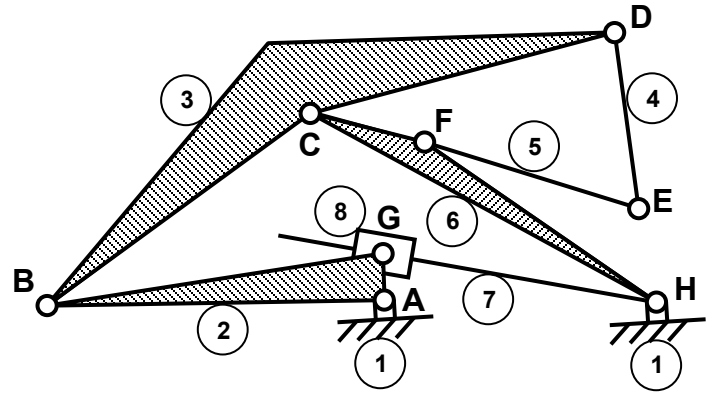
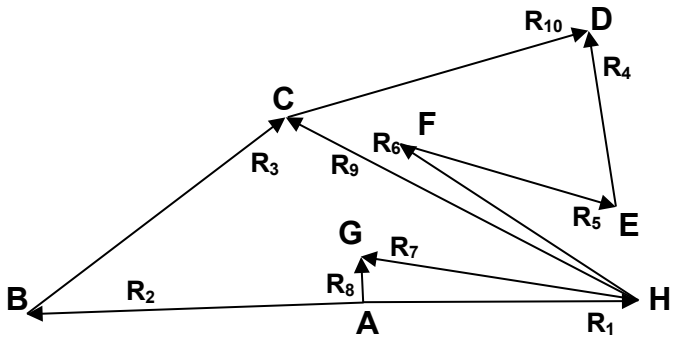
POLYCENTRIC HINGE - SKELETAL DIAGRAM



POLYCENTRIC HINGE - VECTORS

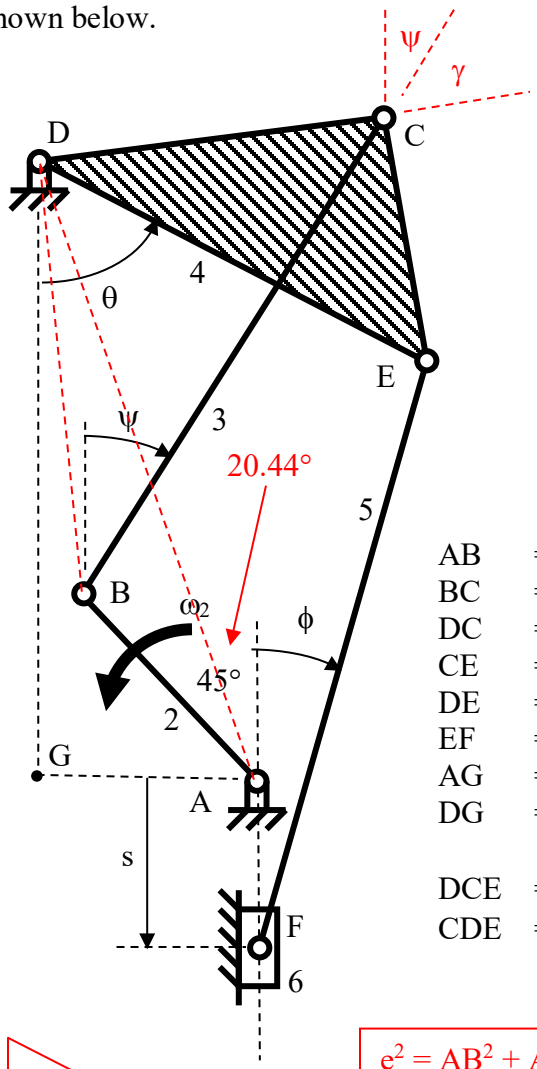


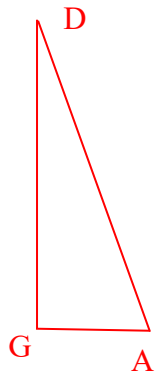
Vector	Position of	Length (mm)		Angle (deg)	
1	H wrt A	21.67	constant	0.00	constant
2	B wrt A	26.69	constant	181.61	var
3	C wrt B	26.00	constant	36.95	var
4	D wrt E	14.00	constant	99.38	driver
5	E wrt F	17.83	constant	-15.87	var
6	F wrt H	22.47	constant	146.31	θ_9 - 5.33°
7	G wrt H	22.03	var	171.56	var
8	G wrt A	3.23	constant	92.21	θ_2 - 89.40°
9	C wrt H	31.33	constant	151.64	var
10	D wrt C	24.62	constant	15.36	θ_3 - 21.59°

$$\overline{R}_2 + \overline{R}_3 - \overline{R}_9 - \overline{R}_1 = 0$$
$$\overline{R}_2 + \overline{R}_3 + \overline{R}_{10} - \overline{R}_4 - \overline{R}_5 - \overline{R}_6 - \overline{R}_1 = 0$$
$$\overline{R}_8 - \overline{R}_7 - \overline{R}_1 = 0$$

Sewing Machine

Determine angles θ, ϕ and ψ as well as distance s for this sewing machine linkage at the position shown below.

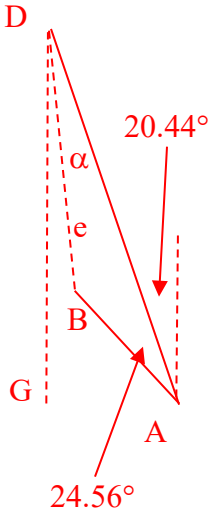


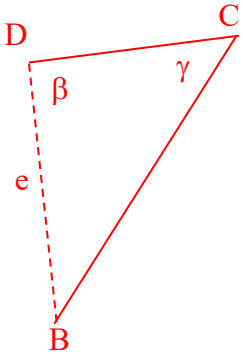


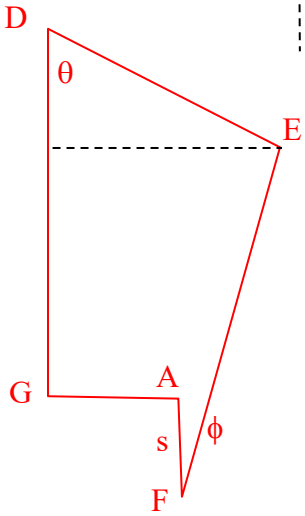
$AD^2 = AG^2 + DG^2$
AD = 4.066 cm

tan GDA = AG / DG
GDA = 20.44°

AB	= 1.60 cm
BC	= 3.57 cm
DC	= 2.24 cm
CE	= 1.60 cm
DE	= 2.74 cm
EF	= 3.81 cm
AG	= 1.42 cm
DG	= 3.81 cm
DCE	= 90°
CDE	= 35.7°





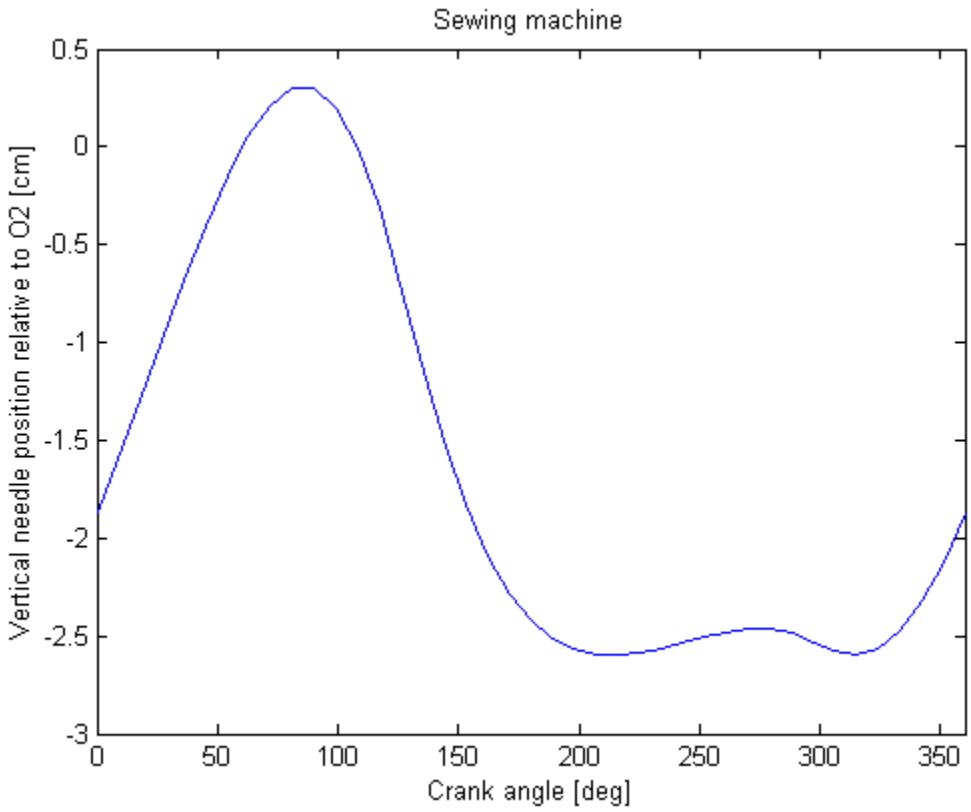


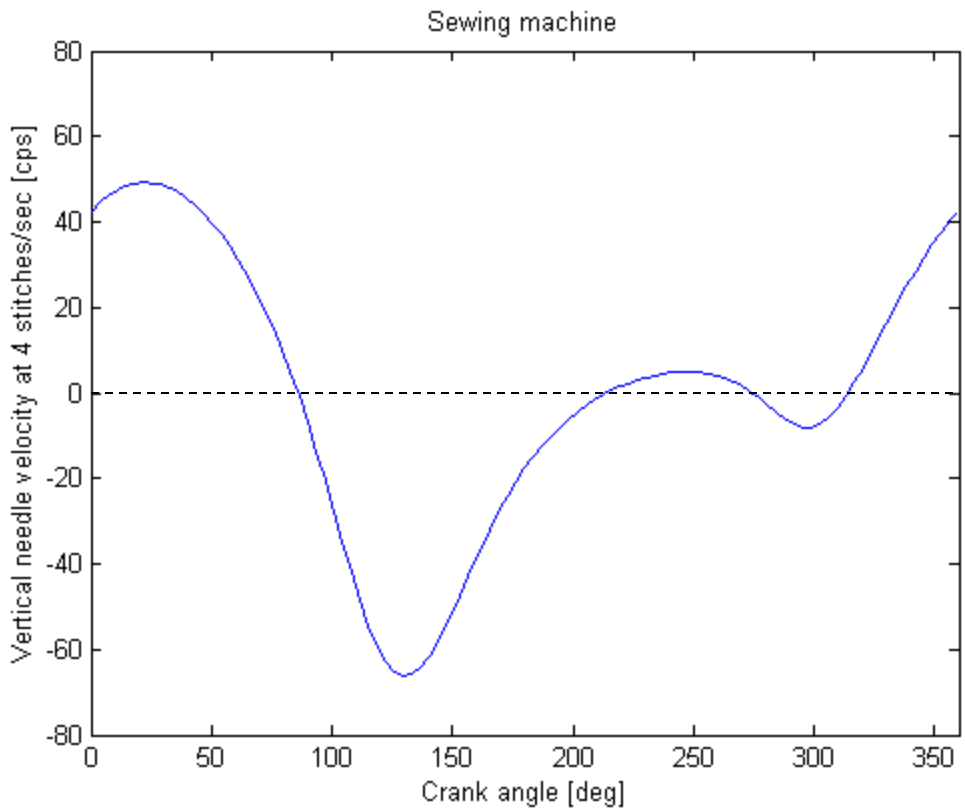
$e^2 = AB^2 + AD^2 - 2\,(AB)\,(AD)\cos 24.56°$ e = 2.694 cm
$\sin\alpha / AB = \sin 24.56° / e$ α = 14.29°

$e^2 = BC^2 + CD^2 - 2\,(BC)\,(CD)\cos\gamma$ γ = 48.94°
$\sin\beta / BC = \sin\gamma / e$ β = 87.77° or 92.23° ??? check which one
$BC^2 = e^2 + CD^2 + \ - 2\,e\,(CD)\cos\beta$ β = 92.23°

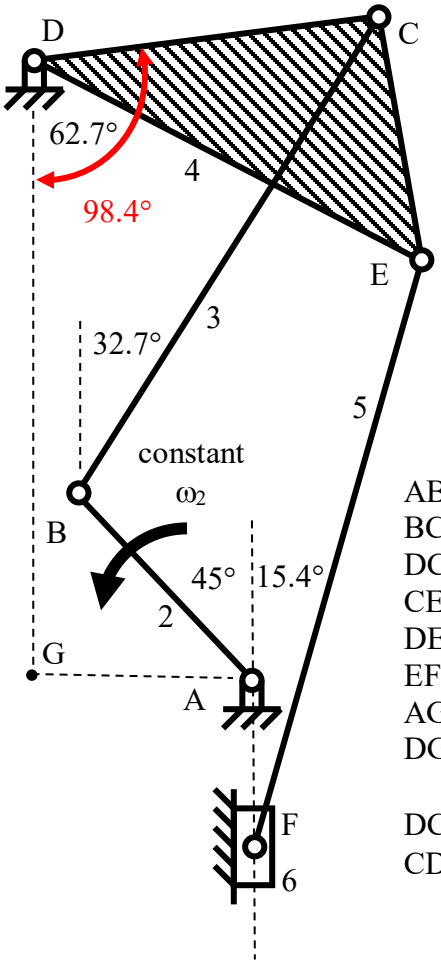
GDA - α + β = CDG = θ + CDE θ = 62.65°
θ + CDE + γ + ψ = 180° ψ = 32.68°

$DE \sin\theta = AG + EF \sin\phi$ ϕ = 15.43°
$s + DG = DE\cos\theta + EF\cos\phi$ s = 1.12 cm


Sewing machine
Vertical needle position relative to O2 [cm]
0.5
0
-0.5
-1
-1.5
-2
-2.5
-3
0
50
100
150
200
250
300
350
Crank angle [deg]


Sewing machine
Vertical needle velocity at 4 stitches/sec [cps]
80
60
40
20
0
-20
-40
-60
-80
0
50
100
150
200
250
300
350
Crank angle [deg]

Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



AB	= 1.60 cm
BC	= 3.57 cm
DC	= 2.24 cm
CE	= 1.60 cm
DE	= 2.74 cm
EF	= 3.81 cm
AG	= 1.42 cm
DG	= 3.81 cm
DCE	= 90°
CDE	= 35.7°

ω_2 = 4 stitches/sec = $+8\pi$ rad/sec

$V_B = AB\ \omega_2 = 40.21$ cps

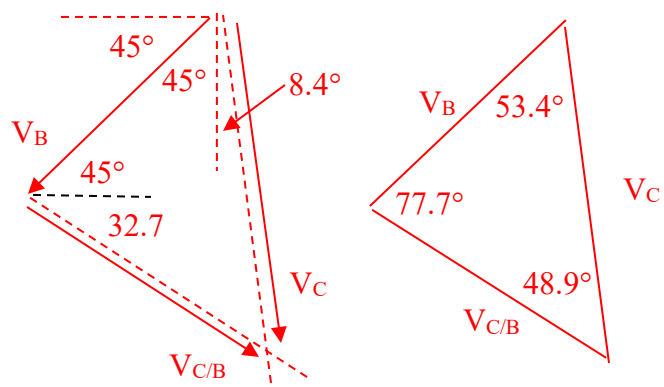
$\overline{V}_C = \overline{V}_B + \overline{V}_{C/B}$

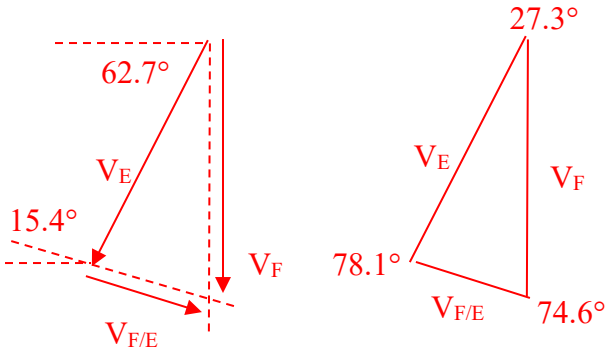
?	AB ω_2	?
⊥CD	⊥AB	⊥BC

V_B / sin 48.9° = V_C / sin 77.7° = $V_{C/B}$ / sin 53.4°

V_C = 52.31 cps $\omega_4 = V_C$ / CD = 23.27 rad/sec CW

$V_{C/B}$ = 42.84 cps $\omega_3 = V_{C/B}$ / BC = 12.00 rad/sec CW





$V_E = DE\ \omega_4 = 63.77$ cps

$\overline{V}_F = \overline{V}_E + \overline{V}_{F/E}$

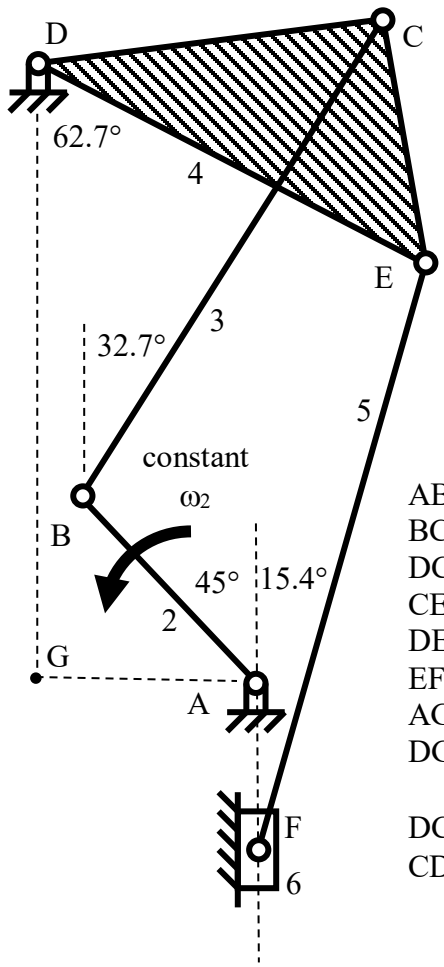
?	DE ω_4	?
vertical	⊥DE	⊥EF

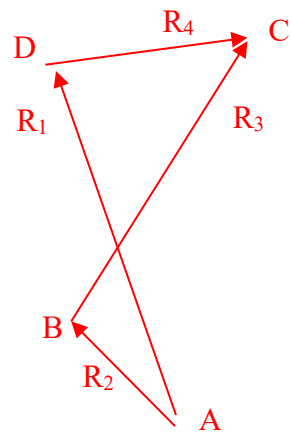
V_E / sin 74.6° = V_F / sin 78.1° = $V_{F/E}$ / sin 27.3°

V_F = 64.72 cps

$V_{F/E}$ = 30.34 cps $\omega_5 = V_{F/E}$ / EF = 7.96 rad/sec CCW

Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.





AB	= 1.60 cm
BC	= 3.57 cm
DC	= 2.24 cm
CE	= 1.60 cm
DE	= 2.74 cm
EF	= 3.81 cm
AG	= 1.42 cm
DG	= 3.81 cm
DCE	= 90°
CDE	= 35.7°

$\omega_2 = +8\pi$ rad/sec

	r [cm]	θ [deg]
1	4.07	110.4
2	1.60	135.0
3	3.57	57.3
4	2.24	8.4

$\omega_3 = -r_2\ \omega_2 \sin(\theta_2-\theta_4) \ /\ r_3 \sin(\theta_3-\theta_4) = -12.00$ rad/s

$\omega_4 = -r_2\ \omega_2 \sin(\theta_2-\theta_3) \ /\ r_4 \sin(\theta_3-\theta_4) = -23.27$ rad/s

$\omega_2^* = \omega_4 = -23.27$ rad/sec

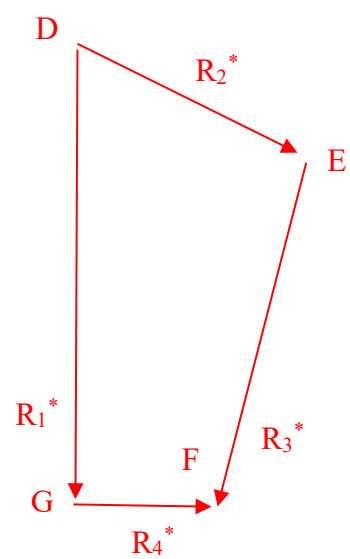
	r^* [cm]	θ^* [deg]
1	4.93	270
2	2.74	332.7
3	3.81	254.6
4	1.42	0

$\omega_3^* = -\ r_2^*\ \omega_2^* \cos(\theta_2^*-\theta_1^*) \ /\ r_3^*\ \cos(\theta_3^*-\theta_1^*) = +\ 7.96$ rad/s

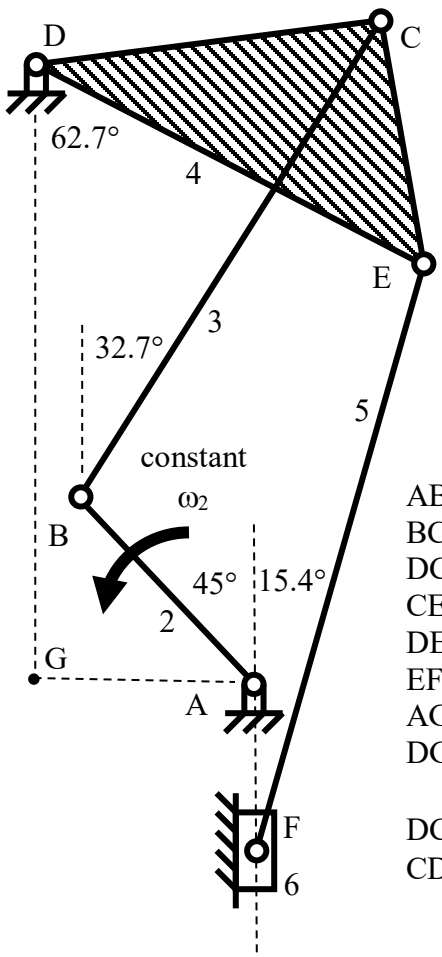
$\dot{r}_1^* = -\ r_2^*\ \omega_2^*\ \sin(\theta_2^*-\theta_3^*) \ /\ \cos(\theta_3^*-\theta_1^*) = +\ 64.71$cps

$\omega_5 = \omega_3^* = 7.96$ rad/sec CCW

$\overline{V}_F = 64.71$ cps down

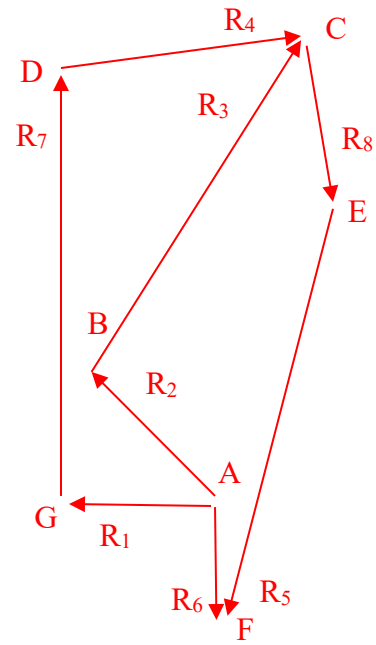


Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



AB = 1.60 cm
BC = 3.57 cm
DC = 2.24 cm
CE = 1.60 cm
DE = 2.74 cm
EF = 3.81 cm
AG = 1.42 cm
DG = 3.81 cm

DCE = 90°
CDE = 35.7°



	r [cm]	θ [deg]
1	1.42 constant	180 constant
2	1.60 constant	135 driver
3	3.57 constant	57.3 variable
4	2.24 constant	8.4 variable
5	3.81 constant	254.6 variable
6	1.12 variable	270 constant
7	3.81 constant	90 constant
8	1.60 constant	θ_4 - 90 constraint

$$\overline{R}_2 + \overline{R}_3 - \overline{R}_4 - \overline{R}_7 - \overline{R}_1 = 0$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} - r_4 e^{j\theta_4} - r_7 e^{j\theta_7} - r_1 e^{j\theta_1} = 0$$

$$j\, r_2\, \dot{\theta}_2\, e^{j\theta_2} + j\, r_3\, \dot{\theta}_3\, e^{j\theta_3} - j\, r_4\, \dot{\theta}_4\, e^{j\theta_4} = 0$$

REAL: $-r_2\dot{\theta}_2 \sin\theta_2 - r_3\dot{\theta}_3 \sin\theta_3 + r_4\dot{\theta}_4 \sin\theta_4 = 0$

IMAG: $j\, r_2\, \dot{\theta}_2 \cos\theta_2 + j\, r_3\dot{\theta}_3 \cos\theta_3 - j\, r_4\dot{\theta}_4 \cos\theta_4 = 0$

$$\overline{R}_2 + \overline{R}_3 + \overline{R}_8 + \overline{R}_5 - \overline{R}_6 = 0$$

$$r_2 e^{j\theta_2} + r_3 e^{j\theta_3} + r_8 e^{j\theta_8} + r_5 e^{j\theta_5} - r_6 e^{j\theta_6} = 0$$

$$j\, r_2\, \dot{\theta}_2\, e^{j\theta_2} + j\, r_3\, \dot{\theta}_3\, e^{j\theta_3} + j\, r_8\, \dot{\theta}_8\, e^{j\theta_8} + j\, r_5\, \dot{\theta}_5\, e^{j\theta_5} - j\, r_6\, \dot{\theta}_6\, e^{j\theta_6} = 0 \qquad \text{FOR} \qquad \dot{\theta}_8 = \dot{\theta}_4$$

REAL: $-r_2\dot{\theta}_2 \sin\theta_2 - r_3\dot{\theta}_3 \sin\theta_3 - r_8\dot{\theta}_4 \sin\theta_8 - r_5\dot{\theta}_5 \sin\theta_5 + \dot{r}_6 \cos\theta_6 = 0$

IMAG: $j\, r_2\, \dot{\theta}_2 \cos\theta_2 + j\, r_3\dot{\theta}_3 \cos\theta_3 + j\, r_8\, \dot{\theta}_4 \cos\theta_8 + j\, r_5\dot{\theta}_5 \cos\theta_5 - j\, \dot{r}_6 \cos\theta_6 = 0$

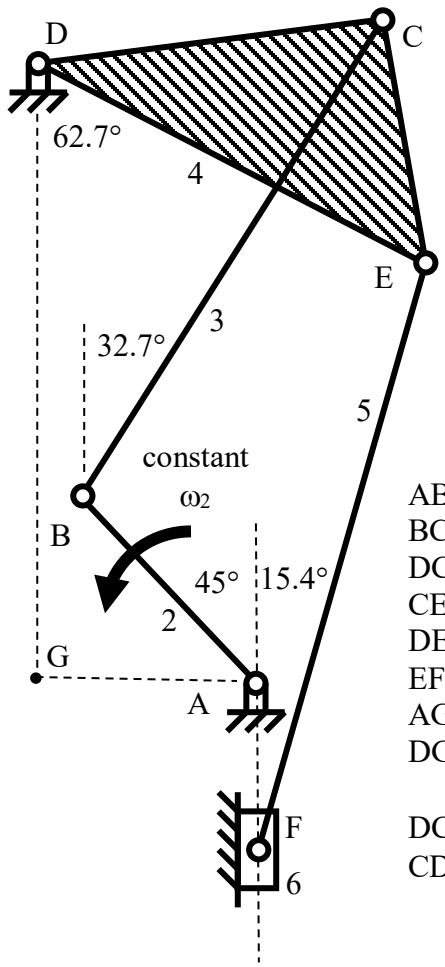
$$\begin{bmatrix} -r_3\sin\theta_3 & r_4\sin\theta_4 & 0 & 0 \\ r_3\cos\theta_3 & -r_4\cos\theta_4 & 0 & 0 \\ -r_3\sin\theta_3 & -r_8\sin\theta_8 & -r_5\sin\theta_5 & 0 \\ r_3\cos\theta_3 & r_8\cos\theta_8 & r_5\cos\theta_5 & 1 \end{bmatrix} \begin{Bmatrix} \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot r_6 \end{Bmatrix} = \begin{Bmatrix} r_2\dot\theta_2\sin\theta_2 \\ -r_2\dot\theta_2\cos\theta_2 \\ r_2\dot\theta_2\sin\theta_2 \\ -r_2\dot\theta_2\cos\theta_2 \end{Bmatrix}$$

$$\begin{bmatrix} -3.004 & 0.327 & 0 & 0 \\ 1.929 & -2.216 & 0 & 0 \\ -3.004 & 1.583 & 3.673 & 0 \\ 1.929 & 0.234 & -1.012 & 1 \end{bmatrix} \begin{Bmatrix} \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot r_6 \end{Bmatrix} = \begin{Bmatrix} 28.43 \\ 28.43 \\ 28.43 \\ 28.43 \end{Bmatrix}$$

using MATLAB

$$\begin{Bmatrix} \dot\theta_3 \\ \dot\theta_4 \\ \dot\theta_5 \\ \dot r_6 \end{Bmatrix} = \begin{Bmatrix} -12.00 \text{ rad/sec} \\ -23.27 \text{ rad/sec} \\ +7.96 \text{ rad/sec} \\ +65.07 \text{ cm/sec} \end{Bmatrix}$$

Determine the angular acceleration of links 2, 3, 4 and 5 as well as the acceleration of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.



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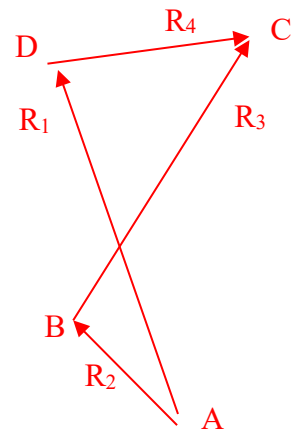
BC	= 3.57 cm
DC	= 2.24 cm
CE	= 1.60 cm
DE	= 2.74 cm
EF	= 3.81 cm
AG	= 1.42 cm
DG	= 3.81 cm
DCE	= 90°
CDE	= 35.7°



$\omega_2 = +8\pi$ rad/sec $\alpha_2 = 0$ angular velocities from velocity solution

	r [cm]	θ [deg]	$\dot{\theta}$ [rad/sec]	$r\dot{\theta}^2$ [cpss]	$\ddot{\theta}$ [rad/s/s]	$r\ddot{\theta}$ [cpss]
1	4.07	110.4				
2	1.60	135.0	+25.13	+1010.6	0	0
3	3.57	57.3	-12.00	+514.1	?	
4	2.24	8.4	-23.27	+1212.9	?	

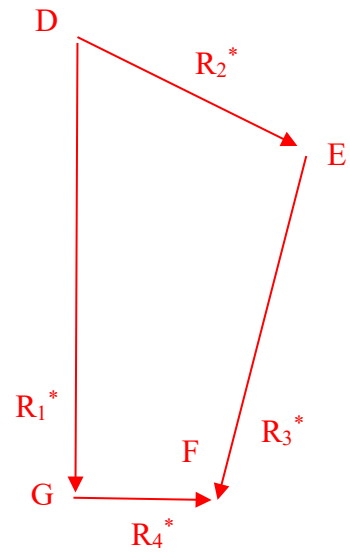
$$\begin{bmatrix} -r_3\sin\theta_3 & r_4\sin\theta_4 \\ r_3\cos\theta_3 & -r_4\cos\theta_4 \end{bmatrix}\begin{Bmatrix}\ddot{\theta}_3\\ \ddot{\theta}_4\end{Bmatrix} = \begin{Bmatrix} r_2\ddot{\theta}_2\sin\theta_2 + r_2\dot{\theta}_2^2\cos\theta_2 + r_3\dot{\theta}_3^2\cos\theta_3 - r_4\dot{\theta}_4^2\cos\theta_4 \\ -r_2\ddot{\theta}_2\cos\theta_2 + r_2\dot{\theta}_2^2\sin\theta_2 + r_3\dot{\theta}_3^2\sin\theta_3 - r_4\dot{\theta}_4^2\sin\theta_4 \end{Bmatrix}$$

$$\begin{bmatrix} -3.004 & 0.327 \\ 1.929 & -2.216 \end{bmatrix}\begin{Bmatrix}\ddot{\theta}_3\\ \ddot{\theta}_4\end{Bmatrix} = \begin{Bmatrix} -1636.8 \text{ cpss} \\ 970.0 \text{ cpss} \end{Bmatrix} \qquad \begin{Bmatrix}\ddot{\theta}_3\\ \ddot{\theta}_4\end{Bmatrix} = \begin{bmatrix} -0.3677 & -0.0543 \\ -0.3201 & -0.4985 \end{bmatrix}\begin{Bmatrix} -1636.8 \\ 970.0 \end{Bmatrix} = \begin{Bmatrix} 549.3 \text{ rad/s/s} \\ 40.4 \text{ rad/s/s} \end{Bmatrix}$$

closed form

$\ddot{\theta}_3 = (- r_2\ \ddot{\theta}_2 \sin(\theta_2-\theta_4) - r_2\ \dot{\theta}_2^2 \cos(\theta_2-\theta_4) - r_3\ \dot{\theta}_3^2\ \cos(\theta_3-\theta_4) + r_4\ \dot{\theta}_4^2\ ) / r_3 \sin(\theta_3-\theta_4) = 549.3$ rad/s/s

$\ddot{\theta}_4 = (- r_2\ \ddot{\theta}_2 \sin(\theta_2-\theta_3) - r_2\ \dot{\theta}_2^2 \cos(\theta_2-\theta_3) - r_3\ \dot{\theta}_3^2\ + r_4\ \dot{\theta}_4^2\ \cos(\theta_4-\theta_3)) / r_4 \sin(\theta_3-\theta_4) = 40.4$ rad/s/s



$\omega_2^* = \omega_4 = -23.27$ rad/sec $\alpha_2^* = \alpha_4 = +40.4$ rad/s/s $\omega_3^* = \omega_5 = -23.27$ rad/sec

	r^* [cm]	θ^* [deg]	$\dot{\theta}^*$ [rad/sec]	$r\dot{\theta}^{2*}$ [cpss]	$\ddot{\theta}^*$ [rad/s/s]	$r\ddot{\theta}^*$ [cpss]
1	4.93	270				
2	2.74	332.7	-23.27	+1483.7	+40.4	+110.7
3	3.81	254.6	+7.96	+241.4	?	
4	1.42	0				

$$\begin{bmatrix} -r_3 \sin\theta_3 & -\cos\theta_1 \\ r_3 \cos\theta_3 & -\sin\theta_1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{Bmatrix} r_2\ddot{\theta}_2 \sin\theta_2 + r_2\dot{\theta}_2^2 \cos\theta_2 + r_3\dot{\theta}_3^2 \cos\theta_3 \\ -r_2\ddot{\theta}_2 \cos\theta_2 + r_2\dot{\theta}_2^2 \sin\theta_2 + r_3\dot{\theta}_3^2 \sin\theta_3 \end{Bmatrix}$$

$$\begin{bmatrix} -3.673 & 0 \\ -1.012 & 1 \end{bmatrix} \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{Bmatrix} +1203.6 \text{ cpss} \\ -1011.6 \text{ cpss} \end{Bmatrix} \qquad \begin{Bmatrix} \ddot{\theta}_3 \\ \ddot{r}_1 \end{Bmatrix} = \begin{bmatrix} 0.2723 & 0 \\ 0.2755 & 1 \end{bmatrix} \begin{Bmatrix} +1203.6 \text{ cpss} \\ -1011.6 \text{ cpss} \end{Bmatrix} = \begin{Bmatrix} +327.7 \text{ rad/s/s} \\ -680.0 \text{ cm/s/s} \end{Bmatrix}$$