ME 481 – H13 **Name _________________________**

1) Develop a Working Model (WM) simulation of a simple pendulum as shown in Notes_07_03 with angular motion of ±10 degrees of swing. Use m = 0.46 lbm, a = 3.7 inches and J_G = 1.5 lbm.in^2. Provide MATLAB graphs of angular position, velocity and acceleration as a function of time for at least 5 oscillations. Attach a screen shot of your mechanism. Determine period of oscillation from your graphs and calculate period of oscillation using Notes_07_03.

τ_{WM} 0.686 sec τ_{CALC} 0.6845 sec

τ_{WM} measured between successive peaks in MATLAB plot

2) Produce a MATLAB phase plane plot of angular velocity as a function of angular position. Discuss the shape of this plot.

ellipse – repeated again and again

3) Provide a MATLAB FFT plot of frequency content for angular acceleration of your pendulum simulation using angular motion of ±10 degrees. Determine natural frequency and the period of oscillation from the FFT. Show your work.

f_N 1.356 Hz τ_{FFT} 0.7375 sec very coarse spectral resolution

4) Repeat part 3) for ±80 degrees of pendulum motion. Discuss the difference in the FFT plots.

f_N 1.212 Hz τ_{FFT} 0.8251 sec

two more smaller peaks at third and fifth harmonic
different spectral resolution due to different number of samples

EXTRA CREDIT

Provide a MATLAB plot of frequency content for piston acceleration from H03. Discuss where you would expect peaks in this plot.

peaks at first, second and fourth harmonics
WM noise floor about 1e-3 of max values
noise floor for exact geometric equations about 1e-6 of max values

$$J_G + m a^2 = \frac{m g a \tau^2}{4\pi^2}$$

$$\tau = \sqrt{\frac{4\pi^2 \left(J_G + m a^2\right)}{m g a}} = \sqrt{\frac{4\pi^2 \left(1.5\ \text{lbm.in}^2 + (0.46\ \text{lbm})(3.7\ \text{in})^2\right)}{(0.46\ \text{lbm})(3.7\ \text{in})}\left(\frac{\text{sec}^2}{386\ \text{in}}\right)} = 0.6845\ \text{sec}$$

$f_N = 1 / \tau = 1.4609$ Hz

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```matlab
% test_fft.m - example use of FFT
% HJSIII - 20.04.06

clear

% read time domain data here
% must also define time step h

% alternately create synthetic signal for testing
% synthetic - 30 Hz sine, +/- 5 mm, 0.001 sec time step
h = 0.001;                                     % time step [sec]
f_synthetic = 30;                              % synthetic frequency [Hz]
x_max = 5;                                     % size synthetic signal [mm]
t = [ 0:(1999) ]' * h;                         % synthetic time [sec]
x = x_max * sin( 2 * pi * f_synthetic * t );  % synthetic signal [mm]
% bottom - creating synthetic signal

% synthetic square wave
%x = sign( x );

% find number of samples and sampling frequency
n = length( x );  % number of samples
fs = 1 / h;       % sampling frequency [Hz]

% FFT
% MATLAB FFT must be scaled by 2/n - DC component must be scaled scaled by 1/n
a = fft(x) * 2 / n;                     % complex number - units [mm]
a(1) = a(1) / 2;                        % offset at frequency of 0 Hz [mm]
amp = abs( a );                         % amplitude at each frequency [mm]
phase = angle( a ) * 180 / pi;          % phase angle [deg]
df = fs / n;                            % frequency resolution between spectral bands [Hz]
freq = [ 0:(n-1) ]' * df;               % all frequencies [Hz]

% find peaks and list
[ peaks, i_locations ] = findpeaks( amp, 'MinPeakHeight', 0.01 );  % ignore tiny values
disp( ' ' )
disp( ' freq [Hz] peak [mm]' )
disp( [ freq(i_locations) peaks ] )     % units [Hz] [mm]

% plot time domain, amplitude, phase
figure( 1 )
subplot( 3,1,1 )
  plot( t, x )
  xlabel( 'Time (sec)' )
  ylabel( 'Signal [mm]' )

subplot( 3,1,2 )
  plot( freq, amp )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Amplitude [mm]' )

subplot( 3,1,3 )
  plot( freq, phase )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Phase [deg]' )

% bottom - test_fft
```

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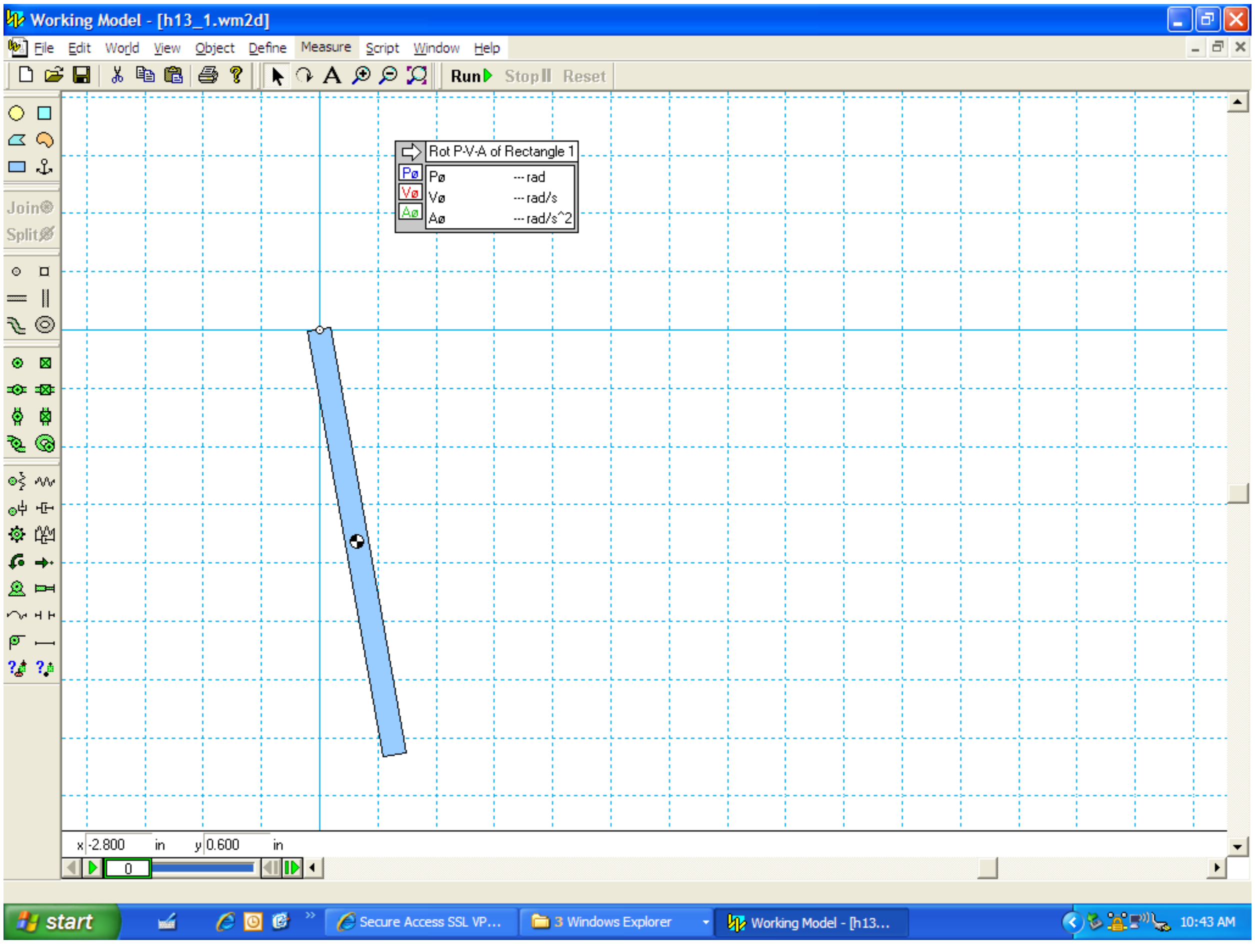
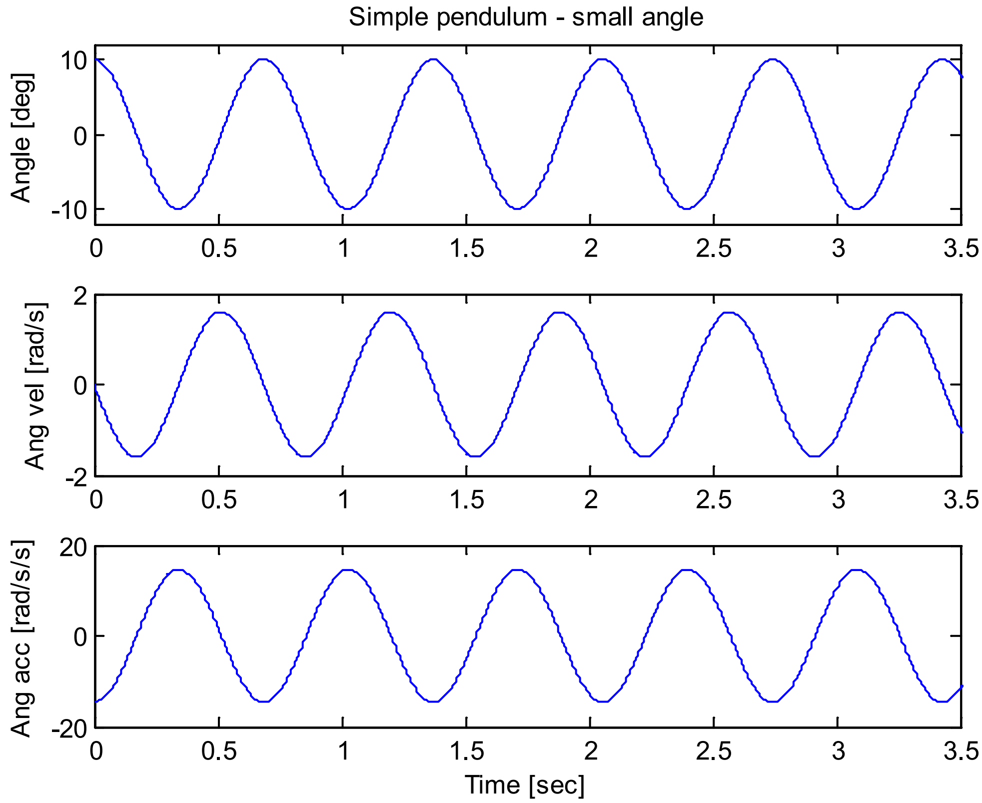
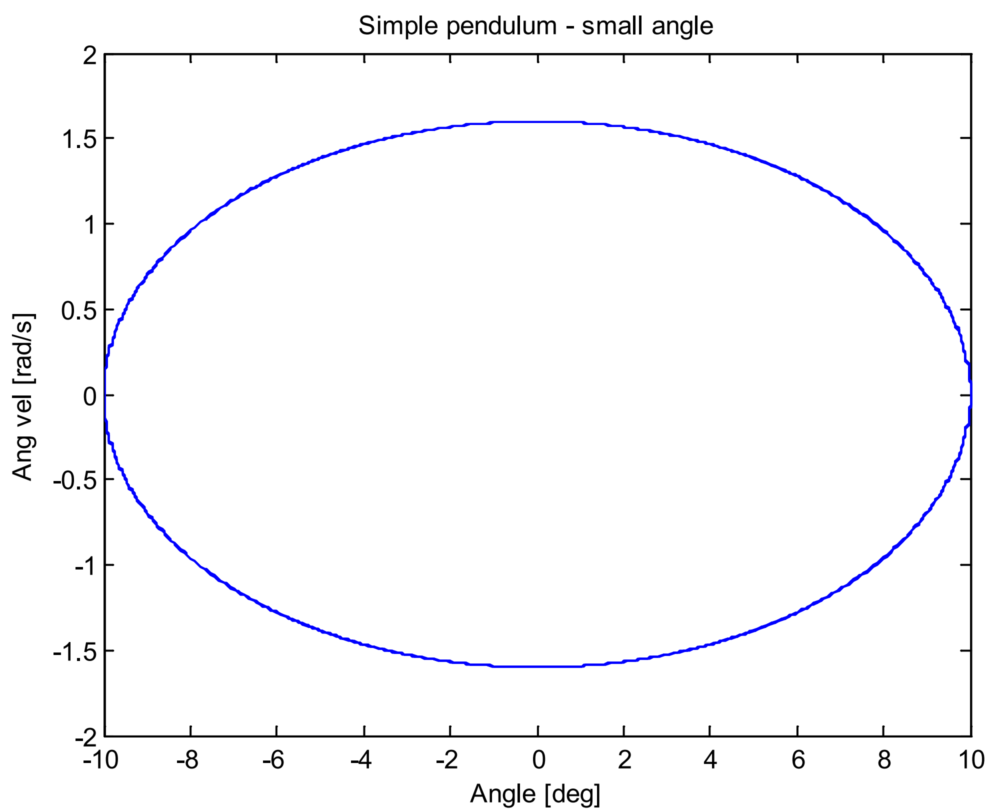



Simple pendulum - small angle
Angle [deg]
10
0
-10
Ang vel [rad/s]
2
0
-2
Ang acc [rad/s/s]
20
0
-20
0
0.5
1
1.5
2
2.5
3
3.5
Time [sec]


Simple pendulum - small angle
Ang vel [rad/s]
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-10
-8
-6
-4
-2
0
2
4
6
8
10
Angle [deg]

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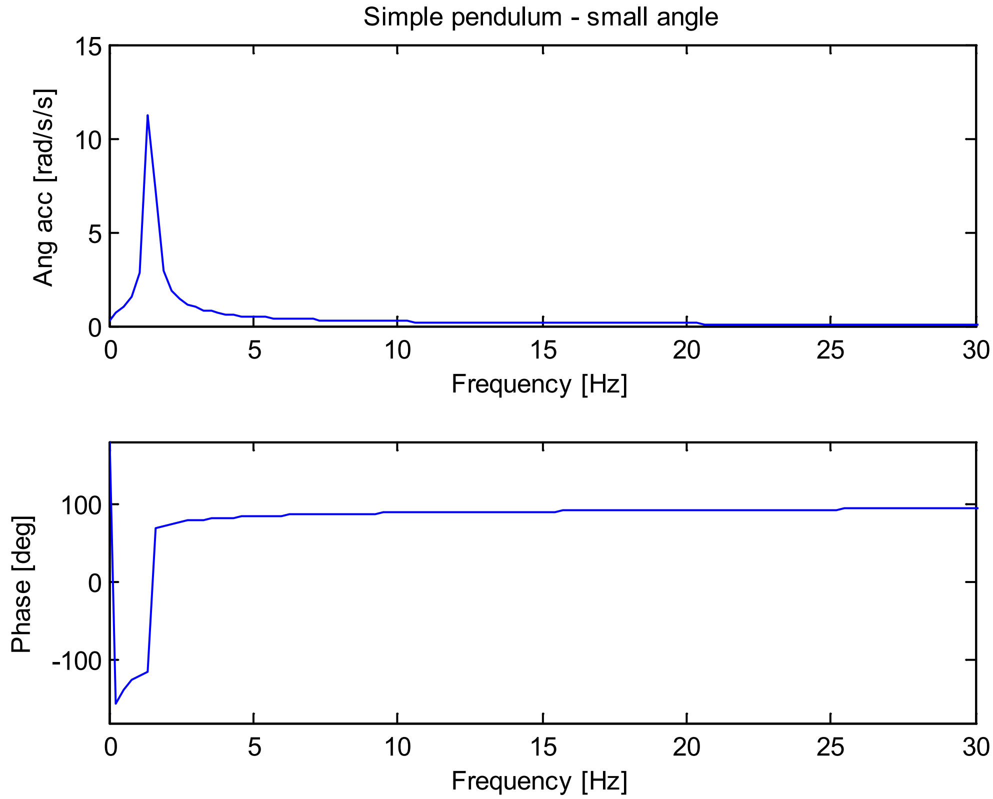

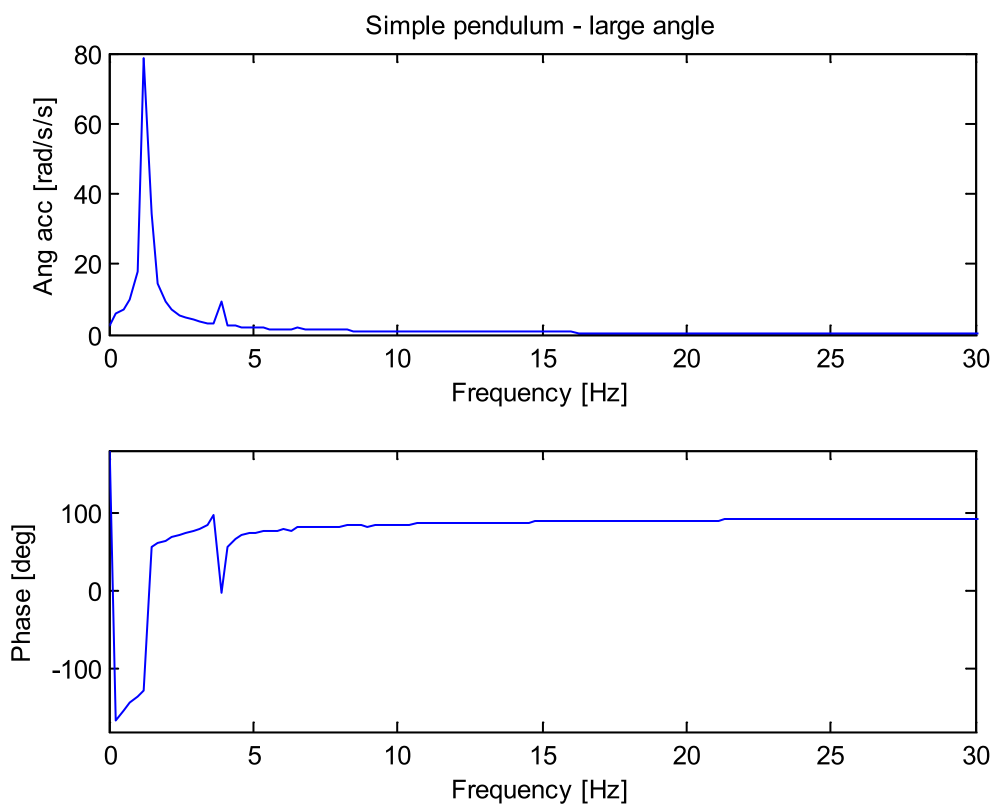

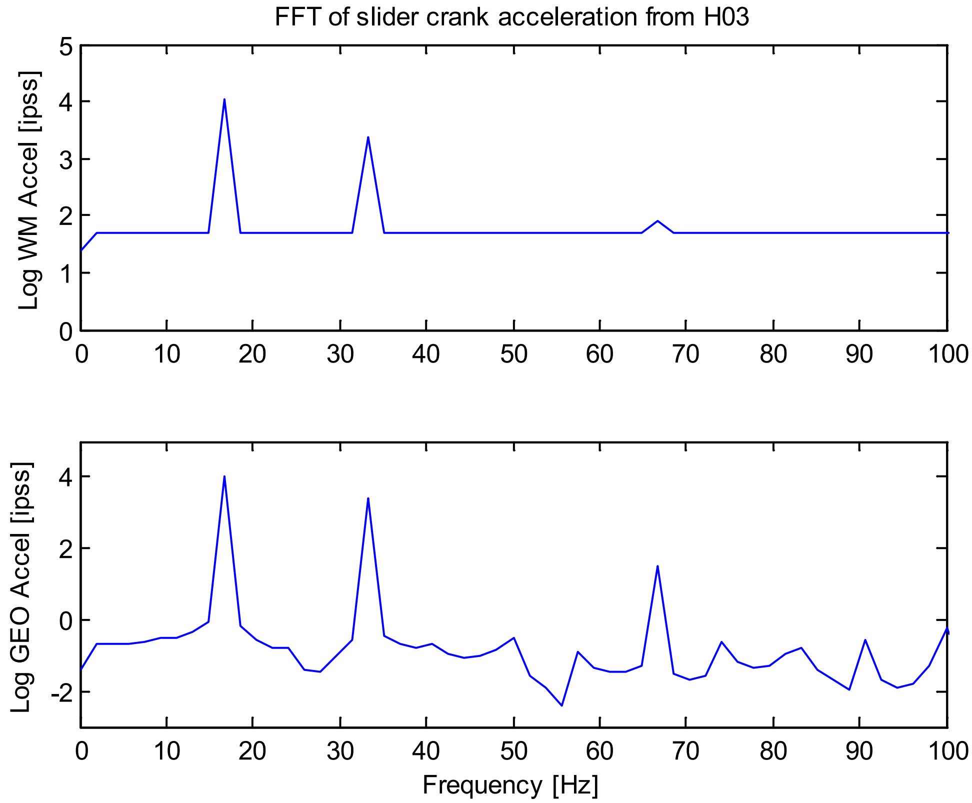
FFT of slider crank acceleration from H03
Log WM Accel [ipss]
Log GEO Accel [ipss]
Frequency [Hz]
0
10
20
30
40
50
60
70
80
90
100
0
1
2
3
4
5
-2
0
2
4

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```
% h13.m - ME 481 HW03 - read WM data for pendulum
% HJSIII, 13.04.06

% general constants
d2r = pi / 180;

% pendulum constants
m = 0.46;   % [lbm]
a = 3.7;    % [in]
JG = 1.5;   % [lbm.in.in]
g = 386;    % [ips2]

tau = sqrt( 4*pi*pi*( JG + m*a*a ) / ( m * g * a ) )
fn = 1 / tau

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% load WM data for pendulum with small angle
load h13_1_cut.txt;

t = h13_1_cut(:,1);
th = h13_1_cut(:,2);
phi_deg = th/d2r + 90;
phid = h13_1_cut(:,3);
phidd = h13_1_cut(:,4);

% plot small angle results
figure( 1 )
  clf
  subplot( 3, 1, 1 )
  plot( t,phi_deg,'b' )
  axis( [ 0 3.5  -12 12 ] )
  ylabel( 'Angle [deg]' )
  title( 'Simple pendulum - small angle' )

  subplot( 3, 1, 2 )
  plot( t,phid,'b' )
  axis( [ 0 3.5  -2 2 ] )
  ylabel( 'Ang vel [rad/s]' )

  subplot( 3, 1, 3 )
  plot( t,phidd,'b' )
  axis( [ 0 3.5  -20 20 ] )
  xlabel( 'Time [sec]' )
  ylabel( 'Ang acc [rad/s/s]' )

figure( 2 )
  clf
  plot( phi_deg, phid, 'b' )
  axis( [ -10 10  -2 2 ] )
  xlabel( 'Angle [deg]' )
  ylabel( 'Ang vel [rad/s]' )
  title( 'Simple pendulum - small angle' )

% time samples
h = t(2) - t(1);      % [sec]
fs = 1 / h;           % [Hz]
n = length( phidd );  % length of sample

% MATLAB FFT scaled by 2/n - DC component scaled by 1/n
a = fft( phidd ) * 2 / n;          % complex number
a(1) = a(1) / 2;                   % units [rad/s/s]
amp = abs( a );                    % units [rad/s/s]
phase = angle( a ) * 180 / pi;     % units [deg]
df = fs / n;                       % units [Hz]
freq = [ 0:(n-1) ]' * df;          % units [Hz]

% plot amplitude
figure( 3 )
  clf
  subplot( 2,1,1 )
  plot( freq, amp )              % units [Hz] [rad/s/s]
```

```
  axis( [ 0 30  0 15 ] )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Ang acc [rad/s/s]' )
  title( 'Simple pendulum - small angle' )

% plot phase
  subplot( 2,1,2 )
  plot( freq, phase )                    % units [Hz] [deg]
  axis( [ 0 30  -180 180 ] )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Phase [deg]' )

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% load WM data for pendulum with large angle
load h13_4_cut.txt;

t = h13_4_cut(:,1);
th = h13_4_cut(:,2);
phi_deg = th/d2r + 90;
phid = h13_4_cut(:,3);
phidd = h13_4_cut(:,4);

% time samples
h = t(2) - t(1);      % [sec]
fs = 1 / h;           % [Hz]
n = length( phidd );  % length of sample

% MATLAB FFT scaled by 2/n - DC component scaled by 1/n
a = fft( phidd ) * 2 / n;         % complex number
a(1) = a(1) / 2;                  % units [rad/s/s]
amp = abs( a );                   % units [rad/s/s]
phase = angle( a ) * 180 / pi;    % units [deg]
df = fs / n;                      % units [Hz]
freq = [ 0:(n-1) ]' * df;         % units [Hz]

% plot amplitude
figure( 4 )
  clf
  subplot( 2,1,1 )
  plot( freq, amp )              % units [Hz] [rad/s/s]
  axis( [ 0 30  0 80 ] )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Ang acc [rad/s/s]' )
  title( 'Simple pendulum - large angle' )

% plot phase
  subplot( 2,1,2 )
  plot( freq, phase )                    % units [Hz] [deg]
  axis( [ 0 30  -180 180 ] )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Phase [deg]' )

%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%
% load WM data for slider crank from H03
load h03_cut.txt;

t = h03_cut(:,1);
th = h03_cut(:,2);
sWM = h03_cut(:,4);
vWM = h03_cut(:,5);
aWM = h03_cut(:,6);

% time samples
h = t(2) - t(1);      % [sec]
fs = 1 / h;
n = length( aWM );

% MATLAB FFT scaled by 2/n - DC component scaled by 1/n
a = fft( aWM ) * 2 / n;                % complex number
a(1) = a(1) / 2;                       % units [ipss]
amp = abs( a );                        % units [ipss]
```

```
phase = angle( a ) * 180 / pi;          % units [deg]
df = fs / n;                            % units [Hz]
freq = [ 0:(n-1) ]' * df;               % units [Hz]

% plot WM
figure( 5 )
  clf
  subplot( 2, 1, 1 )
  plot( freq, log10(amp) )              % units [Hz] [ipss]
  axis( [ 0  100  0  5 ] )
%  xlabel( 'Frequency [Hz]' )
  ylabel( 'Log WM Accel [ipss]' )
  title( 'FFT of slider crank acceleration from H03' )

% slider crank mechanism constants
R = 1.97 / 2;         % crank length [inch]
L = 4.33;             % conn-rod length [inch]
w2 = 1000 *2*pi /60;  % crank speed [rad/sec]
thdot = w2;
thddot = 0;

% geometric equations
phi = asin(  R*sin(th) / L );
sGEO = R*cos(th) + L*cos(phi);

phidot = R*thdot*cos(th) /L ./cos(phi);
vGEO = -R*thdot*sin(th) - L*phidot.*sin(phi);

phiddot = ( R*thddot*cos(th) -R*thdot*thdot*sin(th) +L.*phidot.*phidot.*sin(phi) ) ...
                 /L ./cos(phi);
aGEO = -R*thddot*sin(th) -R*thdot*thdot*cos(th) -L*phiddot.*sin(phi) ...
                -L*phidot.*phidot.*cos(phi);

% MATLAB FFT scaled by 2/n - DC component scaled by 1/n
a = fft( aGEO ) * 2 / n;                 % complex number
a(1) = a(1) / 2;                        % units [ipss]
amp = abs( a );                         % units [ipss]
phase = angle( a ) * 180 / pi;          % units [deg]
df = fs / n;                            % units [Hz]
freq = [ 0:(n-1) ]' * df;               % units [Hz]

% plot geometric equation
subplot( 2, 1, 2 )
  plot( freq, log10(amp) )              % units [Hz] [ipss]
  axis( [ 0  100  -3  5 ] )
  xlabel( 'Frequency [Hz]' )
  ylabel( 'Log GEO Accel [ipss]' )

% bottom of h13
```