Two-dimensional position, velocity and acceleration solutions

Kinematically driven motion

1.0 Initialize

1.1 constants

1.2 global locations of fixed points $\{r_1\}^P$

1.3 local blueprint locations $\{s_i\}'^{\,P}$

2.0 Initial estimates

2.1 global locations of origins $\{r_i\}$

2.2 orientation angles ϕ_i

2.3 assemble into $\{q\} = \begin{Bmatrix} \{r_2\} \\ \phi_2 \\ \{r_3\} \\ \phi_3 \\ \vdots \end{Bmatrix}$

3.0 Explicit time loop

4.0 Position solution

4.1 rip new values for $\{r_i\}$ and ϕ_i from $\{q\}_k$

4.2 form attitude matrices $[A_i]$ $[B_i]$ $\{\hat{f}_i\}$ $\{\hat{g}_i\}$

4.3 compute global locations for all points $\{r_i\}^P$

4.4 assemble position constraints $\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{DRIVER} \end{Bmatrix}$

4.5 assemble Jacobian

$$[\Phi_q] = \begin{bmatrix} [\Phi_{r2}]_{KINEMATIC} & [\Phi_{\phi 2}]_{KINEMATIC} & [\Phi_{r3}]_{KINEMATIC} & [\Phi_{\phi 3}]_{KINEMATIC} & \cdots \\ [\Phi_{r2}]_{DRIVER} & [\Phi_{\phi 2}]_{DRIVER} & [\Phi_{r3}]_{DRIVER} & [\Phi_{\phi 3}]_{DRIVER} & \cdots \end{bmatrix}$$

4.6 Newton-Raphson update $\{q\}_{k+1} = \{q\}_k - \left[\Phi_q\right]^{-1}\{\Phi\}$

4.7 check convergence - repeat 4.1 through 4.7 if needed

5.0 Velocity solution

5.1 assemble $\{\nu\} = \begin{Bmatrix} \{\nu\}_{\text{KINEMATIC}} \\ \{\nu\}_{\text{DRIVER}} \end{Bmatrix}$ $\quad \{\nu\}_{\text{KINEMATIC}} = \{0\}$

5.2 compute $\{\dot{q}\} = \left[\Phi_q\right]^{-1}\{\nu\}$ for $\{\dot{q}\} = \begin{Bmatrix} \{\dot{r}_2\} \\ \dot{\phi}_2 \\ \{\dot{r}_3\} \\ \dot{\phi}_3 \\ \vdots \end{Bmatrix}$

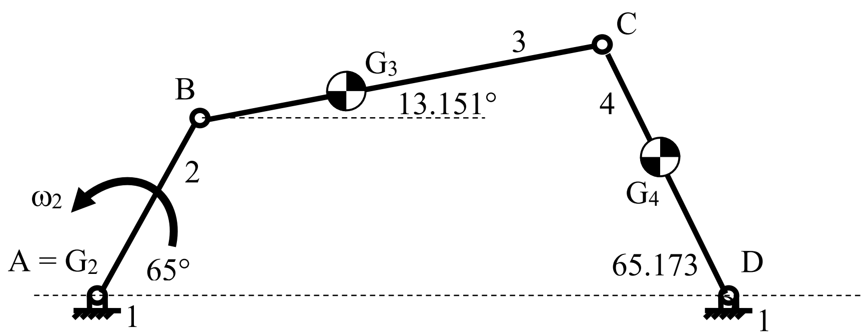
6.0 Acceleration solution

6.1 rip new values for $\{\dot{r}_i\}$ and $\dot{\phi}_i$ from $\{\dot{q}\}$

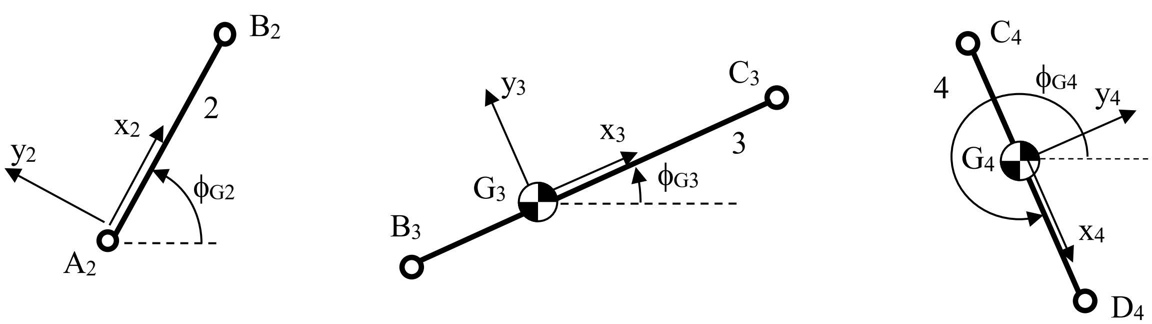
6.2 assemble $\{\gamma\} = \begin{Bmatrix} \{\gamma\}_{\text{KINEMATIC}} \\ \{\gamma\}_{\text{DRIVER}} \end{Bmatrix}$

6.3 compute $\{\ddot{q}\} = \left[\Phi_q\right]^{-1}\{\gamma\}$ for $\{\ddot{q}\} = \begin{Bmatrix} \{\ddot{r}_2\} \\ \ddot{\phi}_2 \\ \{\ddot{r}_3\} \\ \ddot{\phi}_3 \\ \vdots \end{Bmatrix}$

Four Bar Example



AB = 30 cm
BC = 60 cm
CD = 45 cm
AD = 90 cm
BG_3 = 23 cm
DG_4 = 24 cm
$\phi_2 = 65°$
$\phi_3 = 13.151°$
$\phi_4 = -65.173°$



<u>blueprint information</u>

$$\{s_2\}'^{A} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \{s_2\}'^{B} = \begin{Bmatrix} AB \\ 0 \end{Bmatrix} \qquad \{s_3\}'^{B} = \begin{Bmatrix} -BG_3 \\ 0 \end{Bmatrix} \qquad \{s_3\}'^{C} = \begin{Bmatrix} CG_3 \\ 0 \end{Bmatrix}$$

$$\{s_4\}'^{C} = \begin{Bmatrix} -CG_4 \\ 0 \end{Bmatrix} \qquad \{s_4\}'^{D} = \begin{Bmatrix} DG_4 \\ 0 \end{Bmatrix} \qquad \{r_1\}^{A} = \begin{Bmatrix} 0 \\ 0 \end{Bmatrix} \qquad \{r_1\}^{D} = \begin{Bmatrix} AD \\ 0 \end{Bmatrix}$$

<u>adjust</u> $\{r_2\} \quad \phi_2 \quad \{r_3\} \quad \phi_3 \quad \{r_4\} \quad \phi_4$

<u>check</u> $\{r_2\}^{A} = \{r_1\}^{A} \qquad \{r_3\}^{B} = \{r_2\}^{B} \qquad \{r_4\}^{C} = \{r_3\}^{C} \qquad \{r_4\}^{D} = \{r_1\}^{D}$

<u>driver</u> $\phi_2 = \phi_{2\_START} + \omega_2 t$

<u>adjust</u>

$$\{q\} = \begin{Bmatrix} \{q_2\} \\ \{q_3\} \\ \{q_4\} \end{Bmatrix} = \begin{Bmatrix} \{r_2\} \\ \phi_2 \\ \{r_3\} \\ \phi_3 \\ \{r_4\} \\ \phi_4 \end{Bmatrix} = \begin{Bmatrix} x_2 \\ y_2 \\ \phi_2 \\ x_3 \\ y_3 \\ \phi_3 \\ x_4 \\ y_4 \\ \phi_4 \end{Bmatrix}$$

<u>desire</u>

$$\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{\Phi\}_{REV\_A} \\ \{\Phi\}_{REV\_B} \\ \{\Phi\}_{REV\_C} \\ \{\Phi\}_{REV\_D} \\ \Phi_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{r_2\}^A - \{r_1\}^A \\ \{r_3\}^B - \{r_2\}^B \\ \{r_4\}^C - \{r_3\}^C \\ \{r_4\}^D - \{r_1\}^D \\ \phi_2 - \phi_{2\_START} - \omega_2 t \end{Bmatrix} = \{0\}$$

constraints

$$\{\Phi\} = \begin{Bmatrix} \{r_2\} + [A_2]\{s_2\}'^A - \{r_1\}^A \\ \{r_3\} + [A_3]\{s_3\}'^B - \{r_2\} - [A_2]\{s_2\}'^B \\ \{r_4\} + [A_4]\{s_4\}'^C - \{r_3\} - [A_3]\{s_3\}'^C \\ \{r_4\} + [A_4]\{s_4\}'^D - \{r_1\}^D \\ \phi_2 - \phi_{2\_START} - \omega_2 t \end{Bmatrix} = \{0\}$$

Newton-Raphson position solution $\{q\}_{k+1} = \{q\}_k - [\Phi_q]^{-1}\{\Phi\}_k$

velocity solution $\left[\Phi_q\right]\{\dot{q}\} = \{\nu\}$ $\{\nu\}_{KINEMATIC} = \{0\}$ $\{\nu\}_{DRIVER} = -\{\Phi_t\}$

$$\left[\Phi_q\right]\{\dot{q}\} = \begin{Bmatrix} \{\nu\}_{KINEMATIC} \\ \{\nu\}_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{\nu\}_{REV\_A} \\ \{\nu\}_{REV\_B} \\ \{\nu\}_{REV\_C} \\ \{\nu\}_{REV\_D} \\ \nu_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{0_{2x1}\} \\ \{0_{2x1}\} \\ \{0_{2x1}\} \\ \{0_{2x1}\} \\ \omega_2 \end{Bmatrix}$$

$$\{\dot{q}\} = \left[\Phi_q\right]^{-1} \begin{Bmatrix} \{0_{8x1}\} \\ \omega_2 \end{Bmatrix}$$

acceleration solution $\left[\Phi_q\right]\{\ddot{q}\} = \{\gamma\}$

$$\{\Phi\}_{REV} = \{r_j\}^P - \{r_i\}^P = \{0_{2x1}\}$$

$$\{\gamma\}_{REV} = \dot{\phi}_j^{\,2}\left[A_j\right]\{s_j\}'^P - \dot{\phi}_i^{\,2}\left[A_i\right]\{s_i\}'^P \qquad \{\gamma\}_{DRIVER} = -\{\Phi_{tt}\}$$

need $\{\dot{r}_i\}$ and $\dot{\phi}_i$ from $\{\dot{q}\}$

$$\left[\Phi_q\right]\{\ddot{q}\} = \begin{Bmatrix} \{\gamma\}_{KINEMATIC} \\ \{\gamma\}_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{\gamma\}_{REV\_A} \\ \{\gamma\}_{REV\_B} \\ \{\gamma\}_{REV\_C} \\ \{\gamma\}_{REV\_D} \\ \gamma_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \dot{\phi}_2^{\,2}\left[A_2\right]\{s_2\}'^A \\ \dot{\phi}_3^{\,2}\left[A_3\right]\{s_3\}'^B - \dot{\phi}_2^{\,2}\left[A_2\right]\{s_2\}'^B \\ \dot{\phi}_4^{\,2}\left[A_4\right]\{s_4\}'^C - \dot{\phi}_3^{\,2}\left[A_3\right]\{s_3\}'^C \\ \dot{\phi}_4^{\,2}\left[A_4\right]\{s_4\}'^D \\ 0 \end{Bmatrix}$$

$$\{\ddot{q}\} = \left[\Phi_q\right]^{-1}\{\gamma\}$$

$$\{\Phi\} = \begin{Bmatrix} \{r_2\} + [A_2]\{s_2\}'^{A} - \{r_1\}^{A} \\ \{r_3\} + [A_3]\{s_3\}'^{B} - \{r_2\} - [A_2]\{s_2\}'^{B} \\ \{r_4\} + [A_4]\{s_4\}'^{C} - \{r_3\} - [A_3]\{s_3\}'^{C} \\ \{r_4\} + [A_4]\{s_4\}'^{D} - \{r_1\}^{D} \\ \phi_2 - \phi_{2\_START} - \omega_2 t \end{Bmatrix} = \{0\}$$

$$[\Phi_q] = \begin{bmatrix} [I_2] & [B_2]\{s_2\}'^{A} & [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] \\ -[I_2] & -[B_2]\{s_2\}'^{B} & [I_2] & [B_3]\{s_3\}'^{B} & [0_{2x2}] & [0_{2x1}] \\ [0_{2x2}] & [0_{2x1}] & -[I_2] & -[B_3]\{s_3\}'^{C} & [I_2] & [B_4]\{s_4\}'^{C} \\ [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] & [I_2] & [B_4]\{s_4\}'^{D} \\ [0_{1x2}] & 1 & [0_{1x2}] & 0 & [0_{1x2}] & 0 \end{bmatrix}$$

<u>using blueprint information</u>

$$
\left[\Phi_q\right]=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & +BG_2S\phi_2 & 1 & 0 & +BG_3S\phi_3 & 0 & 0 & 0 \\
0 & -1 & -BG_2C\phi_2 & 0 & 1 & -BG_3C\phi_3 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & +CG_3S\phi_3 & 1 & 0 & +CG_4S\phi_4 \\
0 & 0 & 0 & 0 & -1 & -CG_3C\phi_3 & 0 & 1 & -CG_4C\phi_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -DG_4S\phi_4 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & +DG_4C\phi_4 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

<u>evaluating terms</u>

$$
\left[\Phi_q\right]=\begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
-1 & 0 & +27.189 & 1 & 0 & +5.233 & 0 & 0 & 0 \\
0 & -1 & -12.678 & 0 & 1 & -22.397 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & 0 & +8.418 & 1 & 0 & -19.059 \\
0 & 0 & 0 & 0 & -1 & -36.030 & 0 & 1 & -8.817 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & +21.782 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & +10.077 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

AB = 30 cm
BC = 60 cm
CD = 45 cm
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DG_4 = 24 cm
$\phi_2 = 65°$
$\phi_3 = 13.151°$
$\phi_4 = -65.173°$

```
JAC = [ 1.0000         0         0         0         0         0         0         0         0 ;
             0    1.0000         0         0         0         0         0         0         0 ;
        1.0000         0   27.1890    1.0000         0    5.2330         0         0         0 ;
             0   -1.0000  -12.6780         0    1.0000  -22.3970         0         0         0 ;
             0         0         0   -1.0000         0    8.4180    1.0000         0  -19.0590 ;
             0         0         0         0   -1.0000  -36.0300         0    1.0000   -8.8817 ;
             0         0         0         0         0         0    1.0000         0   21.7820 ;
             0         0         0         0         0         0         0    1.0000   10.0770 ;
             0         0    1.0000         0         0         0         0         0         0 ];

det(JAC) = -2.6450e+03
```

transpose

$$\left[\Phi_q\right]^T = \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +27.189 & -12.678 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & +5.233 & -22.397 & +8.418 & -36.030 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -19.059 & -8.817 & +21.782 & +10.077 & 0 \end{bmatrix}$$

from Notes_06_01 matrix static force analysis

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 27.189 & -12.678 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 5.233 & -22.396 & 8.418 & -36.030 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -19.059 & -8.817 & 21.782 & 10.077 & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 102.589 \\ -109.433 \\ 0 \\ 157.973 \\ 122.656 \\ 0 \end{Bmatrix}$$

$$\left[\Phi_q\right]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & +BG_2S\phi_2 & 1 & 0 & +BG_3S\phi_3 & 0 & 0 & 0 \\ 0 & -1 & -BG_2C\phi_2 & 0 & 1 & -BG_3C\phi_3 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & +CG_3S\phi_3 & 1 & 0 & +CG_4S\phi_4 \\ 0 & 0 & 0 & 0 & -1 & -CG_3C\phi_3 & 0 & 1 & -CG_4C\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -DG_4S\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & +DG_4C\phi_4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

shift origins to A, B and C

use $BG_2 = r_2$ $BG_3 = 0$ $CG_3 = r_3$ $CG_4 = 0$ $DG_4 = r_4$

$$\left[\Phi_q\right]=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & r_2S\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -r_2C\phi_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & r_3S\phi_3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -r_3C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -r_4S\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & r_4C\phi_4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\det\left[\Phi_q\right]=\det\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & r_2 S\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -r_2 C\phi_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & r_3 S\phi_3 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -r_3 C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & -r_4 S\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & r_4 C\phi_4 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{pivot } 1,1=+1$$

$$\det\left[\Phi_q\right]=\det\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & r_2 S\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & -r_2 C\phi_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & r_3 S\phi_3 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -r_3 C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -r_4 S\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & r_4 C\phi_4 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{pivot } 1,1=+1$$

$$\det\left[\Phi_q\right]=\det\begin{bmatrix} r_2 S\phi_2 & 1 & 0 & 0 & 0 & 0 & 0 \\ -r_2 C\phi_2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & r_3 S\phi_3 & 1 & 0 & 0 \\ 0 & 0 & -1 & -r_3 C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & -r_4 S\phi_4 \\ 0 & 0 & 0 & 0 & 0 & 1 & r_4 C\phi_4 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \qquad \text{pivot } 7,1=+1$$

$$\det\left[\Phi_q\right] = \det\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ -1 & 0 & r_3S\phi_3 & 1 & 0 & 0 \\ 0 & -1 & -r_3C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & -r_4S\phi_4 \\ 0 & 0 & 0 & 0 & 1 & r_4C\phi_4 \end{bmatrix} \qquad \text{pivot } 1,1 = +1$$

$$\det\left[\Phi_q\right] = \det\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & r_3S\phi_3 & 1 & 0 & 0 \\ -1 & -r_3C\phi_3 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & -r_4S\phi_4 \\ 0 & 0 & 0 & 1 & r_4C\phi_4 \end{bmatrix} \qquad \text{pivot } 1,1 = +1$$

$$\det\left[\Phi_q\right] = \det\begin{bmatrix} r_3S\phi_3 & 1 & 0 & 0 \\ -r_3C\phi_3 & 0 & 1 & 0 \\ 0 & 1 & 0 & -r_4S\phi_4 \\ 0 & 0 & 1 & r_4C\phi_4 \end{bmatrix}$$

$$\det\left[\Phi_q\right] = \left(+r_3S\phi_3\right)\det\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & -r_4S\phi_4 \\ 0 & 1 & r_4C\phi_4 \end{bmatrix} + \left(-1\right)\left(-r_3C\phi_3\right)\det\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & -r_4S\phi_4 \\ 0 & 1 & r_4C\phi_4 \end{bmatrix}$$

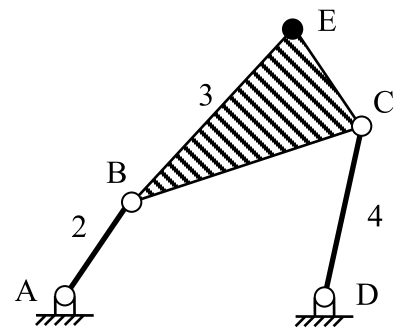
$$\det\left[\Phi_q\right] = \left(+r_3S\phi_3\right)\left(-1\left(r_4C\phi_4\right)\right) + \left(-1\right)\left(-r_3C\phi_3\right)\left(-1\left(-r_4S\phi_4\right)\right) = -r_3r_4S\phi_3C\phi_4 + r_3r_4C\phi_3S\phi_4$$

$$\det\left[\Phi_q\right] = -r_3r_4S\left(\phi_3 - \phi_4\right) = r_3r_4S\gamma \qquad \gamma = \phi_4 - \phi_3$$

r_3 = BC = 60 cm r_4 = CD = 45 cm ϕ_3 = 13.151° ϕ_4 = - 65.173° $\det\left[\Phi_q\right]$ = -2.6441e+03

Four bar

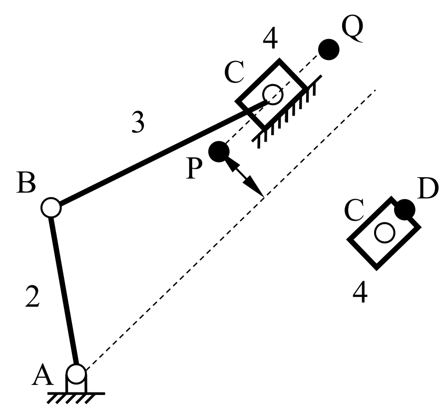
$$\{q\}=\begin{Bmatrix}\{r_2\}\\ \phi_2\\ \{r_3\}\\ \phi_3\\ \{r_4\}\\ \phi_4\end{Bmatrix}\qquad \{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ \{\Phi\}_{REV\_C}\\ \{\Phi\}_{REV\_D}\\ \Phi_{DRIVER}\end{Bmatrix}$$



$$\Phi_{DRIVER}=\phi_2-\phi_{2\_START}-\omega_2 t$$
$$\Phi_{DRIVER}=\phi_4-\phi_{4\_CENTER}-\Delta\phi_4\sin(2\pi f t)$$
$$\Phi_{DRIVER}=\phi_3-c_0-c_2 t-c_2 t^2-c_3 t^3$$
$$\Phi_{DRIVER}=\phi_4-\phi_3-\text{cons tan t}$$
$$\Phi_{DRIVER}=x_3{}^E-x_{3\_START}{}^E-vx_3{}^E t$$

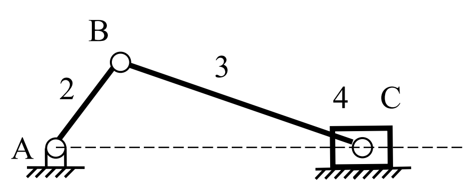
General slider crank

$$\{q\}=\begin{Bmatrix}\{r_2\}\\ \phi_2\\ \{r_3\}\\ \phi_3\\ \{r_4\}\\ \phi_4\end{Bmatrix}\qquad \{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ \{\Phi\}_{REV\_C}\\ \left(\{r_4\}^D-\{r_4\}^C\right)\text{parallel}\left(\{r_1\}^Q-\{r_1\}^P\right)\\ \left(\{r_4\}^C-\{r_1\}^P\right)\text{parallel}\left(\{r_1\}^Q-\{r_1\}^P\right)\\ \Phi_{DRIVER}\end{Bmatrix}$$



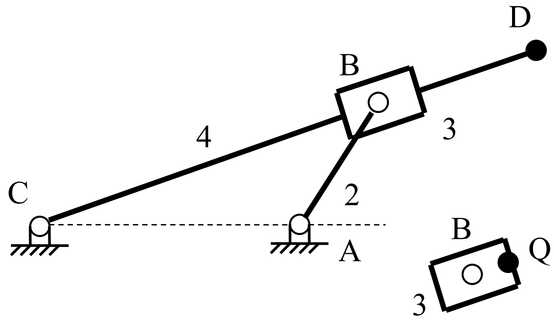
In-line slider crank

$$\{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ \{\Phi\}_{REV\_C}\\ y_4\\ \phi_4\\ \Phi_{DRIVER}\end{Bmatrix}$$



Inverted slider crank

$$\{q\}=\begin{Bmatrix}\{r_2\}\\ \phi_2\\ \{r_3\}\\ \phi_3\\ \{r_4\}\\ \phi_4\end{Bmatrix}\qquad \{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ \{\Phi\}_{REV\_C}\\ \left(\{r_3\}^Q-\{r_3\}^B\right)\text{parallel}\left(\{r_4\}^D-\{r_4\}^C\right)\\ \left(\{r_3\}^B-\{r_4\}^C\right)\text{parallel}\left(\{r_4\}^D-\{r_4\}^C\right)\\ \Phi_{DRIVER}\end{Bmatrix}$$



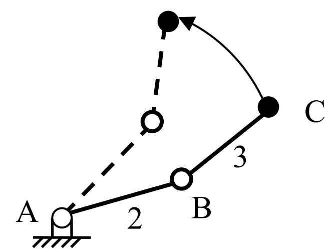
Two link manipulator – joint interpolated motion

$$\{q\}=\begin{Bmatrix}\{r_2\}\\ \phi_2\\ \{r_3\}\\ \phi_3\end{Bmatrix}\qquad \{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ \phi_2-\phi_{2\_START}-\omega_2 t\\ \theta-\theta_{START}-\omega_\theta t\end{Bmatrix}$$

$$\omega_2=\left(\phi_{2\_END}-\phi_{2\_START}\right)/\Delta t$$

$$\theta=\phi_3-\phi_2$$

$$\omega_\theta=\left(\theta_{END}-\theta_{START}\right)/\Delta t$$



Two link manipulator – straight line interpolated motion

$$\{q\}=\begin{Bmatrix}\{r_2\}\\ \phi_2\\ \{r_3\}\\ \phi_3\end{Bmatrix}\qquad \{\Phi\}=\begin{Bmatrix}\{\Phi\}_{REV\_A}\\ \{\Phi\}_{REV\_B}\\ x_{C3}-x_{C3\_START}-v_{C3X}\ t\\ y_{C3}-y_{C3\_START}-v_{C3Y}\ t\end{Bmatrix}$$

$$v_{C3X}=\left(x_{C3\_END}-x_{C3\_START}\right)/\Delta t$$

$$v_{C3X}=\left(y_{C3\_END}-y_{C3\_START}\right)/\Delta t$$