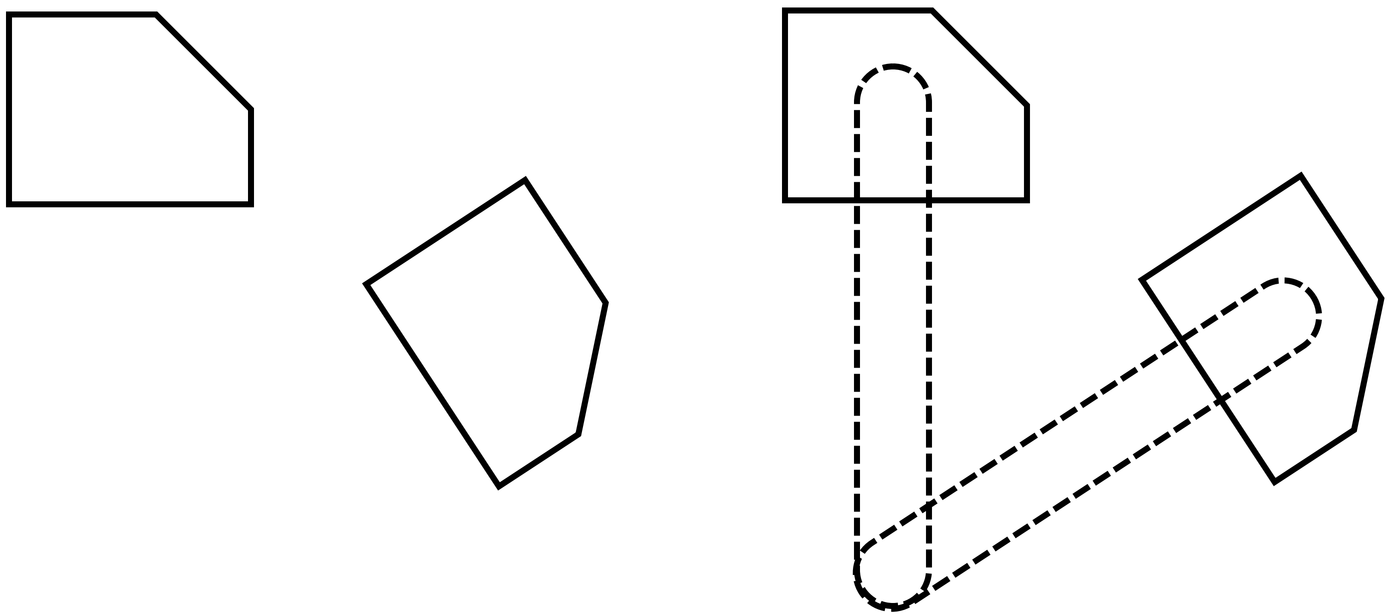
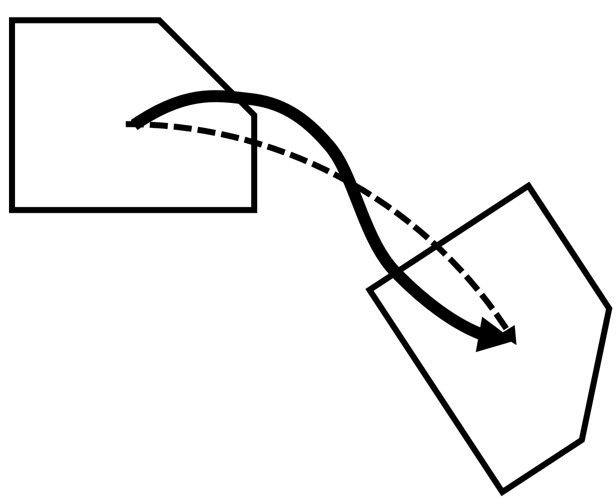
Finite Center of Rotation



Instantaneous Center of Rotation



Instantaneous Centers

Instant center - unique point at which two objects in general planar motion have the same velocity

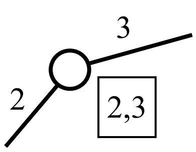
Absolute center – instant center between an object and ground

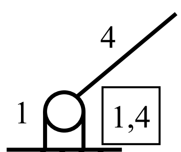
Relative center – instant center between two moving objects

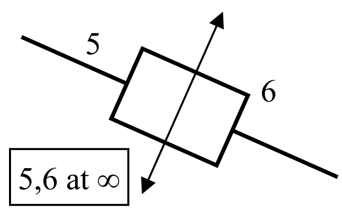
Velocity of a point – tangent to the path of the point, perpendicular to the line that joins the point and the relative instant center of rotation

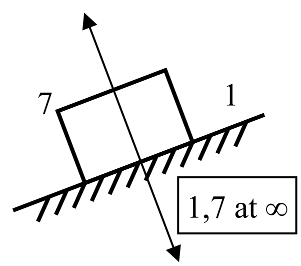
Kennedy-Aronhold theorem – relative instant centers between three bodies are collinear

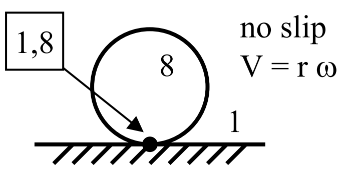
J1 joints define instant centers





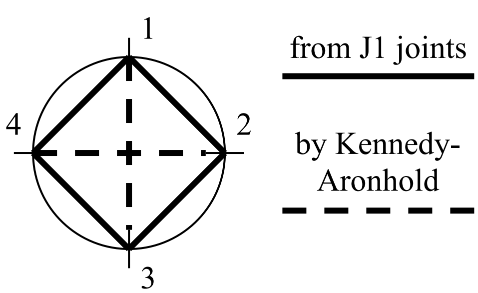


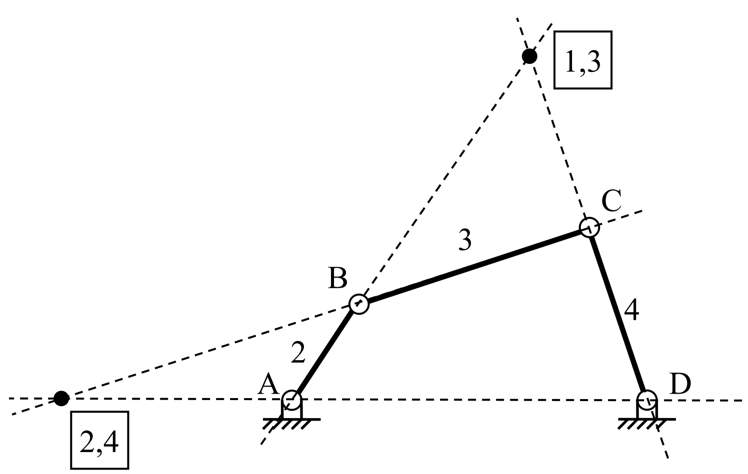




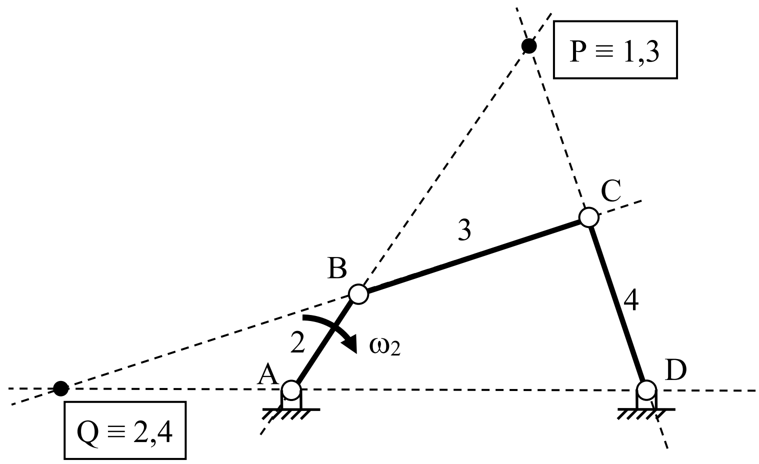
1,2 and 2,3 are collinear with 1,3
1,4 and 3,4 are collinear with 1,3

1,2 and 1,4 are collinear with 2,4
2,3 and 3,4 are collinear with 2,4

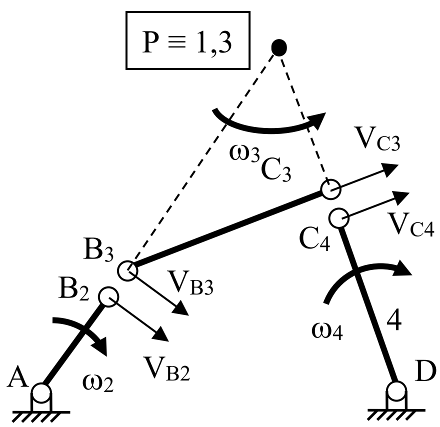




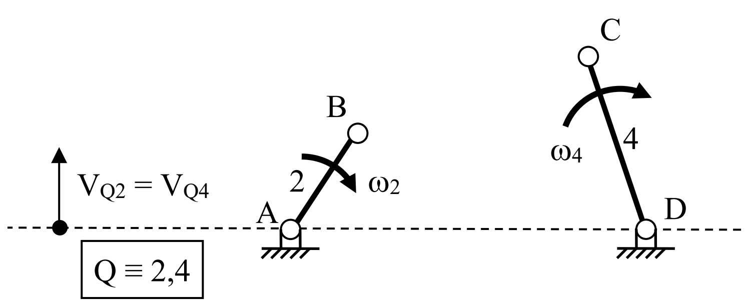
Velocity Transfer



$V_{B2} = \omega_2\ (AB)$
$V_{B2} = V_{B3}$
$V_{B3} = \omega_3\ (BP)$
$V_{C3} = \omega_3\ (CP)$
$V_{C3} = V_{C4}$
$V_{C4} = \omega_4\ (CD)$

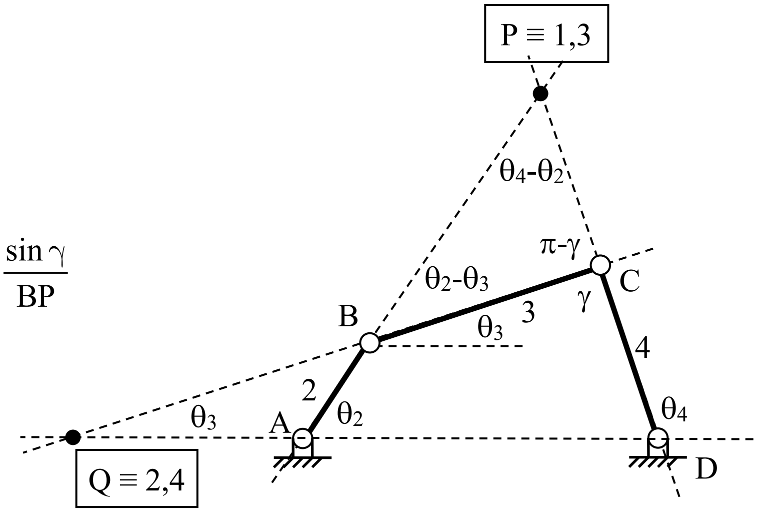


$V_{Q2} = \omega_2\ (AQ)$
$V_{Q2} = V_{Q4}$
$V_{Q4} = \omega_4\ (DQ)$



$r_1 = AD \quad r_2 = AB \quad r_3 = BC \quad r_4 = CD$

$\gamma = \theta_4 - \theta_3 \qquad \sin(\pi - \gamma) = \sin\gamma$



triangle PBC

$$\frac{\sin(\theta_4 - \theta_2)}{r_3} = \frac{\sin(\theta_2 - \theta_3)}{CP} = \frac{\sin(\pi - \gamma)}{BP} = \frac{\sin\gamma}{BP}$$

$$BP = \frac{r_3 \sin\gamma}{\sin(\theta_4 - \theta_2)} \qquad CP = \frac{r_3 \sin(\theta_2 - \theta_3)}{\sin(\theta_4 - \theta_2)}$$

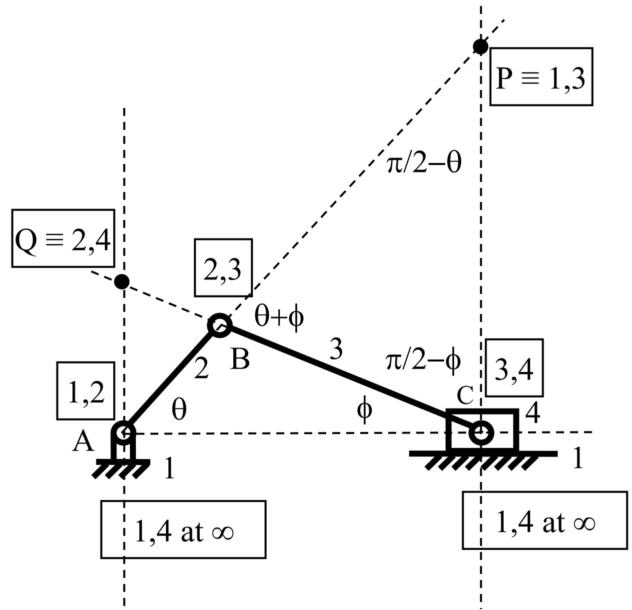
$$V_{B2} = \omega_2 AB = V_{B3} = \omega_3 BP \qquad \omega_3 = \omega_2 \frac{AB}{BP} = \omega_2 \frac{r_2 \sin(\theta_4 - \theta_2)}{r_3 \sin\gamma}$$

$$V_{C3} = \omega_3 CP = V_{C4} = \omega_4 CD \qquad \omega_4 = \omega_3 \frac{CP}{CD} = \omega_3 \frac{r_3 \sin(\theta_2 - \theta_3)}{r_4 \sin(\theta_4 - \theta_2)} = \omega_2 \frac{r_2 \sin(\theta_2 - \theta_3)}{r_4 \sin\gamma}$$

Note: will not provide correct ± signs for CW and CCW

triangle QDC

$$\frac{\sin\theta_3}{r_4} = \frac{\sin\gamma}{DQ} \qquad DQ = \frac{r_4 \sin\gamma}{\sin\theta_3} \qquad AQ = DQ - r_4$$



$R = AB \quad L = BC \quad s = AC$

$$\sin\phi = \frac{R}{L}\sin\theta \qquad s = R\cos\theta + L\cos\phi$$

$V_{B2} = \omega_2\,(AB)$
$V_{B2} = V_{B3}$
$V_{B3} = \omega_3\,(BP)$
$V_{C3} = \omega_3\,(CP)$
$V_{C3} = V_{C4}$

$V_{Q2} = \omega_2\,(AQ)$
$V_{Q2} = V_{Q4}$
$V_{Q4} = V_{C4}$

triangle PBC

$$\frac{\sin\left(\frac{\pi}{2}-\theta\right)}{L} = \frac{\sin(\theta+\phi)}{CP} = \frac{\sin\left(\frac{\pi}{2}-\phi\right)}{BP}$$

$$BP = \frac{L\sin\left(\frac{\pi}{2}-\phi\right)}{\sin\left(\frac{\pi}{2}-\theta\right)} = \frac{L\cos\phi}{\cos\theta} \qquad CP = \frac{L\sin(\theta+\phi)}{\sin\left(\frac{\pi}{2}-\theta\right)} = \frac{L\sin(\theta+\phi)}{\cos\theta}$$

$$V_{B2} = \dot\theta\,AB = V_{B3} = \dot\phi\,BP \qquad \dot\phi = \dot\theta\,\frac{AB}{BP} = \dot\theta\,\frac{R\cos\theta}{L\cos\phi}$$

$$V_{C3} = \dot\phi\,CP = V_{C4} \qquad V_C = \dot\phi\,CP = \dot\phi\,\frac{L\sin(\theta+\phi)}{\cos\theta} = R\dot\theta\,\frac{\sin(\theta+\phi)}{\cos\phi}$$

Note: will not provide correct ± signs for CW and CCW

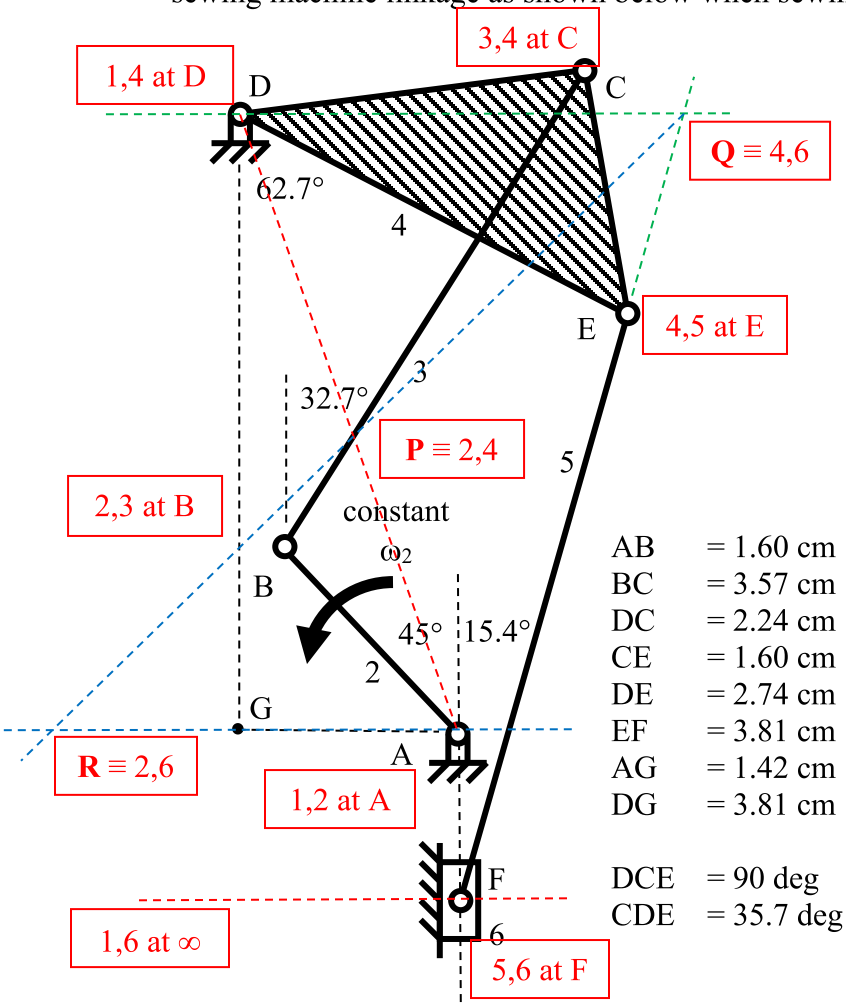
triangle QAC

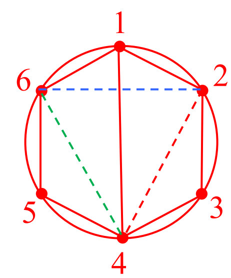
$$AQ = s\tan\phi$$

$$V_{Q2} = \dot\theta\,AQ = V_{Q4} = V_{C4} \qquad V_C = \dot\theta\,s\tan\phi = \dot\theta\left(R\cos\theta + L\cos\phi\right)\frac{\sin\phi}{\cos\phi} = R\dot\theta\,\frac{\sin(\theta+\phi)}{\cos\phi}$$

Sewing Machine

Determine the angular velocity of links 2, 3, 4 and 5 as well as the velocity of needle 6 for the sewing machine linkage as shown below when sewing at 4 stitches per second constant speed.





1,2 and 1,4 intersect 2,4 ≡ P
2,3 and 3,4

1,6 and 1,4 intersect 4,6 ≡ Q
4,5 and 5,6

1,2 and 1,6 intersect 2,6 ≡ R
2,4 and 4,6

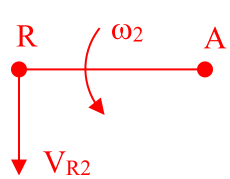
AB	= 1.60 cm
BC	= 3.57 cm
DC	= 2.24 cm
CE	= 1.60 cm
DE	= 2.74 cm
EF	= 3.81 cm
AG	= 1.42 cm
DG	= 3.81 cm
DCE	= 90 deg
CDE	= 35.7 deg

AG = 1.15 inch on page = 1.42 cm actual
AR = 2.10 inch on page = 2.59 cm actual

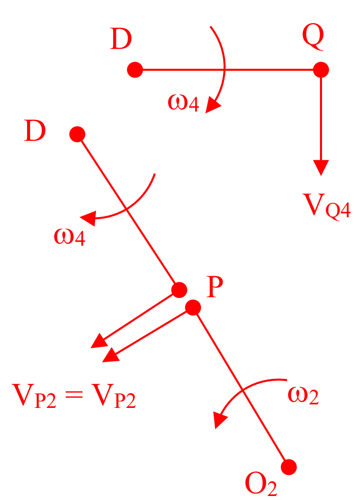
AP = 1.64 inch on page = 2.00 cm actual
DP = 1.76 inch on page = 2.17 cm actual
DQ = 2.36 inch on page = 2.88 cm actual

$\omega_2 = 8\pi$ rad/s CCW

$V_{R2} = \omega_2 (AR) = 65.09$ cps
$V_{R2} = V_{R6}$
$V_{R6} = V_{F6}$

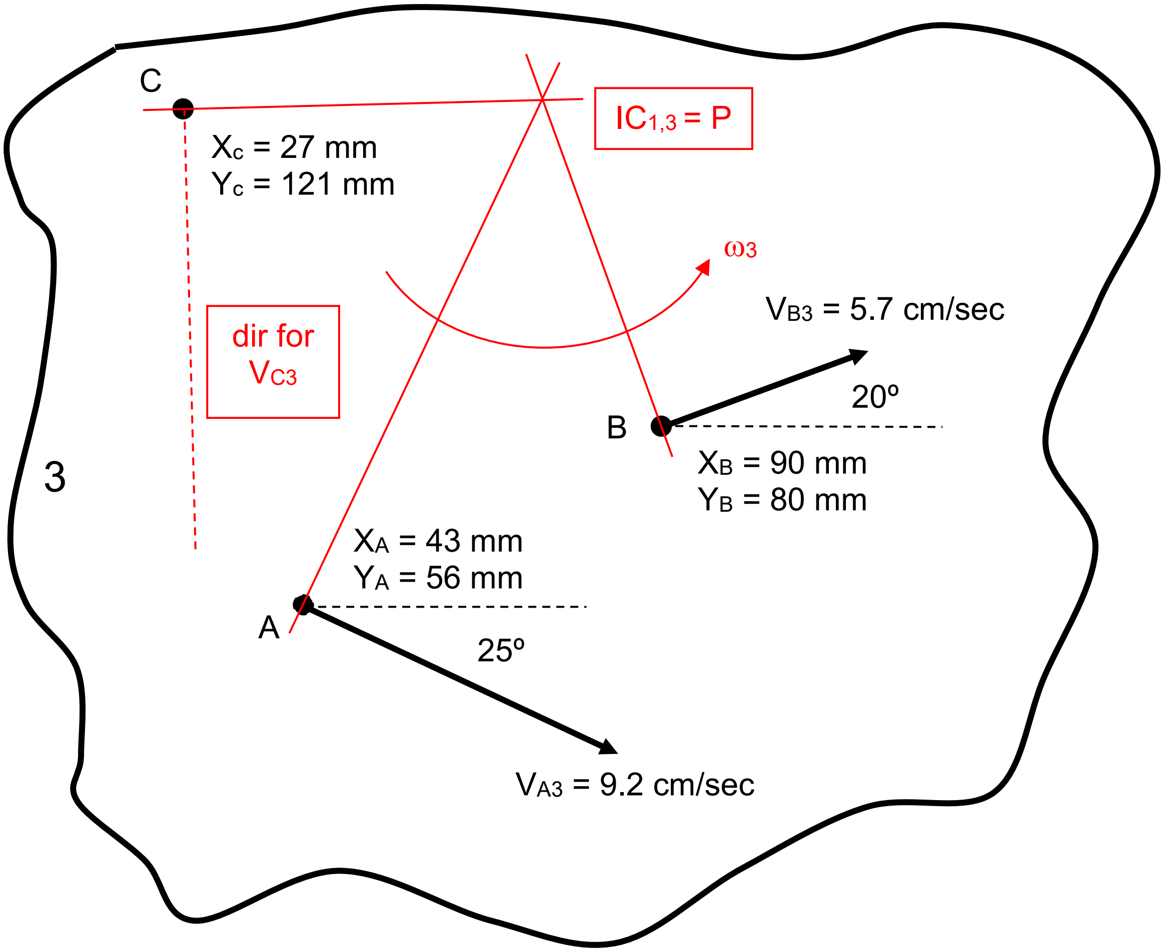


$V_{P2} = \omega_2 (AP) = 50.27$ cps
$V_{P2} = V_{P4}$
$V_{P4} = \omega_4 (DP)$ $\omega_4 = 23.16$ rad/s CW
$V_{Q4} = \omega_4 (DQ) = 66.71$ cps
$V_{Q4} = V_{Q6}$
$V_{Q6} = V_{F6}$
(2.4 % different from complex numbers)



Rigid Body

Determine the velocity of point C on rigid body link 3. The rigid body and the velocity vectors are drawn to scale. Link 3 is NOT pinned to the ground. Show your work.



define P at $IC_{1,3}$	$V_{A3} \perp AP$	$V_{B3} \perp BP$	$V_{C3} \perp CP$
measurements	AP = 73.7 mm	BP = 45.8 mm	CP = 47.5 mm $\angle$ 182º

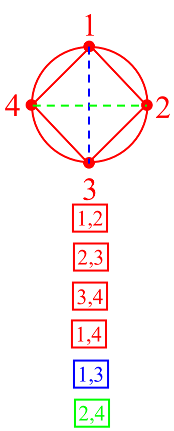
$V_{A3} = \omega_3\,(AP)$ ω_3 = 1.253 rad/sec CCW

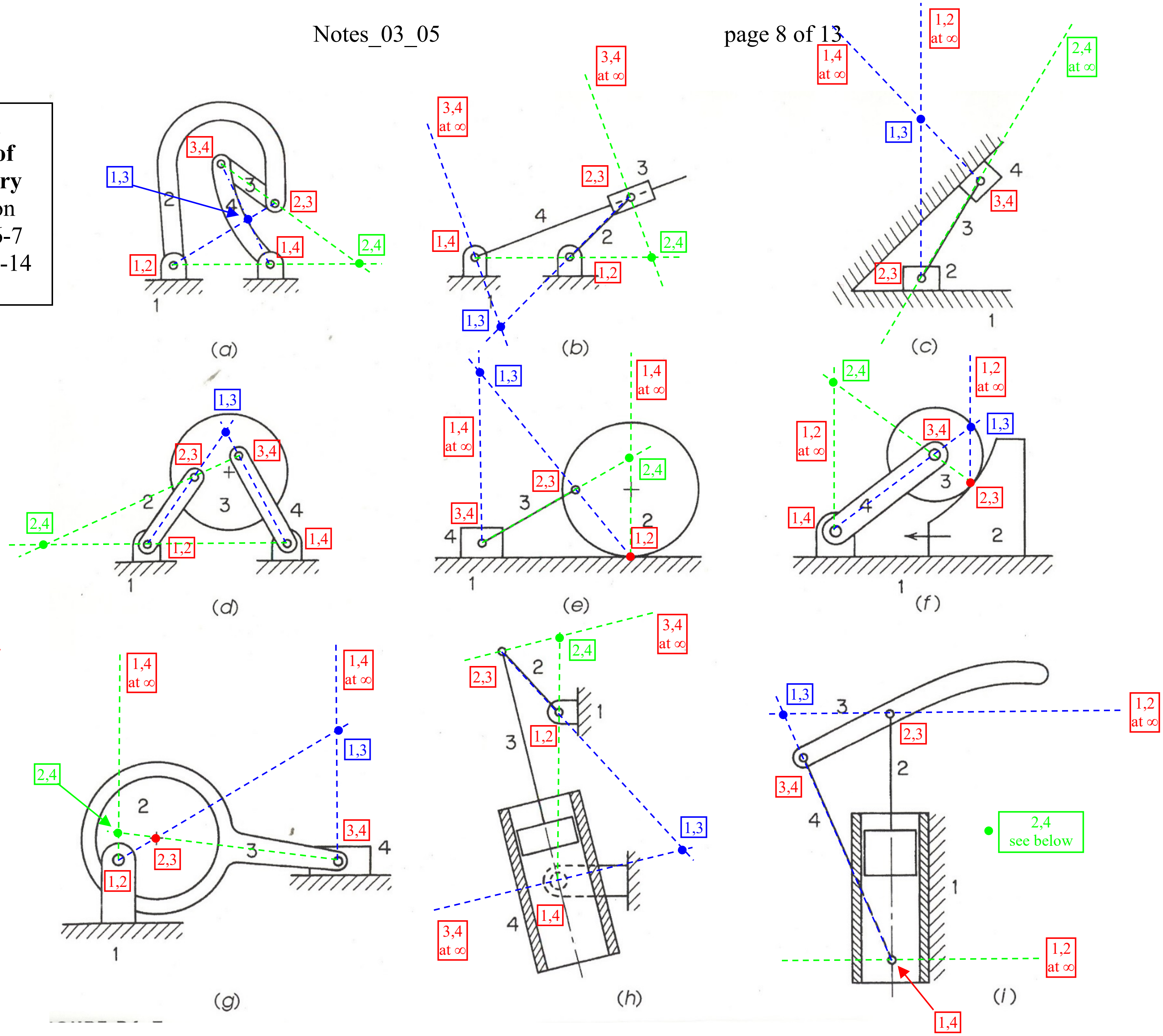
$V_{B3} = \omega_3\,(BP)$ ω_3 = 1.244 rad/sec CCW

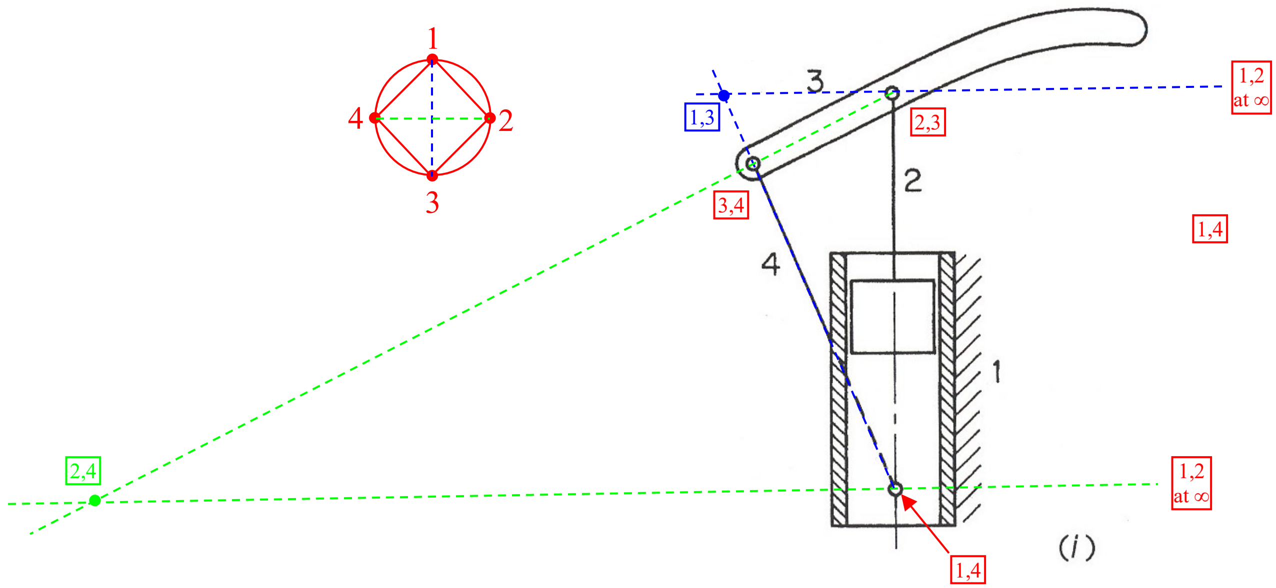
use ω_3 = 1.249 rad/sec CCW average

$V_{C3} = \omega_3\,(CP)$ = 59.3 mmps $\angle$ 272º

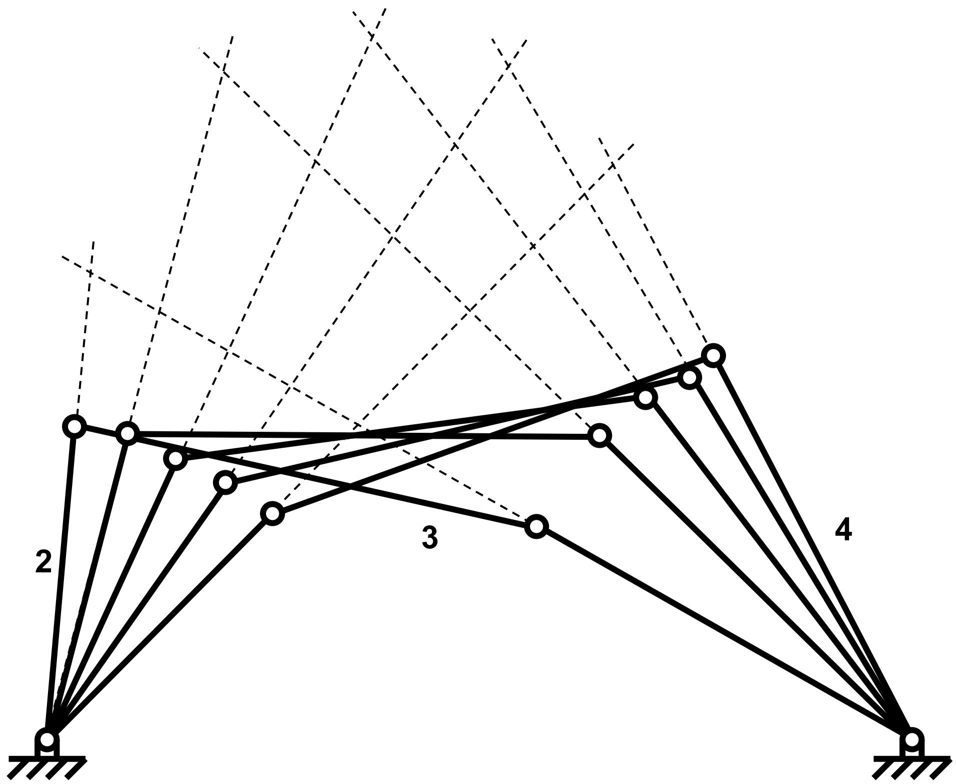
Norton
Design of Machinery
3rd edition
Figure P6-7
Problem 6-14

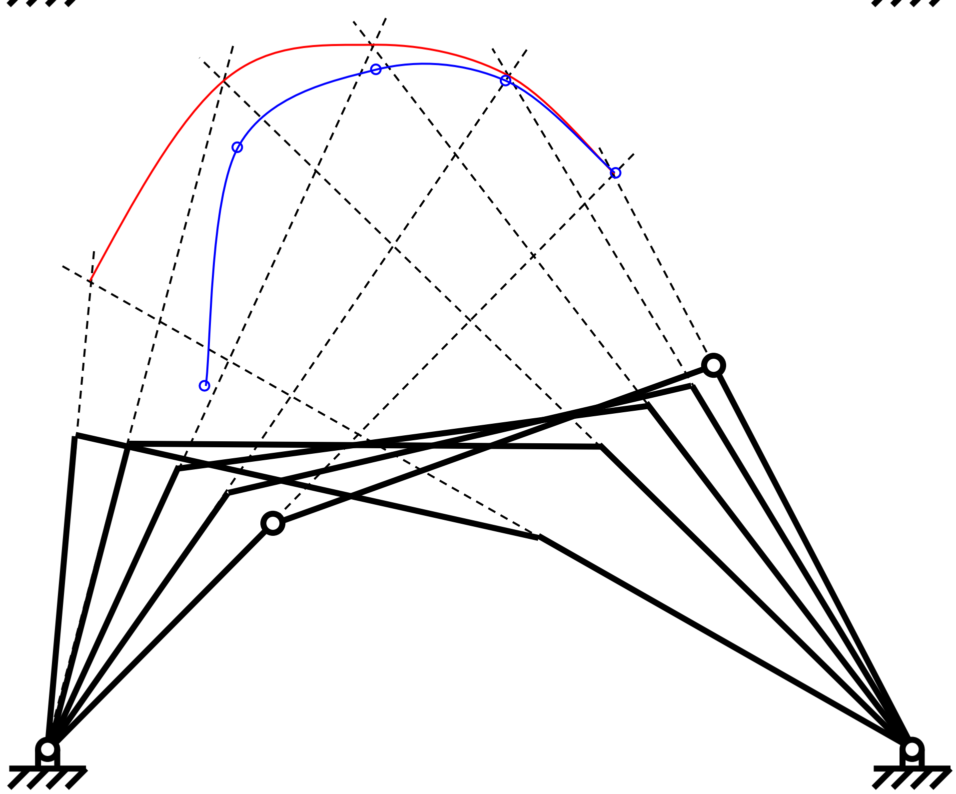


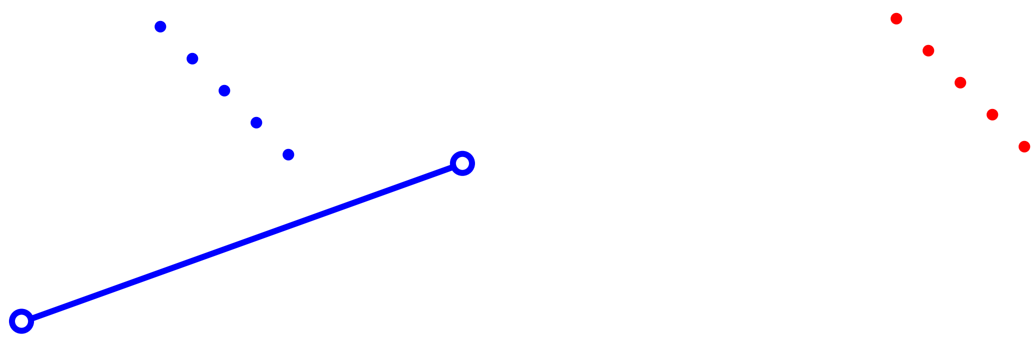



1
2
3
4
3
2
4
1
1,3
2,3
3,4
1,2
at ∞
1,4
2,4
1,2
at ∞
1,4
(i)

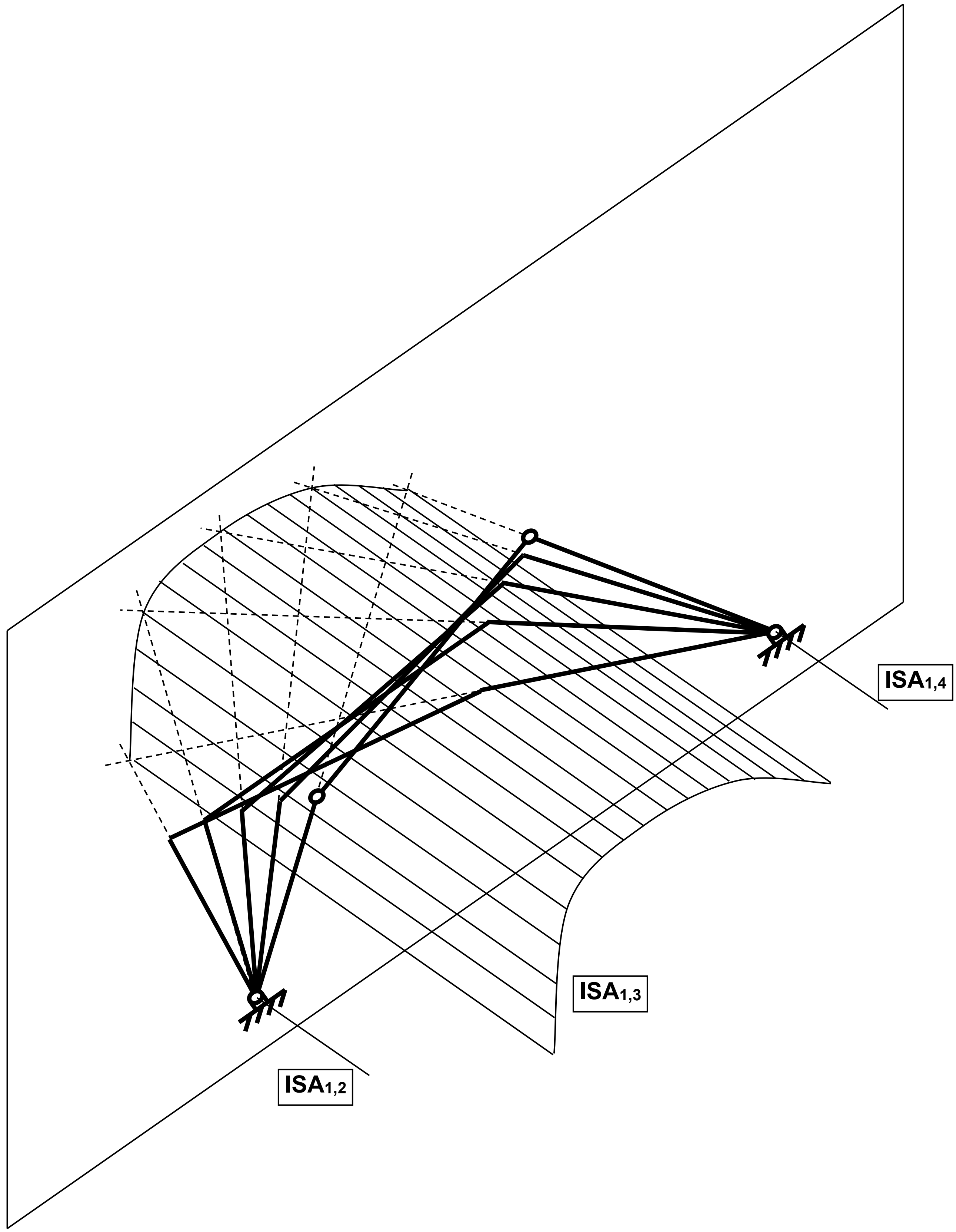
Centrodes of Four Bar







Instantaneous Screw Axes



Knee ACL and PCL