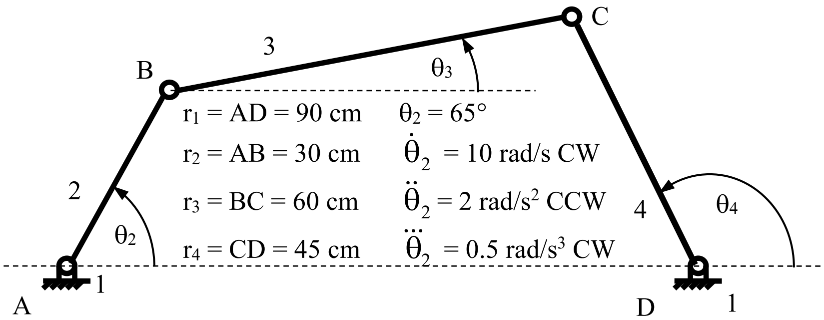
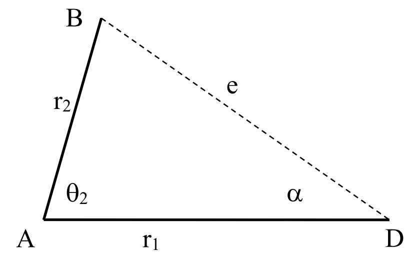
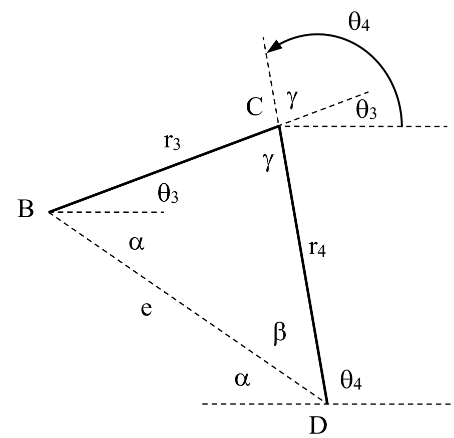
Geometric Kinematics for Four Bar





POSITION ANALYSIS



$$e^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2$$
$$e = \sqrt{r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2}$$

$$\sin\theta_2 / e = \sin\alpha / r_2$$
$$\sin\alpha = r_2 \sin\theta_2 / e$$

$$e^2 = r_3^2 + r_4^2 - 2r_3 r_4 \cos\gamma$$
$$\cos\gamma = \left(r_3^2 + r_4^2 - e^2\right) / 2r_3 r_4$$

$$\sin\gamma / e = \sin\beta / r_3$$
$$\sin\beta = r_3 \sin\gamma / e$$

$$\alpha + \beta + \theta_4 = 180°$$
$$\theta_4 = 180° - \alpha - \beta$$

$$\theta_4 = \theta_3 + \gamma$$
$$\theta_3 = \theta_4 - \gamma$$

VELOCITY ANALYSIS

$$d/dt\left(e^2 = r_1^2 + r_2^2 - 2r_1 r_2 \cos\theta_2\right)$$
$$2e\dot{e} = 2r_1 r_2 \dot{\theta}_2 \sin\theta_2$$
$$\dot{e} = r_1 r_2 \dot{\theta}_2 \sin\theta_2 / e$$

$$d/dt\left(r_2 \sin\theta_2 = e \sin\alpha\right)$$
$$r_2 \dot{\theta}_2 \cos\theta_2 = \dot{e}\sin\alpha + e\dot{\alpha}\cos\alpha$$
$$\dot{\alpha} = \left(r_2 \dot{\theta}_2 \cos\theta_2 - \dot{e}\sin\alpha\right) / e\cos\alpha$$

$$d/dt\left(e^2 = r_3^2 + r_4^2 - 2r_3r_4\cos\gamma\right)$$
$$2e\dot{e} = 2r_3r_4\dot{\gamma}\sin\gamma$$
$$\dot{\gamma} = e\dot{e}/r_3r_4\sin\gamma$$

$$d/dt\left(r_3\sin\gamma = e\sin\beta\right)$$
$$r_3\dot{\gamma}\cos\gamma = \dot{e}\sin\beta + e\dot{\beta}\cos\beta$$
$$\dot{\beta} = \left(r_3\dot{\gamma}\cos\gamma - \dot{e}\sin\beta\right)/e\cos\beta$$

$$d/dt\left(\theta_4 = 180° - \alpha - \beta\right)$$
$$\dot{\theta}_4 = -\dot{\alpha} - \dot{\beta}$$

$$d/dt\left(\theta_3 = \theta_4 - \gamma\right)$$
$$\dot{\theta}_3 = \dot{\theta}_4 - \dot{\gamma}$$

ACCELERATION ANALYSIS

$$d/dt\left(e\dot{e} = r_1r_2\dot{\theta}_2\sin\theta_2\right)$$
$$\dot{e}^2 + e\ddot{e} = r_1r_2\ddot{\theta}_2\sin\theta_2 + r_1r_2\dot{\theta}_2^2\cos\theta_2$$
$$\ddot{e} = \left(r_1r_2\ddot{\theta}_2\sin\theta_2 + r_1r_2\dot{\theta}_2^2\cos\theta_2 - \dot{e}^2\right)/e$$

$$r_2\dot{\theta}_2\cos\theta_2 = \dot{e}\sin\alpha + e\dot{\alpha}\cos\alpha$$
$$r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 = \ddot{e}\sin\alpha + 2\dot{e}\dot{\alpha}\cos\alpha + e\ddot{\alpha}\cos\alpha - e\dot{\alpha}^2\sin\alpha$$
$$\ddot{\alpha} = \left(r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 - \ddot{e}\sin\alpha - 2\dot{e}\dot{\alpha}\cos\alpha + e\dot{\alpha}^2\sin\alpha\right)/e\cos\alpha$$

$$d/dt\left(e\dot{e} = r_3r_4\dot{\gamma}\sin\gamma\right)$$
$$\dot{e}^2 + e\ddot{e} = r_3r_4\ddot{\gamma}\sin\gamma + r_3r_4\dot{\gamma}^2\cos\gamma$$
$$\ddot{\gamma} = \left(\dot{e}^2 + e\ddot{e} - r_3r_4\dot{\gamma}^2\cos\gamma\right)/r_3r_4\sin\gamma$$

$$d/dt\left(r_3\dot{\gamma}\cos\gamma = \dot{e}\sin\beta + e\dot{\beta}\cos\beta\right)$$
$$r_3\ddot{\gamma}\cos\gamma - r_3\dot{\gamma}^2\sin\gamma = \ddot{e}\sin\beta + 2\dot{e}\dot{\beta}\cos\beta + e\ddot{\beta}\cos\beta - e\dot{\beta}^2\sin\beta$$
$$\ddot{\beta} = \left(r_3\ddot{\gamma}\cos\gamma - r_3\dot{\gamma}^2\sin\gamma - \ddot{e}\sin\beta - 2\dot{e}\dot{\beta}\cos\beta + e\dot{\beta}^2\sin\beta\right)/e\cos\beta$$

$$d/dt\left(\dot{\theta}_4 = -\dot{\alpha} - \dot{\beta}\right)$$
$$\ddot{\theta}_4 = -\ddot{\alpha} - \ddot{\beta}$$

$$d/dt\left(\dot{\theta}_3 = \dot{\theta}_4 - \dot{\gamma}\right)$$
$$\ddot{\theta}_3 = \ddot{\theta}_4 - \ddot{\gamma}$$

JERK ANALYSIS

$$d/dt\left(\dot{e}^2 + e\ddot{e} = r_1 r_2 \ddot{\theta}_2 \sin\theta_2 + r_1 r_2 \dot{\theta}_2^2 \cos\theta_2\right)$$
$$3\dot{e}\ddot{e} + e\dddot{e} = r_1 r_2 \dddot{\theta}_2 \sin\theta_2 + 3 r_1 r_2 \dot{\theta}_2 \ddot{\theta}_2 \cos\theta_2 - r_1 r_2 \dot{\theta}_2^3 \sin\theta_2$$
$$\dddot{e} = \left(r_1 r_2 \dddot{\theta}_2 \sin\theta_2 + 3 r_1 r_2 \dot{\theta}_2 \ddot{\theta}_2 \cos\theta_2 - r_1 r_2 \dot{\theta}_2^3 \sin\theta_2 - 3\dot{e}\ddot{e}\right)/ e$$

$$d/dt\left(r_2 \ddot{\theta}_2 \cos\theta_2 - r_2 \dot{\theta}_2^2 \sin\theta_2 = \ddot{e}\sin\alpha + 2\dot{e}\dot{\alpha}\cos\alpha + e\ddot{\alpha}\cos\alpha - e\dot{\alpha}^2 \sin\alpha\right)$$
$$r_2 \dddot{\theta}_2 \cos\theta_2 - 3 r_2 \dot{\theta}_2 \ddot{\theta}_2 \sin\theta_2 - r_2 \dot{\theta}_2^3 \cos\theta_2 = \dddot{e}\sin\alpha + 3\ddot{e}\dot{\alpha}\cos\alpha + 3\dot{e}\ddot{\alpha}\cos\alpha$$
$$- 3\dot{e}\dot{\alpha}^2 \sin\alpha + e\dddot{\alpha}\cos\alpha - 3e\dot{\alpha}\ddot{\alpha}\sin\alpha - e\dot{\alpha}^3 \cos\alpha$$
$$\dddot{\alpha} = \begin{pmatrix} r_2 \dddot{\theta}_2 \cos\theta_2 - 3 r_2 \dot{\theta}_2 \ddot{\theta}_2 \sin\theta_2 - r_2 \dot{\theta}_2^3 \cos\theta_2 + 3e\dot{\alpha}\ddot{\alpha}\sin\alpha \\ + e\dot{\alpha}^3 \cos\alpha - 3\ddot{e}\dot{\alpha}\cos\alpha - 3\dot{e}\ddot{\alpha}\cos\alpha + 3e\dot{\alpha}^2 \sin\alpha - \dddot{e}\sin\alpha \end{pmatrix} / e\cos\alpha$$

$$d/dt\left(\dot{e}^2 + e\ddot{e} = r_3 r_4 \ddot{\gamma}\sin\gamma + r_3 r_4 \dot{\gamma}^2 \cos\gamma\right)$$
$$3\dot{e}\ddot{e} + e\dddot{e} = r_3 r_4 \dddot{\gamma}\sin\gamma + 3 r_3 r_4 \dot{\gamma}\ddot{\gamma}\cos\gamma - r_3 r_4 \dot{\gamma}^3 \sin\gamma$$
$$\dddot{\gamma} = \left(3\dot{e}\ddot{e} + e\dddot{e} - 3 r_3 r_4 \dot{\gamma}\ddot{\gamma}\cos\gamma + r_3 r_4 \dot{\gamma}^3 \sin\gamma\right)/ r_3 r_4 \sin\gamma$$

$$d/dt\left(r_3 \ddot{\gamma}\cos\gamma - r_3 \dot{\gamma}^2 \sin\gamma = \ddot{e}\sin\beta + 2\dot{e}\dot{\beta}\cos\beta + e\ddot{\beta}\cos\beta - e\dot{\beta}^2 \sin\beta\right)$$
$$r_3 \dddot{\gamma}\cos\gamma - 3 r_3 \dot{\gamma}\ddot{\gamma}\sin\gamma - r_3 \dot{\gamma}^3 \cos\gamma = \dddot{e}\sin\beta + 3\ddot{e}\dot{\beta}\cos\beta + 3\dot{e}\ddot{\beta}\cos\beta$$
$$- 3\dot{e}\dot{\beta}^2 \sin\beta + e\dddot{\beta}\cos\beta - 3e\dot{\beta}\ddot{\beta}\sin\beta - e\dot{\beta}^3 \cos\beta$$
$$\dddot{\beta} = \begin{pmatrix} r_3 \dddot{\gamma}\cos\gamma - 3 r_3 \dot{\gamma}\ddot{\gamma}\sin\gamma - r_3 \dot{\gamma}^3 \cos\gamma - \dddot{e}\sin\beta - 3\ddot{e}\dot{\beta}\cos\beta \\ - 3\dot{e}\ddot{\beta}\cos\beta + 3\dot{e}\dot{\beta}^2 \sin\beta + 3e\dot{\beta}\ddot{\beta}\sin\beta + e\dot{\beta}^3 \cos\beta \end{pmatrix} / e\cos\beta$$

$$d/dt\left(\ddot{\theta}_4 = -\ddot{\alpha} - \ddot{\beta}\right)$$
$$\dddot{\theta}_4 = -\dddot{\alpha} - \dddot{\beta}$$

$$d/dt\left(\ddot{\theta}_3 = \ddot{\theta}_4 - \ddot{\gamma}\right)$$
$$\dddot{\theta}_3 = \dddot{\theta}_4 - \dddot{\gamma}$$

SNAP ANALYSIS

$$d/dt\left(3\dot{e}\ddot{e} + e\dddot{e} = r_1 r_2 \dddot{\theta}_2 \sin\theta_2 + 3 r_1 r_2 \dot{\theta}_2 \ddot{\theta}_2 \cos\theta_2 - r_1 r_2 \dot{\theta}_2^3 \sin\theta_2\right)$$

$$3\ddot{e}^2 + 4\dot{e}\dddot{e} + e\ddddot{e} = r_1 r_2 \ddddot{\theta}_2 \sin\theta_2 + 4 r_1 r_2 \dddot{\theta}_2 \dot{\theta}_2 \cos\theta_2 + 3 r_1 r_2 \ddot{\theta}_2^2 \cos\theta_2 - 6 r_1 r_2 \dot{\theta}_2^2 \ddot{\theta}_2 \sin\theta_2 - r_1 r_2 \dot{\theta}_2^4 \cos\theta_2$$

$$\ddddot{e} = \left(r_1 r_2 \ddddot{\theta}_2 \sin\theta_2 + 4 r_1 r_2 \dddot{\theta}_2 \dot{\theta}_2 \cos\theta_2 + 3 r_1 r_2 \ddot{\theta}_2^2 \cos\theta_2 - 6 r_1 r_2 \dot{\theta}_2^2 \ddot{\theta}_2 \sin\theta_2 - r_1 r_2 \dot{\theta}_2^4 \cos\theta_2 - 3\ddot{e}^2 - 4\dot{e}\dddot{e}\right)/ e$$

$$\mathrm{d}/\mathrm{dt}\begin{pmatrix} r_2\dddot{\theta}_2\cos\theta_2 - 3r_2\dot{\theta}_2\ddot{\theta}_2\sin\theta_2 - r_2\dot{\theta}_2^3\cos\theta_2 = \dddot{e}\sin\alpha + 3\ddot{e}\dot{\alpha}\cos\alpha + 3\dot{e}\ddot{\alpha}\cos\alpha \\ -3\dot{e}\dot{\alpha}^2\sin\alpha + e\dddot{\alpha}\cos\alpha - 3e\dot{\alpha}\ddot{\alpha}\sin\alpha - e\dot{\alpha}^3\cos\alpha \end{pmatrix}$$

$$\begin{aligned}
&r_2\ddddot{\theta}_2\cos\theta_2 - r_2\dddot{\theta}_2\dot{\theta}_2\sin\theta_2 - 3r_2\ddot{\theta}_2^2\sin\theta_2 - 3r_2\dot{\theta}_2\dddot{\theta}_2\sin\theta_2 - 3r_2\dot{\theta}_2^2\ddot{\theta}_2\cos\theta_2 - 3r_2\dot{\theta}_2^2\ddot{\theta}_2\cos\theta_2 + r_2\dot{\theta}_2^4\sin\theta_2 \\
&= \ddddot{e}\sin\alpha + \dddot{e}\dot{\alpha}\cos\alpha + 3\dddot{e}\dot{\alpha}\cos\alpha + 3\ddot{e}\ddot{\alpha}\cos\alpha - 3\ddot{e}\dot{\alpha}^2\sin\alpha + 3\ddot{e}\ddot{\alpha}\cos\alpha + 3\dot{e}\dddot{\alpha}\cos\alpha - 3\dot{e}\dot{\alpha}\ddot{\alpha}\sin\alpha \\
&-3\ddot{e}\dot{\alpha}^2\sin\alpha - 6\dot{e}\dot{\alpha}\ddot{\alpha}\sin\alpha - 3\dot{e}\dot{\alpha}^3\cos\alpha + \dot{e}\dddot{\alpha}\cos\alpha + e\ddddot{\alpha}\cos\alpha - e\dot{\alpha}\dddot{\alpha}\sin\alpha \\
&-3\dot{e}\dot{\alpha}\ddot{\alpha}\sin\alpha - 3e\ddot{\alpha}^2\sin\alpha - 3e\dot{\alpha}\dddot{\alpha}\sin\alpha - 3e\dot{\alpha}^2\ddot{\alpha}\cos\alpha - \dot{e}\dot{\alpha}^3\cos\alpha - 3e\dot{\alpha}^2\ddot{\alpha}\cos\alpha + e\dot{\alpha}^4\sin\alpha
\end{aligned}$$

$$\begin{aligned}
&r_2\ddddot{\theta}_2\cos\theta_2 - 4r_2\dot{\theta}_2\dddot{\theta}_2\sin\theta_2 - 3r_2\ddot{\theta}_2^2\sin\theta_2 - 6r_2\dot{\theta}_2^2\ddot{\theta}_2\cos\theta_2 + r_2\dot{\theta}_2^4\sin\theta_2 \\
&= \ddddot{e}\sin\alpha + 4\dddot{e}\dot{\alpha}\cos\alpha + 6\ddot{e}\ddot{\alpha}\cos\alpha - 6\ddot{e}\dot{\alpha}^2\sin\alpha + 4\dot{e}\dddot{\alpha}\cos\alpha - 12\dot{e}\dot{\alpha}\ddot{\alpha}\sin\alpha - 4\dot{e}\dot{\alpha}^3\cos\alpha \\
&+ e\ddddot{\alpha}\cos\alpha - 4e\dot{\alpha}\dddot{\alpha}\sin\alpha - 3e\ddot{\alpha}^2\sin\alpha - 6e\dot{\alpha}^2\ddot{\alpha}\cos\alpha + e\dot{\alpha}^4\sin\alpha
\end{aligned}$$

$$\ddddot{\alpha} = \begin{pmatrix} r_2\ddddot{\theta}_2\cos\theta_2 - 4r_2\dot{\theta}_2\dddot{\theta}_2\sin\theta_2 - 3r_2\ddot{\theta}_2^2\sin\theta_2 - 6r_2\dot{\theta}_2^2\ddot{\theta}_2\cos\theta_2 + r_2\dot{\theta}_2^4\sin\theta_2 \\ +4e\dot{\alpha}\dddot{\alpha}\sin\alpha + 3e\ddot{\alpha}^2\sin\alpha + 6e\dot{\alpha}^2\ddot{\alpha}\cos\alpha - e\dot{\alpha}^4\sin\alpha - \ddddot{e}\sin\alpha \\ -4\dddot{e}\dot{\alpha}\cos\alpha - 6\ddot{e}\ddot{\alpha}\cos\alpha + 6\ddot{e}\dot{\alpha}^2\sin\alpha - 4\dot{e}\dddot{\alpha}\cos\alpha + 12\dot{e}\dot{\alpha}\ddot{\alpha}\sin\alpha + 4\dot{e}\dot{\alpha}^3\cos\alpha \end{pmatrix} / e\cos\alpha$$

$$\mathrm{d}/\mathrm{dt}\left(3\dot{e}\ddot{e} + e\dddot{e} = r_3r_4\dddot{\gamma}\sin\gamma + 3r_3r_4\dot{\gamma}\ddot{\gamma}\cos\gamma - r_3r_4\dot{\gamma}^3\sin\gamma\right)$$

$$\begin{aligned}
&3\ddot{e}^2 + 3\dot{e}\dddot{e} + \dot{e}\dddot{e} + e\ddddot{e} = r_3r_4\ddddot{\gamma}\sin\gamma + r_3r_4\dot{\gamma}\dddot{\gamma}\cos\gamma \\
&+ 3r_3r_4\ddot{\gamma}^2\cos\gamma + 3r_3r_4\dot{\gamma}\dddot{\gamma}\cos\gamma - 3r_3r_4\dot{\gamma}^2\ddot{\gamma}\sin\gamma - 3r_3r_4\dot{\gamma}^2\ddot{\gamma}\sin\gamma - r_3r_4\dot{\gamma}^4\cos\gamma
\end{aligned}$$

$$3\ddot{e}^2 + 4\dot{e}\dddot{e} + e\ddddot{e} = r_3r_4\ddddot{\gamma}\sin\gamma + 4r_3r_4\dot{\gamma}\ \dddot{\gamma}\cos\gamma + 3r_3r_4\ddot{\gamma}^2\cos\gamma - 6r_3r_4\dot{\gamma}^2\ddot{\gamma}\sin\gamma - r_3r_4\dot{\gamma}^4\cos\gamma$$

$$\ddddot{\gamma} = \left(3\ddot{e}^2 + 4\dot{e}\dddot{e} + e\ddddot{e} - 4r_3r_4\dot{\gamma}\ \dddot{\gamma}\cos\gamma - 3r_3r_4\ddot{\gamma}^2\cos\gamma + 6r_3r_4\dot{\gamma}^2\ddot{\gamma}\sin\gamma + r_3r_4\dot{\gamma}^4\cos\gamma\right) / r_3r_4\sin\gamma$$

$$\mathrm{d}/\mathrm{dt}\begin{pmatrix} r_3\dddot{\gamma}\cos\gamma - 3r_3\dot{\gamma}\ddot{\gamma}\sin\gamma - r_3\dot{\gamma}^3\cos\gamma = \dddot{e}\sin\beta + 3\ddot{e}\dot{\beta}\cos\beta + 3\dot{e}\ddot{\beta}\cos\beta \\ -3\dot{e}\dot{\beta}^2\sin\beta + e\dddot{\beta}\cos\beta - 3e\dot{\beta}\ddot{\beta}\sin\beta - e\dot{\beta}^3\cos\beta \end{pmatrix}$$

$$\begin{aligned}
&r_3\ddddot{\gamma}\cos\gamma - r_3\dot{\gamma}\ \dddot{\gamma}\sin\gamma - 3r_3\ddot{\gamma}^2\sin\gamma - 3r_3\dot{\gamma}\ \dddot{\gamma}\sin\gamma - 3r_3\dot{\gamma}^2\ \ddot{\gamma}\cos\gamma - 3r_3\dot{\gamma}^2\ddot{\gamma}\cos\gamma + r_3\dot{\gamma}^4\sin\gamma \\
&= \ddddot{e}\sin\beta + \dddot{e}\dot{\beta}\cos\beta + 3\dddot{e}\dot{\beta}\cos\beta + 3\ddot{e}\ddot{\beta}\cos\beta - 3\ddot{e}\dot{\beta}^2\sin\beta + 3\ddot{e}\ddot{\beta}\cos\beta + 3\dot{e}\dddot{\beta}\cos\beta - 3\dot{e}\dot{\beta}\ddot{\beta}\sin\beta \\
&-3\ddot{e}\dot{\beta}^2\sin\beta - 6\dot{e}\dot{\beta}\ddot{\beta}\sin\beta - 3\dot{e}\dot{\beta}^3\cos\beta + \dot{e}\dddot{\beta}\cos\beta + e\ddddot{\beta}\cos\beta - e\dot{\beta}\dddot{\beta}\sin\beta \\
&-3\dot{e}\dot{\beta}\ddot{\beta}\sin\beta - 3e\ddot{\beta}^2\sin\beta - 3e\dot{\beta}\dddot{\beta}\sin\beta - 3e\dot{\beta}^2\ddot{\beta}\cos\beta - \dot{e}\dot{\beta}^3\cos\beta - 3e\dot{\beta}^2\ddot{\beta}\cos\beta + e\dot{\beta}^4\sin\beta
\end{aligned}$$

$$
\begin{aligned}
& r_3\ddddot{\gamma}\cos\gamma - 4r_3\dot{\gamma}\;\dddot{\gamma}\sin\gamma - 3r_3\ddot{\gamma}^2\sin\gamma - 6r_3\dot{\gamma}^2\;\ddot{\gamma}\cos\gamma + r_3\dot{\gamma}^4\sin\gamma \\
& = \ddddot{e}\sin\beta + 4\dddot{e}\dot{\beta}\cos\beta + 6\ddot{e}\ddot{\beta}\cos\beta - 6\ddot{e}\dot{\beta}^2\sin\beta + 4\dot{e}\dddot{\beta}\cos\beta - 12\dot{e}\dot{\beta}\ddot{\beta}\sin\beta - 4\dot{e}\dot{\beta}^3\cos\beta \\
& + e\ddddot{\beta}\cos\beta - 4e\dot{\beta}\dddot{\beta}\sin\beta - 3e\ddot{\beta}^2\sin\beta - 6e\dot{\beta}^2\ddot{\beta}\cos\beta + e\dot{\beta}^4\sin\beta
\end{aligned}
$$

$$
\ddddot{\beta} = \begin{pmatrix} r_3\ddddot{\gamma}\cos\gamma - 4r_3\dot{\gamma}\;\dddot{\gamma}\sin\gamma - 3r_3\ddot{\gamma}^2\sin\gamma - 6r_3\dot{\gamma}^2\;\ddot{\gamma}\cos\gamma + r_3\dot{\gamma}^4\sin\gamma \\ + 4e\dot{\beta}\dddot{\beta}\sin\beta + 3e\ddot{\beta}^2\sin\beta + 6e\dot{\beta}^2\ddot{\beta}\cos\beta - e\dot{\beta}^4\sin\beta - \ddddot{e}\sin\beta \\ - 4\dddot{e}\dot{\beta}\cos\beta - 6\ddot{e}\ddot{\beta}\cos\beta + 6\ddot{e}\dot{\beta}^2\sin\beta - 4\dot{e}\dddot{\beta}\cos\beta + 12\dot{e}\dot{\beta}\ddot{\beta}\sin\beta + 4\dot{e}\dot{\beta}^3\cos\beta \end{pmatrix} / e\cos\beta
$$

$$
d/dt\left(\dddot{\theta}_4 = -\dddot{\alpha} - \dddot{\beta}\right)
$$

$$
\ddddot{\theta}_4 = -\ddddot{\alpha} - \ddddot{\beta}
$$

$$
d/dt\left(\dddot{\theta}_3 = \dddot{\theta}_4 - \dddot{\gamma}\right)
$$

$$
\ddddot{\theta}_3 = \ddddot{\theta}_4 - \ddddot{\gamma}
$$

SAMPLE VALUES FOR FOUR BAR

$\theta_2 = 65°$	$\dot{\theta}_2$ = -10 rad/s	$\ddot{\theta}_2$ = +2 rad/s^2	$\dddot{\theta}_2$ = -0.5 rad/s^3
e = 81.9626 cm	$\dot{e}$ = -298.5547 cps	$\ddot{e}$ = +364.3867 cps^2	$\dddot{e}$ = +32987 cm/s^3
$\alpha = 19.3737°$	$\dot{\alpha}$ = -0.3588 rad/s	$\ddot{\alpha}$ = -38.9682 rad/s^2	$\dddot{\alpha}$ = -363.3719 rad/s^3
$\gamma = 101.6763°$	$\dot{\gamma}$ = -9.2546 rad/s	$\ddot{\gamma}$ = +62.7055 rad/s^2	$\dddot{\gamma}$ = -253.3073 rad/s^3
$\beta = 45.7986°$	$\dot{\beta}$ = +5.7122 rad/s	$\ddot{\beta}$ = -30.8000 rad/s^2	$\dddot{\beta}$ = +125.9668 rad/s^3
$\theta_4 = 114.8278°$	$\dot{\theta}_4$ = -5.3533 rad/s	$\ddot{\theta}_4$ = +69.7682 rad/s^2	$\dddot{\theta}_4$ = +237.4051 rad/s^3
$\theta_3 = 13.1515°$	$\dot{\theta}_3$ = +3.9013 rad/s	$\ddot{\theta}_3$ = +7.0627 rad/s^2	$\dddot{\theta}_3$ = +490.7125 rad/s^3

MATLAB code available in Notes_03_02

Freudenstein's Equations for Four Bar

horizontal and vertical components

$$r_2 \cos\theta_2 + r_3 \cos\theta_3 = r_4 \cos\theta_4 + r_1 \qquad r_2 \sin\theta_2 + r_3 \sin\theta_3 = r_4 \sin\theta_4$$

rearrange

$$r_3 \cos\theta_3 = r_4 \cos\theta_4 - r_2 \cos\theta_2 + r_1 \qquad r_3 \sin\theta_3 = r_4 \sin\theta_4 - r_2 \sin\theta_2$$

square both equations

$$r_3^2 \cos^2\theta_3 = r_4^2 \cos^2\theta_4 + r_2^2 \cos^2\theta_2 + r_1^2 - 2r_2r_4 \cos\theta_2 \cos\theta_4 + 2r_1r_4 \cos\theta_4 - 2r_1r_2 \cos\theta_2$$

$$r_3^2 \sin^2\theta_3 = r_4^2 \sin^2\theta_4 + r_2^2 \sin^2\theta_2 - 2r_2r_4 \sin\theta_2 \sin\theta_4$$

add equations and simplify to remove θ_3

$$r_3^2 = r_4^2 + r_2^2 + r_1^2 - 2r_2r_4 \cos(\theta_2 - \theta_4) + 2r_1r_4 \cos\theta_4 - 2r_1r_2 \cos\theta_2$$

rearrange

$$(2r_1r_4)\cos\theta_4 - (2r_1r_2)\cos\theta_2 + (r_2^2 - r_3^2 + r_4^2 + r_1^2) = (2r_2r_4)\cos(\theta_2 - \theta_4)$$

use same process to remove θ_4

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)$$

$$K_1 \cos\theta_3 + K_4 \cos\theta_2 + K_5 = \cos(\theta_2 - \theta_3)$$

$$K_1 = r_1 / r_2 \qquad K_2 = r_1 / r_4 \qquad K_3 = (r_2^2 - r_3^2 + r_4^2 + r_1^2)/(2r_2r_4)$$

$$K_4 = r_1 / r_3 \qquad K_5 = (r_4^2 - r_1^2 - r_2^2 - r_3^2)/(2r_2r_3)$$

$$K_1 \cos\theta_4 - K_2 \cos\theta_2 + K_3 = \cos\theta_2 \cos\theta_4 + \sin\theta_2 \sin\theta_4$$

$$(\cos\theta_2 - K_1)\cos\theta_4 + (\sin\theta_2)\sin\theta_4 + (K_2 \cos\theta_2 - K_3) = 0$$

$$u = \tan\frac{\theta_4}{2} \qquad \sin\theta_4 = \frac{2u}{1+u^2} \qquad \cos\theta_4 = \frac{1-u^2}{1+u^2} \qquad \tan\theta_4 = \frac{2u}{1-u^2}$$

$$(\cos\theta_2 - K_1)\frac{1-u^2}{1+u^2} + (\sin\theta_2)\frac{2u}{1+u^2} + (K_2 \cos\theta_2 - K_3) = 0$$

$$\left(K_1 + (K_2 - 1)\cos\theta_2 - K_3\right)u^2 + (2\sin\theta_2)u + \left(-K_1 + (K_2 + 1)\cos\theta_2 - K_3\right) = 0$$

$$a = K_1 + (K_2 - 1)\cos\theta_2 - K_3 \qquad b = 2\sin\theta_2 \qquad c = -K_1 + (K_2 + 1)\cos\theta_2 - K_3$$

$$a\,u^2 + b\,u + c = 0 \qquad u = \frac{-b \pm \sqrt{b^2 - 4\,a\,c}}{2\,a} = \tan\frac{\theta_4}{2}$$

$$K_1\cos\theta_3 + K_4\cos\theta_2 + K_5 = \cos\theta_2\cos\theta_3 + \sin\theta_2\sin\theta_3$$

$$(\cos\theta_2 - K_1)\cos\theta_3 + (\sin\theta_2)\sin\theta_3 - (K_4\cos\theta_2 + K_5) = 0$$

$$v = \tan\frac{\theta_3}{2} \qquad \sin\theta_3 = \frac{2v}{1+v^2} \qquad \cos\theta_3 = \frac{1-v^2}{1+v^2} \qquad \tan\theta_3 = \frac{2v}{1-v^2}$$

$$(\cos\theta_2 - K_1)\frac{1-v^2}{1+v^2} + (\sin\theta_2)\frac{2v}{1+v^2} - (K_4\cos\theta_2 + K_5) = 0$$

$$\left(K_1 - (K_4 + 1)\cos\theta_2 - K_5\right)v^2 + (2\sin\theta_2)v - \left(K_1 + (K_4 - 1)\cos\theta_2 + K_5\right) = 0$$

$$f = K_1 - (K_4 + 1)\cos\theta_2 - K_5 \qquad g = 2\sin\theta_2 \qquad h = -K_1 - (K_4 - 1)\cos\theta_2 - K_5$$

$$f\,v^2 + g\,v + h = 0 \qquad v = \frac{-g \pm \sqrt{g^2 - 4\,f\,h}}{2\,f} = \tan\frac{\theta_3}{2}$$

VELOCITY ANALYSIS

$$d/dt\left(K_1\cos\theta_3 + K_4\cos\theta_2 + K_5 = \cos(\theta_2 - \theta_3)\right)$$

$$-K_1\dot\theta_3\sin\theta_3 - K_4\dot\theta_2\sin\theta_2 = -\left(\dot\theta_2 - \dot\theta_3\right)\sin(\theta_2 - \theta_3)$$

$$\dot\theta_3 = \dot\theta_2\,\frac{-K_4\sin\theta_2 + \sin(\theta_2 - \theta_3)}{K_1\sin\theta_3 + \sin(\theta_2 - \theta_3)}$$

$$d/dt\left(K_1\cos\theta_4 - K_2\cos\theta_2 + K_3 = \cos(\theta_2 - \theta_4)\right)$$

$$-K_1\dot\theta_4\sin\theta_4 + K_2\dot\theta_2\sin\theta_2 = -\left(\dot\theta_2 - \dot\theta_4\right)\sin(\theta_2 - \theta_4)$$

$$\dot\theta_4 = \dot\theta_2\,\frac{K_2\sin\theta_2 + \sin(\theta_2 - \theta_4)}{K_1\sin\theta_4 + \sin(\theta_2 - \theta_4)}$$

ACCELERATION ANALYSIS

$$d/dt\left(-K_1\dot{\theta}_3\sin\theta_3 - K_4\dot{\theta}_2\sin\theta_2 = -\left(\dot{\theta}_2-\dot{\theta}_3\right)\sin\left(\theta_2-\theta_3\right)\right)$$

$$-K_1\ddot{\theta}_3\sin\theta_3 - K_1\dot{\theta}_3^{\,2}\cos\theta_3 - K_4\ddot{\theta}_2\sin\theta_2 - K_4\dot{\theta}_2^{\,2}\cos\theta_2 = -\left(\ddot{\theta}_2-\ddot{\theta}_3\right)\sin\left(\theta_2-\theta_3\right) - \left(\dot{\theta}_2-\dot{\theta}_3\right)^2\cos\left(\theta_2-\theta_3\right)$$

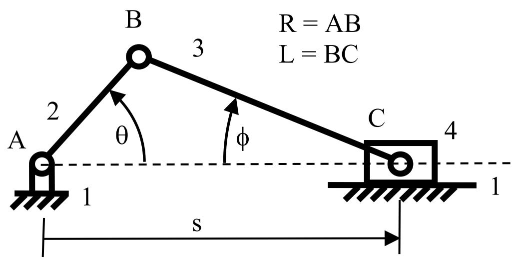
$$\ddot{\theta}_3 = \frac{-\left(K_4\sin\theta_2 - \sin\left(\theta_2-\theta_3\right)\right)\ddot{\theta}_2 - K_1\dot{\theta}_3^{\,2}\cos\theta_3 - K_4\dot{\theta}_2^{\,2}\cos\theta_2 + \left(\dot{\theta}_2-\dot{\theta}_3\right)^2\cos\left(\theta_2-\theta_3\right)}{K_1\sin\theta_3 + \sin\left(\theta_2-\theta_3\right)}$$

$$d/dt\left(-K_1\dot{\theta}_4\sin\theta_4 + K_2\dot{\theta}_2\sin\theta_2 = -\left(\dot{\theta}_2-\dot{\theta}_4\right)\sin\left(\theta_2-\theta_4\right)\right)$$

$$-K_1\ddot{\theta}_4\sin\theta_4 - K_1\dot{\theta}_4^{\,2}\cos\theta_4 + K_2\ddot{\theta}_2\sin\theta_2 + K_2\dot{\theta}_2^{\,2}\cos\theta_2 = -\left(\ddot{\theta}_2-\ddot{\theta}_4\right)\sin\left(\theta_2-\theta_4\right) - \left(\dot{\theta}_2-\dot{\theta}_4\right)^2\cos\left(\theta_2-\theta_4\right)$$

$$\ddot{\theta}_4 = \frac{\left(K_2\sin\theta_2 + \sin\left(\theta_2-\theta_4\right)\right)\ddot{\theta}_2 - K_1\dot{\theta}_4^{\,2}\cos\theta_4 + K_2\dot{\theta}_2^{\,2}\cos\theta_2 + \left(\dot{\theta}_2-\dot{\theta}_4\right)^2\cos\left(\theta_2-\theta_4\right)}{K_1\sin\theta_4 + \sin\left(\theta_2-\theta_4\right)}$$

Geometric Kinematics for Slider Crank



POSITION ANALYSIS

$$R\sin\theta = L\sin\phi$$
$$\sin\phi = R\sin\theta / L$$

$$s = R\cos\theta + L\cos\phi$$

VELOCITY ANALYSIS

$$d/dt\left(R\sin\theta = L\sin\phi\right)$$
$$R\dot{\theta}\cos\theta = L\dot{\phi}\cos\phi$$
$$\dot{\phi} = R\dot{\theta}\cos\theta / L\cos\phi$$

$$d/dt\left(s = R\cos\theta + L\cos\phi\right)$$
$$\dot{s} = -R\dot{\theta}\sin\theta - L\dot{\phi}\sin\phi = -R\dot{\theta}\left(\frac{\sin(\theta+\phi)}{\cos\phi}\right)$$

ACCELERATION ANALYSIS

$$d/dt\left(R\dot{\theta}\cos\theta = L\dot{\phi}\cos\phi\right)$$
$$R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta = L\ddot{\phi}\cos\phi - L\dot{\phi}^2\sin\phi$$
$$\ddot{\phi} = \left(R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta + L\dot{\phi}^2\sin\phi\right)/L\cos\phi$$

$$d/dt\left(\dot{s} = -R\dot{\theta}\sin\theta - L\dot{\phi}\sin\phi\right)$$
$$\ddot{s} = -R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta - L\ddot{\phi}\sin\phi - L\dot{\phi}^2\cos\phi$$

JERK ANALYSIS

$$d/dt\left(R\ddot{\theta}\cos\theta - R\dot{\theta}^2\sin\theta = L\ddot{\phi}\cos\phi - L\dot{\phi}^2\sin\phi\right)$$
$$R\dddot{\theta}\cos\theta - 3R\dot{\theta}\ddot{\theta}\sin\theta - R\dot{\theta}^3\cos\theta = L\dddot{\phi}\cos\phi - 3L\dot{\phi}\ddot{\phi}\sin\phi - L\dot{\phi}^3\cos\phi$$
$$\dddot{\phi} = \left(R\dddot{\theta}\cos\theta - 3R\dot{\theta}\ddot{\theta}\sin\theta - R\dot{\theta}^3\cos\theta + 3L\dot{\phi}\ddot{\phi}\sin\phi + L\dot{\phi}^3\cos\phi\right)/L\cos\phi$$

$$d/dt\left(\ddot{s} = -R\ddot{\theta}\sin\theta - R\dot{\theta}^2\cos\theta - L\ddot{\phi}\sin\phi - L\dot{\phi}^2\cos\phi\right)$$

$$\dddot{s} = -R\dddot{\theta}\sin\theta - 3R\dot{\theta}\ddot{\theta}\cos\theta + R\dot{\theta}^3\sin\theta - L\dddot{\phi}\sin\phi - 3L\dot{\phi}\ddot{\phi}\cos\phi + L\dot{\phi}^3\sin\phi$$

SNAP ANALYSIS

$$d/dt\left(R\dddot{\theta}\cos\theta - 3R\dot{\theta}\ddot{\theta}\sin\theta - R\dot{\theta}^3\cos\theta = L\dddot{\phi}\cos\phi - 3L\dot{\phi}\ddot{\phi}\sin\phi - L\dot{\phi}^3\cos\phi\right)$$

$$R\ddddot{\theta}\cos\theta - 4R\dot{\theta}\dddot{\theta}\sin\theta - 3R\ddot{\theta}^2\sin\theta - 6R\dot{\theta}^2\ddot{\theta}\cos\theta + R\dot{\theta}^4\sin\theta$$

$$= L\ddddot{\phi}\cos\phi - 4L\dot{\phi}\dddot{\phi}\sin\phi - 3L\ddot{\phi}^2\sin\phi - 6L\dot{\phi}^2\ddot{\phi}\cos\phi + L\dot{\phi}^4\sin\phi$$

$$\ddddot{\phi} = \begin{pmatrix} R\ddddot{\theta}\cos\theta - 4R\dot{\theta}\dddot{\theta}\sin\theta - 3R\ddot{\theta}^2\sin\theta - 6R\dot{\theta}^2\ddot{\theta}\cos\theta + R\dot{\theta}^4\sin\theta \\ + 4L\dot{\phi}\dddot{\phi}\sin\phi + 3L\ddot{\phi}^2\sin\phi + 6L\dot{\phi}^2\ddot{\phi}\cos\phi - L\dot{\phi}^4\sin\phi \end{pmatrix} / L\cos\phi$$

$$d/dt\left(\dddot{s} = -R\dddot{\theta}\sin\theta - 3R\dot{\theta}\ddot{\theta}\cos\theta + R\dot{\theta}^3\sin\theta - L\dddot{\phi}\sin\phi - 3L\dot{\phi}\ddot{\phi}\cos\phi + L\dot{\phi}^3\sin\phi\right)$$

$$\ddddot{s} = -R\ddddot{\theta}\sin\theta - 4R\dot{\theta}\dddot{\theta}\cos\theta - 3R\ddot{\theta}^2\cos\theta + 6R\dot{\theta}^2\ddot{\theta}\sin\theta + R\dot{\theta}^4\cos\theta$$

$$- L\ddddot{\phi}\sin\phi - 4L\dot{\phi}\dddot{\phi}\cos\phi - 3L\ddot{\phi}^2\cos\phi + 6L\dot{\phi}^2\ddot{\phi}\sin\phi + L\dot{\phi}^4\cos\phi$$

APPROXIMATE EXPLICIT SOLUTION

$$\cos^2\phi = 1 - \sin^2\phi = 1 - \frac{R^2}{L^2}\sin^2\theta \qquad \text{for} \qquad \sin\phi = \frac{R}{L}\sin\theta$$

$$\cos\phi = \sqrt{1 - \frac{R^2}{L^2}\sin^2\theta} = \sqrt{1 \pm B^2} \qquad \text{for} \qquad B = \frac{R}{L}\sin\theta$$

binomial series $\sqrt{1 \pm B^2} = 1 \pm \frac{1}{2}B^2 - \frac{1}{2\cdot 4}B^4 \pm \frac{1\cdot 3}{2\cdot 4\cdot 6}B^6 - \frac{1\cdot 3\cdot 5}{2\cdot 4\cdot 6\cdot 8}B^8 \pm \cdots$

$$\cos\phi \approx 1 - \frac{R^2}{2L^2}\sin^2\theta$$

$$\sin^2\theta = \frac{1}{2}(1 - \cos 2\theta)$$

$$s \approx R\cos\theta + L\left(1 - \frac{R^2}{4L^2}(1 - \cos 2\theta)\right) = R\cos\theta + L - \frac{R^2}{4L}(1 - \cos 2\theta)$$

$$\dot{s} \approx -R\,\dot{\theta}\sin\theta - \frac{R^2}{2L}\dot{\theta}\sin 2\theta = -R\,\dot{\theta}\left(\sin\theta + \frac{R}{2L}\sin 2\theta\right)$$

$$\ddot{s} \approx -R\,\dot{\theta}^2\left(\cos\theta + \frac{R}{L}\cos 2\theta\right) \qquad \text{for } \ddot{\theta} = 0$$

$$\dddot{s} \approx R\,\dot{\theta}^3\left(\sin\theta + \frac{2R}{L}\sin 2\theta\right) \qquad \text{for } \ddot{\theta} = \dddot{\theta} = 0$$

$$\ddddot{s} \approx R\,\dot{\theta}^3\left(\cos\theta + \frac{4R}{L}\cos 2\theta\right) \qquad \text{for } \ddot{\theta} = \dddot{\theta} = \ddddot{\theta} = 0$$

HIGHER ORDER APPROXIMATION (for $\ddot{\theta} = \dddot{\theta} = 0$)

$$s \approx R\cos\theta + \left(L - \frac{R^2}{4L} - \frac{3R^4}{64L^3} - \frac{5R^6}{256L^5}\right) + R\left(\frac{R}{4L} + \frac{R^3}{16L^3} + \frac{15R^5}{512L^5}\right)\cos 2\theta$$
$$- R\left(\frac{R^3}{64L^3} + \frac{3R^5}{256L^5}\right)\cos 4\theta + R\left(\frac{R^5}{512L^5}\right)\cos 6\theta$$

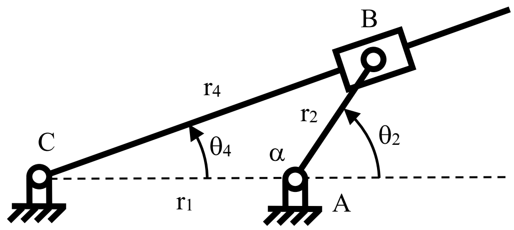
$$\dot{s} \approx -R\dot{\theta}\left[\sin\theta + 2\left(\frac{R}{4L} + \frac{R^3}{16L^3} + \frac{15R^5}{512L^5}\right)\sin 2\theta - 4\left(\frac{R^3}{64L^3} + \frac{3R^5}{256L^5}\right)\sin 4\theta \quad + 6\left(\frac{R^5}{512L^5}\right)\sin 6\theta\right]$$

$$\ddot{s} \approx -R\dot{\theta}^2\left[\cos\theta + 4\left(\frac{R}{4L} + \frac{R^3}{16L^3} + \frac{15R^5}{512L^5}\right)\cos 2\theta - 16\left(\frac{R^3}{64L^3} + \frac{3R^5}{256L^5}\right)\cos 4\theta + 36\left(\frac{R^5}{512L^5}\right)\cos 6\theta\right]$$

$$\dddot{s} \approx +R\dot{\theta}^3\left[\sin\theta + 8\left(\frac{R}{4L} + \frac{R^3}{16L^3} + \frac{15R^5}{512L^5}\right)\sin 2\theta - 64\left(\frac{R^3}{64L^3} + \frac{3R^5}{256L^5}\right)\sin 4\theta + 216\left(\frac{R^5}{512L^5}\right)\sin 6\theta\right]$$

$$\ddddot{s} \approx +R\dot{\theta}^4\left[\cos\theta + 16\left(\frac{R}{4L} + \frac{R^3}{16L^3} + \frac{15R^5}{512L^5}\right)\cos 2\theta - 256\left(\frac{R^3}{64L^3} + \frac{3R^5}{256L^5}\right)\cos 4\theta + 1296\left(\frac{R^5}{512L^5}\right)\cos 6\theta\right]$$

Geometric Kinematics for Inverted Slider Crank



POSITION ANALYSIS

$$\theta_2 + \alpha = 180°$$
$$\alpha = 180° - \theta_2$$

$$r_4^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\alpha$$
$$r_4 = \sqrt{r_1^2 + r_2^2 - 2r_1r_2\cos\alpha}$$

$$r_2\sin\theta_2 = r_4\sin\theta_4$$
$$\sin\theta_4 = r_2\sin\theta_2 / r_4$$

VELOCITY ANALYSIS

$$d/dt\left(\alpha = 180° - \theta_2\right)$$
$$\dot{\alpha} = -\dot{\theta}_2$$

$$d/dt\left(r_4^2 = r_1^2 + r_2^2 - 2r_1r_2\cos\alpha\right)$$
$$2r_4\dot{r}_4 = 2r_1r_2\dot{\alpha}\sin\alpha$$
$$\dot{r}_4 = r_1r_2\dot{\alpha}\sin\alpha / r_4$$

$$d/dt\left(r_2\sin\theta_2 = r_4\sin\theta_4\right)$$
$$r_2\dot{\theta}_2\cos\theta_2 = \dot{r}_4\sin\theta_4 + r_4\dot{\theta}_4\cos\theta_4$$
$$\dot{\theta}_4 = \left(r_2\dot{\theta}_2\cos\theta_2 - \dot{r}_4\sin\theta_4\right) / r_4\cos\theta_4$$

ACCELERATION ANALYSIS

$$d/dt\left(\dot{\alpha} = -\dot{\theta}_2\right)$$
$$\ddot{\alpha} = -\ddot{\theta}_2$$

$$d/dt\left(r_4\dot{r}_4 = r_1r_2\dot{\alpha}\sin\alpha\right)$$
$$\dot{r}_4^2 + r_4\ddot{r}_4 = r_1r_2\ddot{\alpha}\sin\alpha + r_1r_2\dot{\alpha}^2\cos\alpha$$
$$\ddot{r}_4 = \left(r_1r_2\ddot{\alpha}\sin\alpha + r_1r_2\dot{\alpha}^2\cos\alpha - \dot{r}_4^2\right) / r_4$$

$$d/dt\left(r_2\dot{\theta}_2\cos\theta_2 = \dot{r}_4\sin\theta_4 + r_4\dot{\theta}_4\cos\theta_4\right)$$
$$r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 = \ddot{r}_4\sin\theta_4 + 2\dot{r}_4\dot{\theta}_4\cos\theta_4 + r_4\ddot{\theta}_4\cos\theta_4 - r_4\dot{\theta}_4^2\sin\theta_4$$
$$\ddot{\theta}_4 = \left(r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 - \ddot{r}_4\sin\theta_4 - 2\dot{r}_4\dot{\theta}_4\cos\theta_4 + r_4\dot{\theta}_4^2\sin\theta_4\right)/\, r_4\cos\theta_4$$

JERK ANALYSIS

$$d/dt\left(\ddot{\alpha} = -\ddot{\theta}_2\right)$$
$$\dddot{\alpha} = -\dddot{\theta}_2$$

$$d/dt\left(\dot{r}_4^2 + r_4\ddot{r}_4 = r_1r_2\ddot{\alpha}\sin\alpha + r_1r_2\dot{\alpha}^2\cos\alpha\right)$$
$$3\dot{r}_4\ddot{r}_4 + r_4\dddot{r}_4 = r_1r_2\dddot{\alpha}\sin\alpha + 3r_1r_2\dot{\alpha}\ddot{\alpha}\cos\alpha - r_1r_2\dot{\alpha}^3\sin\alpha$$
$$\dddot{r}_4 = \left(r_1r_2\dddot{\alpha}\sin\alpha + 3r_1r_2\dot{\alpha}\ddot{\alpha}\cos\alpha - r_1r_2\dot{\alpha}^3\sin\alpha - 3\dot{r}_4\ddot{r}_4\right)/\, r_4$$

$$d/dt\left(r_2\ddot{\theta}_2\cos\theta_2 - r_2\dot{\theta}_2^2\sin\theta_2 = \ddot{r}_4\sin\theta_4 + 2\dot{r}_4\dot{\theta}_4\cos\theta_4 + r_4\ddot{\theta}_4\cos\theta_4 - r_4\dot{\theta}_4^2\sin\theta_4\right)$$
$$\begin{aligned} r_2\dddot{\theta}_2\cos\theta_2 - 3r_2\dot{\theta}_2\ddot{\theta}_2\sin\theta_2 - r_2\dot{\theta}_2^3\cos\theta_2 &= \dddot{r}_4\sin\theta_4 + 3\ddot{r}_4\dot{\theta}_4\cos\theta_4 + 3\dot{r}_4\ddot{\theta}_4\cos\theta_4 \\ &\quad - 3\dot{r}_4\dot{\theta}_4^2\sin\theta_4 + r_4\dddot{\theta}_4\cos\theta_4 - 3r_4\dot{\theta}_4\ddot{\theta}_4\sin\theta_4 - r_4\dot{\theta}_4^3\cos\theta_4 \end{aligned}$$
$$\dddot{\theta}_4 = \begin{pmatrix} r_2\dddot{\theta}_2\cos\theta_2 - 3r_2\dot{\theta}_2\ddot{\theta}_2\sin\theta_2 - r_2\dot{\theta}_2^3\cos\theta_2 - \dddot{r}_4\sin\theta_4 - 3\ddot{r}_4\dot{\theta}_4\cos\theta_4 \\ -3\dot{r}_4\ddot{\theta}_4\cos\theta_4 + 3\dot{r}_4\dot{\theta}_4^2\sin\theta_4 + 3r_4\dot{\theta}_4\ddot{\theta}_4\sin\theta_4 + r_4\dot{\theta}_4^3\cos\theta_4 \end{pmatrix}/\, r_4\cos\theta_4$$

SNAP ANALYSIS

$$d/dt\left(\dddot{\alpha} = -\dddot{\theta}_2\right)$$
$$\ddddot{\alpha} = -\ddddot{\theta}_2$$

$$d/dt\left(3\dot{r}_4\ddot{r}_4 + r_4\dddot{r}_4 = r_1r_2\dddot{\alpha}\sin\alpha + 3r_1r_2\dot{\alpha}\ddot{\alpha}\cos\alpha - r_1r_2\dot{\alpha}^3\sin\alpha\right)$$

$$\begin{aligned} 3\ddot{r}_4^2 + 3\dot{r}_4\dddot{r}_4 + \dot{r}_4\dddot{r}_4 + r_4\ddddot{r}_4 &= r_1r_2\ddddot{\alpha}\sin\alpha + r_1r_2\dot{\alpha}\dddot{\alpha}\cos\alpha \\ &\quad + 3r_1r_2\ddot{\alpha}^2\cos\alpha + 3r_1r_2\dot{\alpha}\dddot{\alpha}\cos\alpha - 3r_1r_2\dot{\alpha}^2\ddot{\alpha}\sin\alpha - 3r_1r_2\dot{\alpha}^2\ddot{\alpha}\sin\alpha - r_1r_2\dot{\alpha}^4\cos\alpha \end{aligned}$$

$$3\ddot{r}_4^2 + 4\dot{r}_4\dddot{r}_4 + r_4\ddddot{r}_4 = r_1r_2\ddddot{\alpha}\sin\alpha + 4r_1r_2\dot{\alpha}\dddot{\alpha}\cos\alpha + 3r_1r_2\ddot{\alpha}^2\cos\alpha - 6r_1r_2\dot{\alpha}^2\ddot{\alpha}\sin\alpha - r_1r_2\dot{\alpha}^4\cos\alpha$$

$$\ddddot{r}_4 = \left(r_1r_2\ddddot{\alpha}\sin\alpha + 4r_1r_2\dot{\alpha}\dddot{\alpha}\cos\alpha + 3r_1r_2\ddot{\alpha}^2\cos\alpha - 6r_1r_2\dot{\alpha}^2\ddot{\alpha}\sin\alpha - r_1r_2\dot{\alpha}^4\cos\alpha - 3\ddot{r}_4^2 - 4\dot{r}_4\dddot{r}_4\right)/\, r_4$$

$$d/dt\begin{pmatrix} r_2\dddot{\theta}_2\cos\theta_2 - 3r_2\dot{\theta}_2\ddot{\theta}_2\sin\theta_2 - r_2\dot{\theta}_2^3\cos\theta_2 = \dddot{r}_4\sin\theta_4 + 3\ddot{r}_4\dot{\theta}_4\cos\theta_4 + 3\dot{r}_4\ddot{\theta}_4\cos\theta_4 \\ -3\dot{r}_4\dot{\theta}_4^2\sin\theta_4 + r_4\dddot{\theta}_4\cos\theta_4 - 3r_4\dot{\theta}_4\ddot{\theta}_4\sin\theta_4 - r_4\dot{\theta}_4^3\cos\theta_4 \end{pmatrix}$$

$$
\begin{aligned}
r_2\dddot{\theta}_2 \cos\theta_2 - 3r_2\dot{\theta}_2\ddot{\theta}_2 \sin\theta_2 - r_2\dot{\theta}_2^3 \cos\theta_2 &= \dddot{r}_4 \sin\theta_4 + 3\ddot{r}_4\dot{\theta}_4 \cos\theta_4 + 3\dot{r}_4\ddot{\theta}_4 \cos\theta_4 \\
&\quad - 3\dot{r}_4\dot{\theta}_4^2 \sin\theta_4 + r_4\dddot{\theta}_4 \cos\theta_4 - 3r_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 - r_4\dot{\theta}_4^3 \cos\theta_4
\end{aligned}
$$

$$
\begin{aligned}
& r_2\ddddot{\theta}_2 \cos\theta_2 - r_2\dot{\theta}_2\dddot{\theta}_2 \sin\theta_2 - 3r_2\ddot{\theta}_2^2 \sin\theta_2 - 3r_2\dot{\theta}_2\dddot{\theta}_2 \sin\theta_2 - 3r_2\dot{\theta}_2^2\ddot{\theta}_2 \cos\theta_2 \\
& - 3r_2\dot{\theta}_2^2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^4 \sin\theta_2 = \ddddot{r}_4 \sin\theta_4 + \dddot{r}_4\dot{\theta}_4 \cos\theta_4 + 3\dddot{r}_4\dot{\theta}_4 \cos\theta_4 + 3\ddot{r}_4\ddot{\theta}_4 \cos\theta_4 - 3\ddot{r}_4\dot{\theta}_4^2 \sin\theta_4 \\
& + 3\ddot{r}_4\ddot{\theta}_4 \cos\theta_4 + 3\dot{r}_4\dddot{\theta}_4 \cos\theta_4 - 3\dot{r}_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 - 3\ddot{r}_4\dot{\theta}_4^2 \sin\theta_4 - 6\dot{r}_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 - 3\dot{r}_4\dot{\theta}_4^3 \cos\theta_4 \\
& + \dot{r}_4\dddot{\theta}_4 \cos\theta_4 + r_4\ddddot{\theta}_4 \cos\theta_4 - r_4\dot{\theta}_4\dddot{\theta}_4 \sin\theta_4 - 3\dot{r}_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 - 3r_4\ddot{\theta}_4^2 \sin\theta_4 - 3r_4\dot{\theta}_4\dddot{\theta}_4 \sin\theta_4 - 3r_4\dot{\theta}_4^2\ddot{\theta}_4 \cos\theta_4 \\
& - \dot{r}_4\dot{\theta}_4^3 \cos\theta_4 - 3r_4\dot{\theta}_4^2\ddot{\theta}_4 \cos\theta_4 + r_4\dot{\theta}_4^4 \sin\theta_4 \\
& r_2\ddddot{\theta}_2 \cos\theta_2 - 4r_2\dot{\theta}_2\dddot{\theta}_2 \sin\theta_2 - 3r_2\ddot{\theta}_2^2 \sin\theta_2 - 6r_2\dot{\theta}_2^2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^4 \sin\theta_2 \\
& = \ddddot{r}_4 \sin\theta_4 + 4\dddot{r}_4\dot{\theta}_4 \cos\theta_4 + 6\ddot{r}_4\ddot{\theta}_4 \cos\theta_4 - 6\ddot{r}_4\dot{\theta}_4^2 \sin\theta_4 + 4\dot{r}_4\dddot{\theta}_4 \cos\theta_4 - 12\dot{r}_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 - 4\dot{r}_4\dot{\theta}_4^3 \cos\theta_4 \\
& + r_4\ddddot{\theta}_4 \cos\theta_4 - 4r_4\dot{\theta}_4\dddot{\theta}_4 \sin\theta_4 - 3r_4\ddot{\theta}_4^2 \sin\theta_4 - 6r_4\dot{\theta}_4^2\ddot{\theta}_4 \cos\theta_4 + r_4\dot{\theta}_4^4 \sin\theta_4
\end{aligned}
$$

$$
\ddddot{\theta}_4 = \begin{pmatrix}
r_2\ddddot{\theta}_2 \cos\theta_2 - 4r_2\dot{\theta}_2\dddot{\theta}_2 \sin\theta_2 - 3r_2\ddot{\theta}_2^2 \sin\theta_2 - 6r_2\dot{\theta}_2^2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^4 \sin\theta_2 \\
+ 4r_4\dot{\theta}_4\dddot{\theta}_4 \sin\theta_4 + 3r_4\ddot{\theta}_4^2 \sin\theta_4 + 6r_4\dot{\theta}_4^2\ddot{\theta}_4 \cos\theta_4 - r_4\dot{\theta}_4^4 \sin\theta_4 \\
- \ddddot{r}_4 \sin\theta_4 - 4\dddot{r}_4\dot{\theta}_4 \cos\theta_4 - 6\ddot{r}_4\ddot{\theta}_4 \cos\theta_4 + 6\ddot{r}_4\dot{\theta}_4^2 \sin\theta_4 \\
- 4\dot{r}_4\dddot{\theta}_4 \cos\theta_4 + 12\dot{r}_4\dot{\theta}_4\ddot{\theta}_4 \sin\theta_4 + 4\dot{r}_4\dot{\theta}_4^3 \cos\theta_4
\end{pmatrix} / r_4 \cos\theta_4
$$