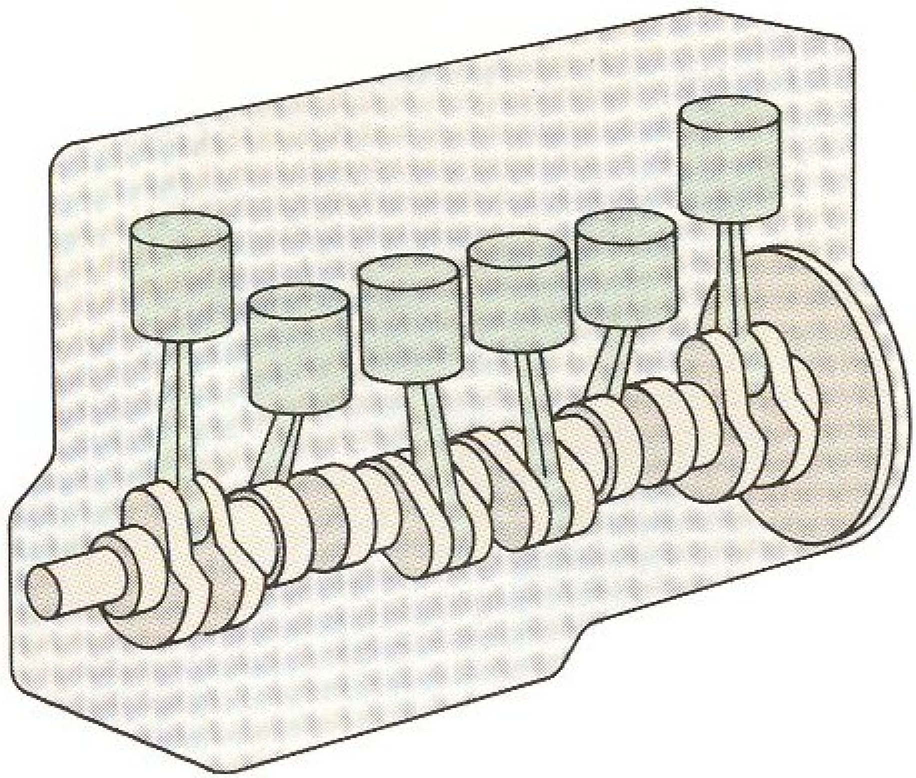
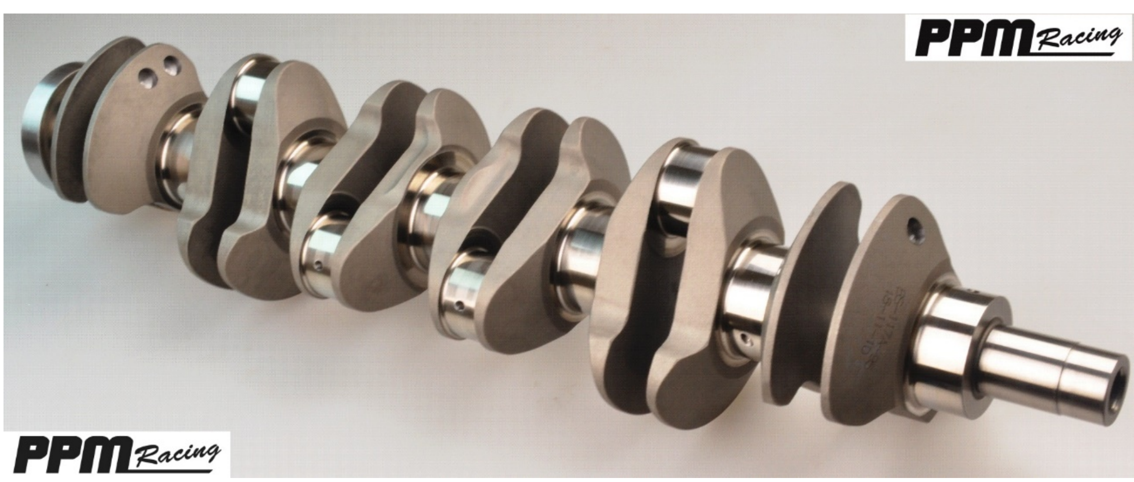
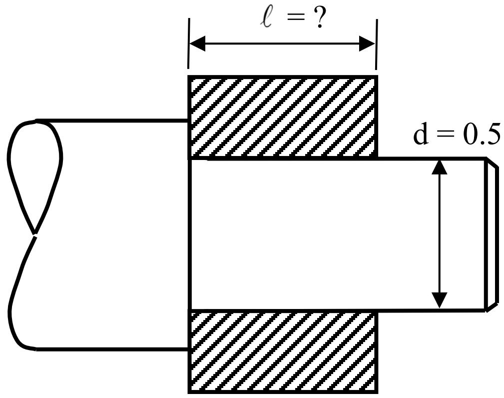
Journal Bearings in In-line Six Cylinder Engine







hydrodynamic journal bearing
1800 rpm
SAE 30 oil @ 180 F°
80 lbf radial load
$O_N = 25$
$c_d / d = 0.002$

$O_N = 25$ (prefer $O_N \leq 30$)

$c_d / d = 0.002$ (prefer $0.001 < c_d / d < 0.003$)

$$O_N = \left(\frac{p_{avg}}{\eta\, n'}\right)\left(\frac{d}{\ell}\right)^2\left(\frac{c_d}{d}\right)^2$$ Eq 11.12c Norton

$$p_{avg} = \frac{P}{d\,\ell}$$ Eq 11.6d Norton

$$O_N = \left(\frac{P\,d}{\eta\, n'\,\ell^3}\right)\left(\frac{c_d}{d}\right)^2 \qquad \ell^3 = \left(\frac{P\,d}{\eta\, n'\, O_N}\right)\left(\frac{c_d}{d}\right)^2$$

$\eta = 2\ \mu\text{reyn}$ Figure 11-1 Norton

$n = 1800$ rpm $n' = 30$ rev/sec

$$\ell^3 = \frac{(80\text{ lbf})(0.5\text{ in})}{(2\ \mu\text{reyn})(25)}\left(\frac{\text{sec}}{30\text{ rev}}\right)(0.002)^2\left(\frac{\text{reyn.in}^2}{\text{lbf.sec}}\right)\left(\frac{10^6\ \mu\text{reyn}}{\text{reyn}}\right) = 0.10667\text{ in}^3 \qquad \ell = 0.474\text{ in}$$

use $\ell = 0.5$ in

$\varepsilon = 0.21394 + 0.38517 \log O_N - 0.0008\,(O_N - 60) = 0.78038$ Eq 11.13b Norton

O_N	20	25	30	35	40
ε	0.74706	0.78038	0.80688	0.82867	0.84701

$c_d / d = 0.002$ $c_d = 0.001$ in $c_r = c_d / 2 = 0.0005$ in

$\varepsilon = e / c_r$ $e = 3.901 \times 10^{-4}$ in Eq 11.3 Norton

$h_{min} = c_r (1 - \varepsilon) = 109.8\ \mu in$ Eq 11.4b Norton

OK $R_q < 30$ to 40 μin for precision milled/ground surface

$$T_s = \frac{\eta\, d^3\ \ell\, n'\, \pi^2}{c_d \sqrt{1-\varepsilon^2}}$$ Eq 11.9c Norton

$$T_s = \frac{(2\ \mu reyn)(0.5\ in)^3 (0.5\ in)\pi^2}{(0.001\ in)\sqrt{1-(0.78038)^2}} \left(\frac{30\ rev}{sec}\right)\left(\frac{lbf.sec}{reyn.in^2}\right)\left(\frac{reyn}{10^6\ \mu reyn}\right) = 0.0592\ in.lbf$$

$$\tan\phi = \frac{\pi\sqrt{1-\varepsilon^2}}{4\varepsilon}$$ $\phi = 32.18°$ Eq 11.8a Norton

$T_r = T_s + P\, e \sin\phi = 0.07867$ in.lbf Eq 11.9a

$P_{LOSS} = T_r\, \omega = 2\, \pi\, T_r\, n' = 14.83$ in.lbf/sec = 0.00225 HP Eq 11.10 Norton

$\mu = 2\, T_r / P\, d = 0.00336$ Eq 11.11

hydrodynamic journal bearing

given
radial load P = 54 lbf
diameter d = 0.591 inch (15 mm)
speed = 1725 rpm
clearance ratio c_d / d = 0.0017
length ratio ℓ / d = 0.75
Ocvirk number O_N = 20

select lubricant at 190° F

n = 1725 rpm   n' = 28.75 rev/sec

c_d / d = 0.0017 c_d = 0.0010 inch

$c_r = c_d / 2 = 0.0005$ inch = 500 μin

$\varepsilon = 0.21394 + 0.38517 \log O_N - 0.0008\,(O_N - 60) = 0.74706$

$h_{min} = c_r\,(1 - \varepsilon) = 126.5$ μin   OK

ℓ / d = 0.75 ℓ = 0.443 inch

$$p_{avg} = \frac{P}{d\,\ell} = \frac{54\text{ lbf}}{(0.591\text{ in})(0.443\text{ in})} = 206\text{ psi}$$

$$O_N = \left(\frac{p_{avg}}{\eta\, n'}\right)\left(\frac{d}{\ell}\right)^2\left(\frac{c_d}{d}\right)^2 \qquad \eta = \left(\frac{p_{avg}}{O_N\, n'}\right)\left(\frac{d}{\ell}\right)^2\left(\frac{c_d}{d}\right)^2$$

$$\eta = \left(\frac{206\text{ lbf}}{\text{in}^2}\right)\left(\frac{1}{20}\right)\left(\frac{\text{sec}}{28.75\text{ rev}}\right)\left(\frac{1}{0.75}\right)^2(0.0017)^2\left(\frac{\text{reyn.in}^2}{\text{lbf.sec}}\right) = 1.841 \times 10^{-6}\text{ reyn} = 1.84\ \mu\text{reyn}$$

Fig 11-1 at 190° F   AGMA 3, SAE 30

$$T_s = \frac{\eta\, d^3\, \ell\, n'\, \pi^2}{c_d\sqrt{1-\varepsilon^2}} \qquad \text{Eq 11.9c Norton}$$

$$T_s = \frac{(1.84\ \mu\text{reyn})(0.591\text{ in})^3(0.443\text{ in})\pi^2}{(0.001\text{ in})\sqrt{1-(0.74706)^2}}\left(\frac{28.75\text{ rev}}{\text{sec}}\right)\left(\frac{\text{lbf.sec}}{\text{reyn.in}^2}\right)\left(\frac{\text{reyn}}{10^6\ \mu\text{reyn}}\right) = 0.07182\text{ in.lbf}$$

$\tan\phi = \dfrac{\pi\sqrt{1-\varepsilon^2}}{4\varepsilon}$ $\phi = 34.95°$ Eq 11.8a Norton

$\varepsilon = e / c_r$ $e = 3.73\text{x}10^{-4}$ in Eq 11.3 Norton

$T_r = T_s + P\ e \sin\phi = 0.08337$ in.lbf Eq 11.9a

$P_{LOSS} = T_r\ \omega = 2\ \pi\ T_r\ n' = 15.06$ in.lbf/sec = 0.002227 HP Eq 11.10 Norton

$\mu = 2\ T_r / P\ d = 0.00522$ Eq 11.11

hydrodynamic journal bearing

given
radial load P = 300 N
diameter d = 20 mm
speed = 1500 rpm
clearance ratio c_d / d = 0.002
length ratio ℓ / d = 0.8
Ocvirk number O_N = 25

select maximum temperature for ISO VG 100 lubricant

n = 1500 rpm   n' = 25 rev/sec

c_d / d = 0.002 c_d = 0.04 mm

$c_r = c_d / 2 = 0.02$ mm

$\varepsilon = 0.21394 + 0.38517 \log O_N - 0.0008\,(O_N - 60) = 0.78038$

$h_{min} = c_r(1 - \varepsilon) = 0.00439$ mm = 4.39 μm = 173 μin   OK

ℓ / d = 0.8 ℓ = 16 mm

$$p_{avg} = \frac{P}{d\,\ell} = \frac{300\text{ N}}{(20\text{ mm})(16\text{ mm})} = 0.9375\text{ MPa} = 0.9375 \times 10^9\text{ mPa}$$

$$O_N = \left(\frac{p_{avg}}{\eta\,n'}\right)\left(\frac{d}{\ell}\right)^2\left(\frac{c_d}{d}\right)^2 \qquad \eta = \left(\frac{p_{avg}}{O_N\,n'}\right)\left(\frac{d}{\ell}\right)^2\left(\frac{c_d}{d}\right)^2$$

$$\eta = \left(\frac{0.9375\text{x}10^9\text{ mPa}}{25}\right)\left(\frac{\text{sec}}{25\text{ rev}}\right)\left(\frac{1}{0.8}\right)^2(0.002)^2\left(\frac{\text{cP}}{\text{mPa.sec}}\right) = 9.375\text{ cP}$$

Fig 11-1   9.4 cP for ISO VG 100   95 deg C

$$T_0 = \frac{\eta\,d^3\,\ell\,n'\,\pi^2}{c_d} \qquad \text{Eq 11.2c Norton}$$

$$T_0 = \frac{(9.375\text{ cP})(20\text{ mm})^3(16\text{ mm})\pi^2}{(0.04\text{ mm})}\left(\frac{25\text{ rev}}{\text{sec}}\right)\left(\frac{\text{mPa.sec}}{\text{cP}}\right)\left(\frac{\text{MPa}}{10^9\text{ mPa}}\right)\left(\frac{\text{N}}{\text{MPa.mm}^2}\right) = 7.402\text{ N.mm}$$

$$T_s = \frac{T_0}{\sqrt{1-\varepsilon^2}}$$ = 11.84 N.mm Eq 11.9d Norton

$$\tan\phi = \frac{\pi\sqrt{1-\varepsilon^2}}{4\varepsilon}$$ $\phi = 32.18°$ Eq 11.8a Norton

$\varepsilon = e / c_r$ $e = 0.0156$ mm Eq 11.3 Norton

$T_r = T_s + P\, e \sin\phi = 11.84$ N.mm + (300N) (0.0156 mm) sin 32.18° = 14.33 N.mm Eq 11.9a

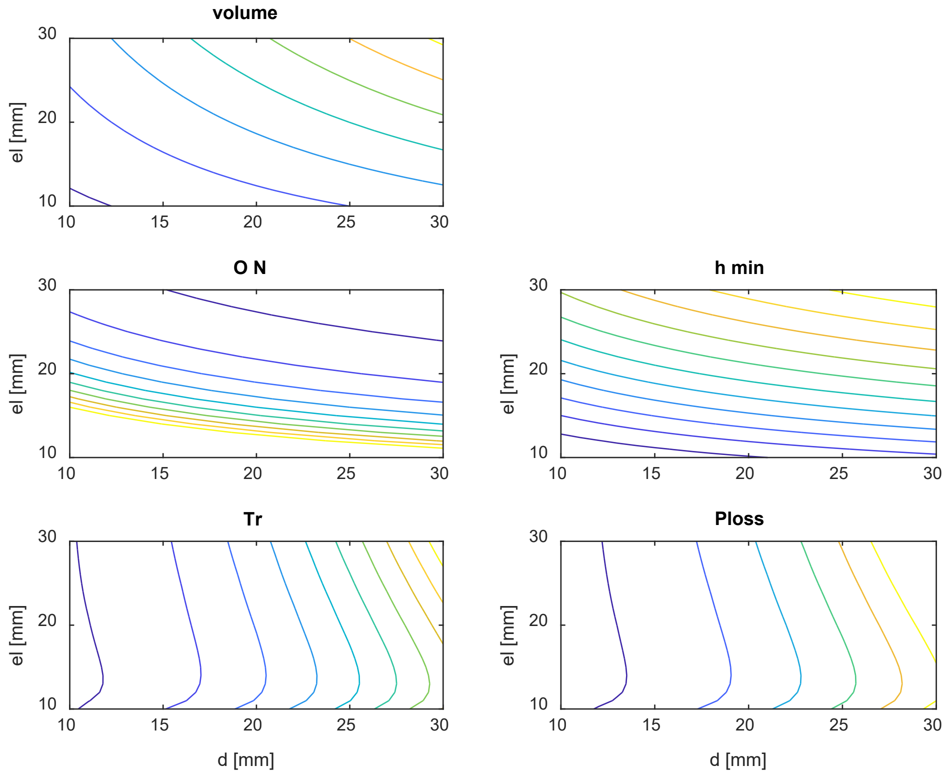
$P_{LOSS} = T_r\, \omega = 2\, \pi\, T_r\, n'$ = 2251 N.mm/sec = 2.251 W = 0.00302 HP Eq 11.10 Norton

$\mu = 2\, T_r / P\, d = 0.00478$ Eq 11.11

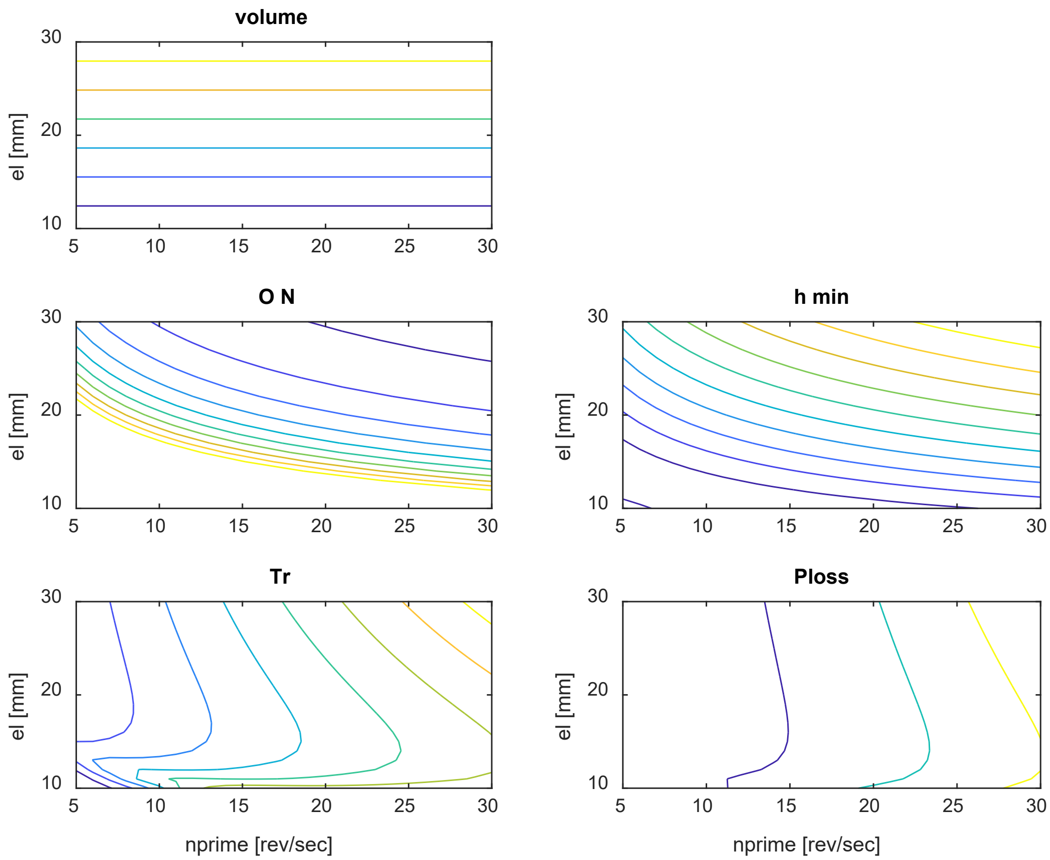
volume of phosphor bronze liner t = 1 mm thick

$V = \pi\, \ell\, [\, (d+t)^2 - d^2\,]/4$

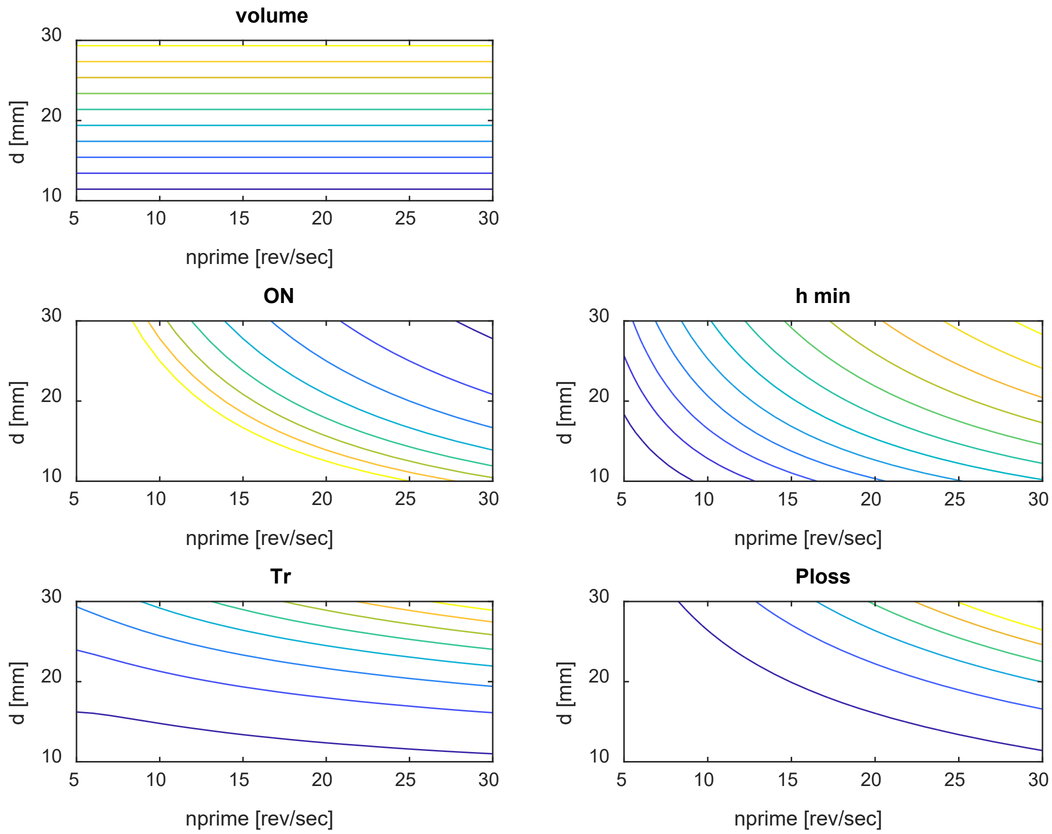
x = d, y = ℓ



x = n', y = ℓ



x = n', y = d


volume
d [mm]
nprime [rev/sec]
ON
d [mm]
nprime [rev/sec]
h min
d [mm]
nprime [rev/sec]
Tr
d [mm]
nprime [rev/sec]
Ploss
d [mm]
nprime [rev/sec]
10
20
30
5
10
15
20
25
30

```
% brg_plots.m - bearing plots
% HJSIII, 19.10.25

clear

% constants
d = 20;               % diameter [mm]
el = 16;              % length [mm]
c_d = 0.04;           % diametral clearance [mm]
c_r = c_d / 2;        % radial clearance [mm]
P = 300;              % radial load [N]
nprime = 25;          % speed [rev/sec]
eta = 9.375;          % viscosity [cP] - ISO VG 100 at 95 deg C
t = 1;                % liner thickness [mm]

% design parameters
d_list = ( 10 : 30 );       % diameter [mm]
el_list = ( 10 : 30 );      % length [mm]
nprime_list = ( 5 : 30 );   % rev/sec

% single values for testing
%d_list = 20;
%el_list = 16;
%nprime_list = 25;

% exhaustive search values
n_d = length( d_list );
n_el = length( el_list );
n_nprime = length( nprime_list );

% column = x, row = y
for j = 1 : n_d,
  d = d_list(j);
%for j = 1 : n_nprime,
%  nprime = nprime_list( j );
  for i = 1 : n_el;
    el = el_list( i );
%  for i = 1 : n_d;
%    d = d_list( i );

% working variables
  p_avg = P /d /el;  % N/mm/mm
  O_N = 1e9 * ( p_avg /eta /nprime ) * (d/el) * (d/el) * (c_d/d) *  (c_d/d);
  eps = 0.21394 + 0.38517*log10(O_N) - 0.0008*(O_N-60);  % eccentricity ratio
  h_min = c_r * (1 - eps) * 1000;                        % minimum film thickness [micro m]
  e = c_r * eps;                                         % eccentricity [mm]
  phi = atan( pi *  sqrt(1-eps*eps) /4 /eps );           % angle for maximum pressure [rad]
  V = pi * el * ( (d+t)*(d+t) - d*d ) /4;               % liner volume [mm^3]
  T0 = eta * d*d*d * el * nprime * pi*pi / c_d /1e9;     % Petroff's torque [N.mm]
  Ts = T0 / sqrt( 1 - eps*eps );                         % stationary torque [N.mm]
  Tr = Ts + P*e*sin(phi);                                % rotating torque [N.mm]
  Ploss = 2 * pi * Tr * nprime /1000;                    % power loss [W]

  V_table(i,j) = V;
  O_N_table(i,j) = O_N;
  h_min_table(i,j) = h_min;
  T_table(i,j) = Tr;
  Ploss_table(i,j) = Ploss;
  end
end

x_list = d_list;
x_str = 'd [mm]';
%x_list = nprime_list;
%x_str = 'nprime [rev/sec]';
y_list = el_list;
y_str = 'el [mm]';
%y_list = d_list;
%y_str = 'd [mm]';

figure( 10 )
```

```
  clf
  subplot( 3, 2, 1 )
    contour( x_list, y_list, V_table )
    title( 'volume' )
    ylabel( y_str )
  subplot( 3, 2, 3 )
    contour( x_list, y_list, O_N_table, [ 5 10 15 20 25 30 35 40 45 50 ] )
    title( 'O N' )
    ylabel( y_str )
  subplot( 3, 2, 4 )
    contour( x_list, y_list, h_min_table )
    title( 'h min' )
    ylabel( y_str )
  subplot( 3, 2, 5 )
    contour( x_list, y_list, T_table )
    title( 'Tr' )
    xlabel( x_str )
    ylabel( y_str )
  subplot( 3, 2, 6 )
    contour( x_list, y_list, Ploss_table, [ 1 2 3 4 5 6 ] )
    title( 'Ploss' )
    xlabel( x_str )
    ylabel( y_str )

% bottom of brg_plots
```

ball bearing selection

$F_r = 1449$ lbf $F_a = 1310$ lbf desire $L_{10} = 100$ for 100×10^6 rev inner race rotating

$P = X V F_r + Y F_a$ Eq. 11.22a

$V = 1$ Fig. 11-24

try $X = 1$ $Y = 0$ $P = F_r = 1449$ lbf

$L_{10} = (C / P)^3$ Eq. 11.20a

$C_{DESIRED} = P (L_{10})^{1/3} = 1449 \text{ lbf} (100)^{1/3} = 6726$ lbf

try 6308 bearing $C = 7350$ lbf $C_0 = 5300$ lbf Fig. 11-23

$F_a / C_0 = 0.2472$ interpolate $e = 0.3681$ Fig. 11-24

$F_a / (V F_r) = 0.9041 > e$ $X = 0.56$ interpolate $Y = 1.198$ Fig. 11-24

$P = X V F_r + Y F_a = 2381$ lbf Eq. 11.22a

$C_{DESIRED} = P (L_{10})^{1/3} = 2381 \text{ lbf} (100)^{1/3} = 11{,}051$ lbf

try 6311 bearing $C = 12{,}900$ lbf $C_0 = 10{,}000$ lbf Fig. 11-23

$F_a / C_0 = 0.131$ interpolate $e = 0.314$ Fig. 11-24

$F_a / (V F_r) = 0.9041 > e$ $X = 0.56$ interpolate $Y = 1.401$ Fig. 11-24

$P = X V F_r + Y F_a = 2647$ lbf Eq. 11.22a

$C_{DESIRED} = P (L_{10})^{1/3} = 2647 \text{ lbf} (100)^{1/3} = 12{,}286$ lbf

OK 6311 bearing $C = 12{,}900 \text{ lbf} > C_{DESIRED}$

helical gear with $\phi = 25°$ pressure angle and $\psi = 20°$ helix angle

$W_T = 2000$ lbf $W_R = W_R \tan \phi = 932.6$ lbf $W_A = W_T \tan \psi = 727.9$ lbf

force on bearing

$F_r = \sqrt{W_T^2 + W_R^2} = 2206$ lbf $F_a = W_A = 727.9$ lbf

desire $L_{10} = 500 \times 10^6$ rev inner race rotating

$P = X\,V\,F_r + Y\,F_a$ Eq. 11.22a

$V = 1$ Fig. 11-24

try $X = 1$ $Y = 0$ $P = F_r = 2206$ lbf

$L_{10} = (C / P)^3$ Eq. 11.20a

$C_{DESIRED} = P\,(L_{10})^{1/3} = 2206 \text{ lbf } (500)^{1/3} = 17{,}509$ lbf

try 6314 bearing $C = 18{,}000$ lbf $C_0 = 14{,}000$ lbf Fig. 11-23

$F_a / C_0 = 0.0520$ interpolate $e = 0.2543$ Fig. 11-24

$F_a / (V\,F_r) = 0.3230 > e$ $X = 0.56$ interpolate $Y = 1.750$ Fig. 11-24

$P = X\,V\,F_r + Y\,F_a = 2509$ lbf Eq. 11.22a

$C_{DESIRED} = P\,(L_{10})^{1/3} = 2509 \text{ lbf } (500)^{1/3} = 19{,}914$ lbf

try 6316 bearing $C = 21{,}200$ lbf $C_0 = 18{,}000$ lbf Fig. 11-23

$F_a / C_0 = 0.04044$ interpolate $e = 0.2378$ Fig. 11-24

$F_a / (V\,F_r) = 0.3230 > e$ $X = 0.56$ interpolate $Y = 1.866$ Fig. 11-24

$P = X\,V\,F_r + Y\,F_a = 2594$ lbf Eq. 11.22a

$C_{DESIRED} = P\,(L_{10})^{1/3} = 2594 \text{ lbf } (500)^{1/3} = 20{,}589$ lbf

OK 6316 bearing $C = 21{,}200 \text{ lbf} > C_{DESIRED}$