Two-Dimensional Vector and Matrix Notation

$\{r_i\}$ global position of the origin of the reference frame attached to body i

$\{r_i\}^P$ global position of point P attached to body i

example $\{r_4\}^B = \begin{Bmatrix} x_4{}^B \\ y_4{}^B \end{Bmatrix}$ global position of point B attached to body 4

$\{\dot{r}_i\}$ global velocity of the origin of the reference frame attached to body i

$\{\dot{r}_i\}^P$ global velocity of point P attached to body i

$\{\ddot{r}_i\}$ global acceleration of the origin of the reference frame attached to body i

$\{\ddot{r}_i\}^P$ global acceleration of point P attached to body i

$\{\dddot{r}_i\}$ global jerk of the origin of the reference frame attached to body i

$\{\dddot{r}_i\}^P$ global jerk of point P attached to body i

$\{\ddddot{r}_i\}$ global snap of the origin of the reference frame attached to body i

$\{\ddddot{r}_i\}^P$ global snap of point P attached to body i

$\{s_i\}^{'P}$ position of point P on body i relative to the reference frame for body i measured in local body-fixed directions

example $\{s_4\}^{'B} = \begin{Bmatrix} x_4{}^{'B} \\ y_4{}^{'B} \end{Bmatrix}$ location of point B on body 4 relative to the reference frame for body 4 measured in local body-fixed directions for body 4

$\{s_i\}^P$ position of point P on body i relative to the reference frame for body i but measured in global directions

$\{d_{ij}\}$ relative location between two points on bodies i and j measured in global directions

example $\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$ relative location of point P on body j with respect to point P on body i measured in global directions

ϕ_i attitude angle for reference frame attached to body i

ϕ_{ij} attitude angle of body j with respect to reference frame attached to body i

example $\phi_{ij} = \phi_j - \phi_i$

$\dot{\phi}_i$ angular velocity of body i

$\ddot{\phi}_i$ angular acceleration of body i

$\dddot{\phi}_i$ angular jerk of body i

$\ddddot{\phi}_i$ angular snap of body i

$\{q_i\}$ generalized coordinates for body i

example $\{q_i\} = \begin{Bmatrix} \{r_i\} \\ \phi_i \end{Bmatrix}$

$\{\dot{q}_i\}$ generalized velocity for body i

$\{\ddot{q}_i\}$ generalized acceleration for body i

$\{\dddot{q}_i\}$ generalized jerk for body i

$\{\ddddot{q}_i\}$ generalized snap for body i

$[A_i]$ orthonormal rotation matrix that describes attitude of body i

example $\lfloor A_i \rfloor = \begin{bmatrix} \cos\phi_i & -\sin\phi_i \\ \sin\phi_i & \cos\phi_i \end{bmatrix}$

example $\{s_i\}^P = \lfloor A_i \rfloor \{s_i\}'^P$ rotation matrix converts information in local body-fixed directions into global directions

$\{\hat{f}_i\}$ global direction of unit vector along local x axis attached to body i

$\{\hat{g}_i\}$ global direction of unit vector along local y axis attached to body i example $\lfloor A_i \rfloor = \left\lfloor \{\hat{f}_i\} \quad \{\hat{g}_i\} \right\rfloor$ unit directions of local axes for body i

$[B_i]$ second rotation matrix

example $\lfloor B_i \rfloor = [R]\lfloor A_i \rfloor = \begin{bmatrix} -\sin\phi & -\cos\phi \\ \cos\phi & -\sin\phi \end{bmatrix}$

$[R]$ rotator matrix $[R] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

$\lfloor A_{ij} \rfloor$ rotation matrix that describes attitude of body j with respect to body i

example $\lfloor A_{ij} \rfloor = \begin{bmatrix} \cos\phi_{ij} & -\sin\phi_{ij} \\ \sin\phi_{ij} & \cos\phi_{ij} \end{bmatrix} = [A_i]^T \lfloor A_j \rfloor$

$\{F_{\text{on } i}\}^P$ force on body i acting through point P measured in global directions

$\{F_{\text{on } i}\}'^P$ force on body i acting through point P measured in body-fixed directions local to body i

$T_{\text{on } i}$ torque on body i

$\{Q_{\text{on } i}\}$ generalized force on body i measured in global directions

example $\{Q_{\text{on } i}\} = \left\{ \begin{matrix} \{F_{\text{on } i}\} \\ T_{\text{on } i} \end{matrix} \right\}$

Numbering and lettering

Bodies should be numbered consecutively beginning with 1. Body 1 is typically reserved for ground.

Points should be lettered.

Point G is typically reserved for the mass center of a body.

Point T is seldom used in that it causes confusion with the vector/matrix transpose operator.

Subscripts and superscripts outside vector/matrix brackets

Post-superscript prime outside vector brackets denotes information measured in local body-fixed directions.

Post-superscript letters outside vector brackets denote information related to a specific point.

Post-subscripts outside vector/matrix brackets are occasionally used for iteration or time indices.

Pre-superscripts and pre-subscripts are typically not used outside brackets.

Subscripts and superscripts inside vector/matrix brackets

Post-superscripts inside vector/matrix brackets are occasionally used for iteration or time indices.

Post-subscript numerals inside vector/matrix brackets are typically used for body numbers.

Post-subscript variables inside vector/matrix brackets denote partial derivative operators.

Pre-superscripts and pre-subscripts are typically not used inside brackets.

General vector/matrix operations

$\{\ \}^{T}, [\ ]^{T}$	vector/matrix transpose
$[\ ]^{-1}$	matrix inverse
$\det[\ ]$	determinant of matrix
$\mathrm{tr}[\ ]$	trace of matrix (sum of diagonal elements)
$\{\mathrm{diag}[\ ]\}$	diagonal elements of matrix rearranged into column vector
$[\mathrm{diag}\{\ \}]$	elements of vector placed into a diagonal matrix
$[\ ]^{n}$	matrix to power n
$\mathrm{norm}\{\ \}$	scalar norm of vector (magnitude)
$\{\hat{a}\}$	unit vector
$[I_n]$	identity matrix of order n
$\{0\},[0]$	vector/matrix of zeros
$[R]$	2x2 skew-symmetric rotation operator $[R]=\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$ (used for cross-product)