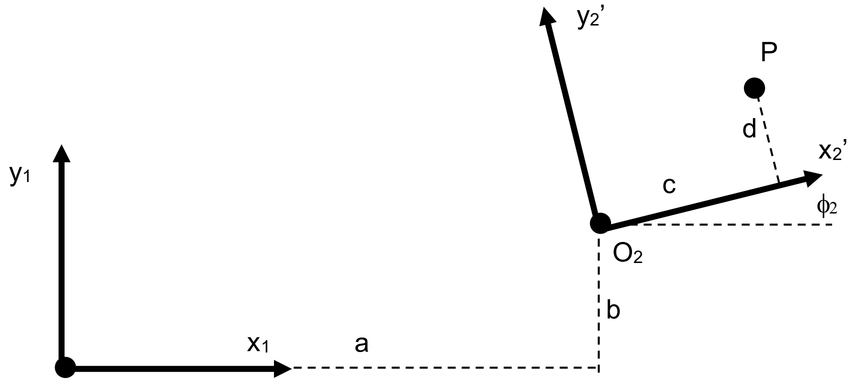
Two-Dimensional Coordinate Transformations



$$\{r_i\}^P = \{r_i\} + [A_i]\{s_i\}'^P$$

$$\begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix}^P = \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix} + \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} x_2 \\ y_2 \end{Bmatrix}'^P$$

$$\begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} + \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix}$$

$$\begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} = \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix} + \begin{bmatrix} \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} & \underline{\hspace{2cm}} \end{bmatrix} \begin{Bmatrix} \underline{\hspace{2cm}} \\ \underline{\hspace{2cm}} \end{Bmatrix}$$

$$[A_i] = \begin{bmatrix} C\phi_i & -S\phi_i \\ S\phi_i & C\phi_i \end{bmatrix}$$

$$[A_i] = \left[\{\hat{f}_i\} \quad \{\hat{g}_i\} \right] \qquad \{\hat{f}\} \text{ and } \{\hat{g}\} \text{ are unit vectors}$$

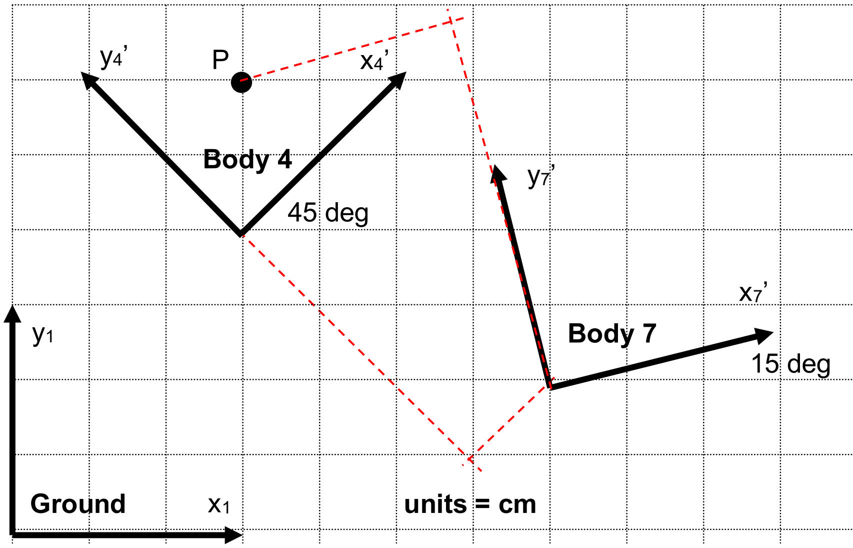
$$\{\hat{f}_i\}' = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} \qquad \{\hat{g}_i\}' = \begin{Bmatrix} 0 \\ 1 \end{Bmatrix} \qquad \{\hat{f}_i\} = [A_i]\{\hat{f}_i\}' \qquad \{\hat{g}_i\} = [A_i]\{\hat{g}_i\}'$$

$$\{s_i\}^P = [A_i]\{s_i\}'^P \qquad \{s_i\}'^P = [A_i]^{-1}\{s_i\}^P$$

[A] matrices are orthonormal $[A]^{-1} = [A]^T$

- all columns are unit vectors
- all columns are mutually orthogonal
- all rows are unit vectors
- all rows are mutually orthogonal
- det([A]) = +1

Provide numerical values for the three coordinate transformations shown below.



$$\{r\}^P = \begin{Bmatrix} 3 \\ 6 \end{Bmatrix} \qquad \{s_4\}'^P = \begin{Bmatrix} \underline{1.4142\ \text{cm}} \\ \underline{1.4142\ \text{cm}} \end{Bmatrix} \qquad \{s_7\}'^P = \begin{Bmatrix} \underline{\sim -2.8\ \text{cm}} \\ \underline{\sim +5\ \text{cm}} \end{Bmatrix}$$

$$\{r_4\}^P = \{r_4\} + [A_4]\{s_4\}'^P$$

$$\{r_4\} = \begin{Bmatrix} \underline{3\ \text{cm}} \\ \underline{4\ \text{cm}} \end{Bmatrix} \qquad [A_4] = \begin{bmatrix} \underline{C(45°)} & \underline{-\ S(45°)} \\ \underline{S(45°)} & \underline{C(45°)} \end{bmatrix} \qquad \phi_4 = +45°$$

$$\{r_7\}^P = \{r_7\} + [A_7]\{s_7\}'^P$$

$$\{r_7\} = \begin{Bmatrix} \underline{7\ \text{cm}} \\ \underline{2\ \text{cm}} \end{Bmatrix} \qquad [A_7] = \begin{bmatrix} \underline{C(15°)} & \underline{-\ S(15°)} \\ \underline{S(15°)} & \underline{C(15°)} \end{bmatrix} \qquad \phi_7 = +15°$$

ϕ_{ij} attitude angle for body j with respect to body i $\qquad \phi_{ij} = \phi_j - \phi_i$

$\lfloor A_{ij} \rfloor$ attitude matrix for body j with respect to body i

$$\lfloor A_{ij} \rfloor = \begin{bmatrix} \cos\phi_{ij} & -\sin\phi_{ij} \\ \sin\phi_{ij} & \cos\phi_{ij} \end{bmatrix} = [A_i]^T \lfloor A_j \rfloor$$

$$[A_{47}] = \begin{bmatrix} \underline{C(-30°)} & \underline{-\ S(-30°)} \\ \underline{S(-30°)} & \underline{C(-30°)} \end{bmatrix} \qquad \phi_{47} = -30°$$

check using MATLAB

$$[A_{47}] = [A_4]^T [A_7]$$

```
% n0402.m - check Notes_04_02
% HJSIII, 14.02.03

clear

% constants
d2r = pi / 180;

% body 4
r4 = [ 3  4 ]';
s4pP=[ 1.4142  1.4142 ]';
phi4 = 45 * d2r;
A4 = [ cos(phi4)  -sin(phi4) ;
       sin(phi4)   cos(phi4) ];
r4P = r4 + A4 *s4pP

% body 7
r7 = [ 7  2 ]';
s7pP=[ -2.8  5 ]';
phi7 = 15 * d2r;
A7 = [ cos(phi7)  -sin(phi7) ;
       sin(phi7)   cos(phi7) ];
r7P = r7 + A7 *s7pP

% relative attitude
A47 = A4' * A7

% bottom of n0402


+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++

>> n0402

r4P =

    3.0000
    6.0000


r7P =

    3.0013
    6.1049


A47 =

    0.8660    0.5000
   -0.5000    0.8660
```