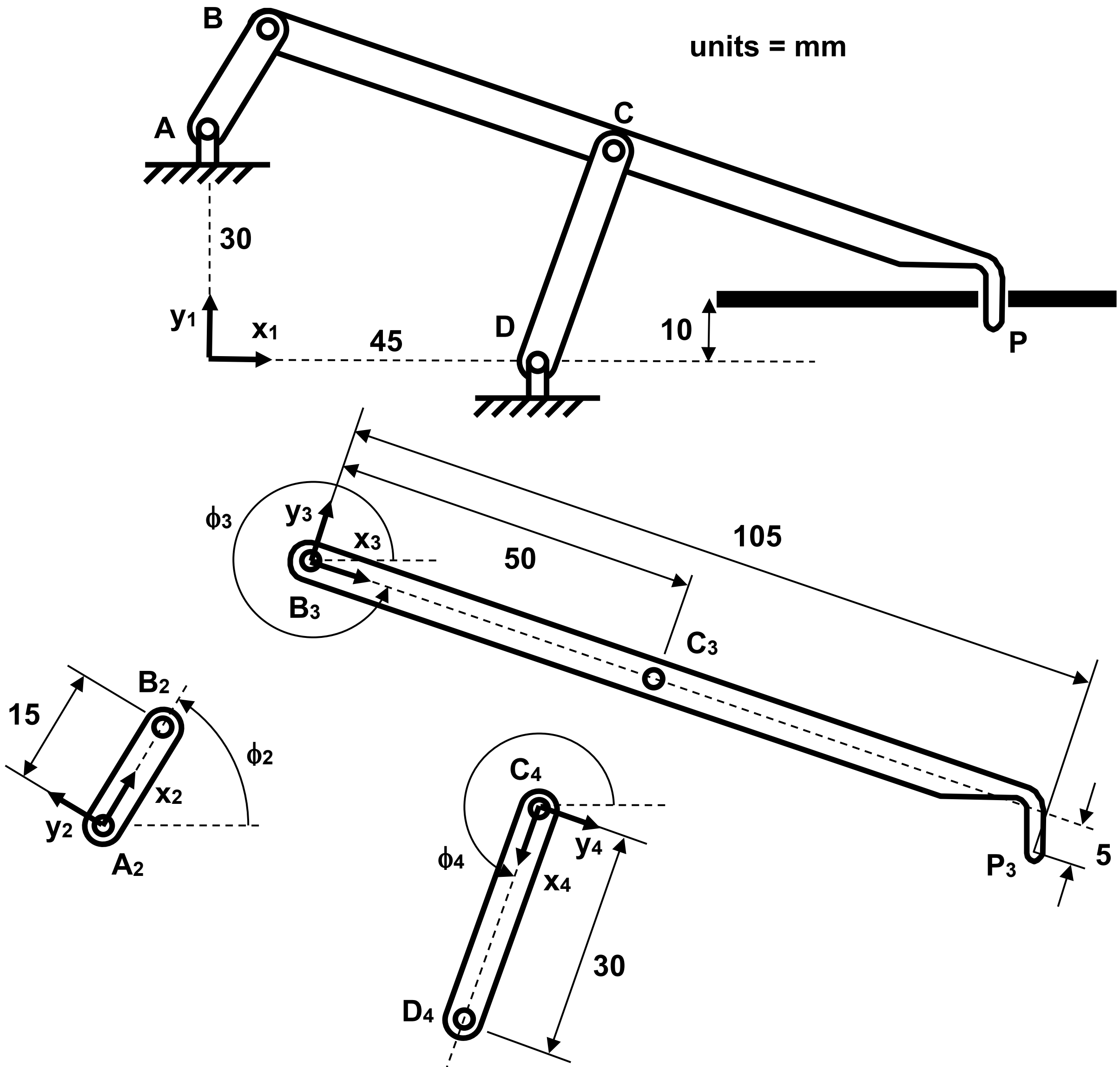
D-mechanism



CONSTANT LOCAL BODY-FIXED LOCATIONS OF SPECIFIC POINTS

	$\{s_1\}'$	$\{s_2\}'$	$\{s_3\}'$	$\{s_4\}'$
A	$\{\ 0\ ,\ 30\ \}^T$	$\{\ 0\ ,\ 0\ \}^T$		
B		$\{\ AB\ ,\ 0\ \}^T$	$\{\ 0\ ,\ 0\ \}^T$	
C			$\{\ BC\ ,\ 0\ \}^T$	$\{\ 0\ ,\ 0\ \}^T$
D	$\{\ 45\ ,\ 0\ \}^T$			$\{\ CD\ ,\ 0\ \}^T$
P			$\{\ 105\ ,\ -5\ \}^T$	

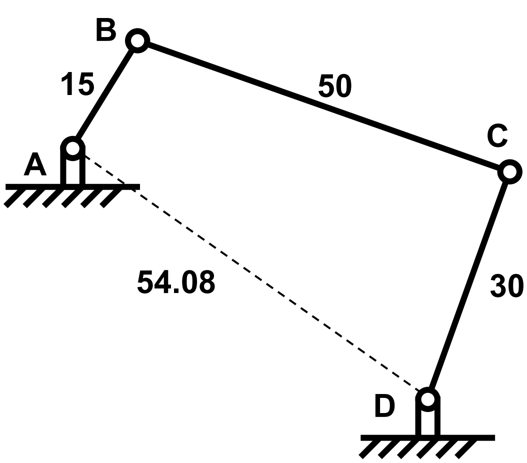
$\{q\} = \{\ x_2\ \ y_2\ \ \phi_2\ \ x_3\ \ y_3\ \ \phi_3\ \ x_4\ \ y_4\ \ \phi_4\ \}^T$

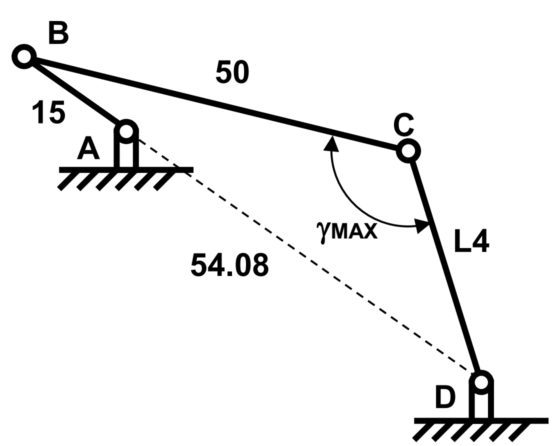
ESTIMATED GLOBAL POSE OF COORDINATE FRAMES

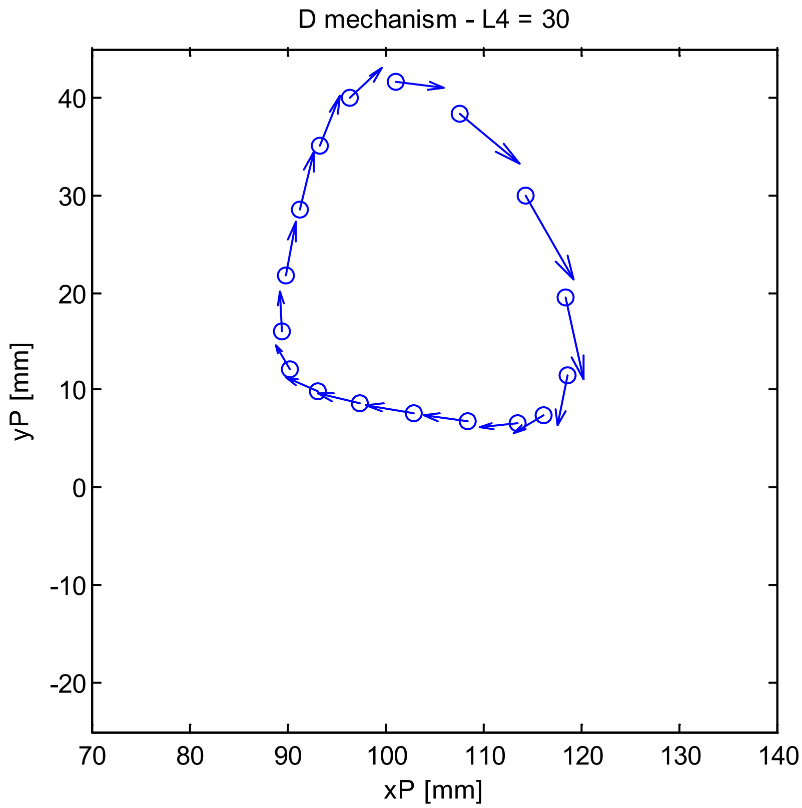
Link	1	2	3	4
Origin $\{r_i\}$	$\{\ 0, 0\ \}^T$	$\{\ 0\ ,\ 30\ \}^T$	$\{\ 7.5\ ,\ 43\ \}^T$	$\{\ 55\ ,\ 28\ \}^T$
Angle ϕ_i	0 deg	60 deg	-15 deg	-110 deg

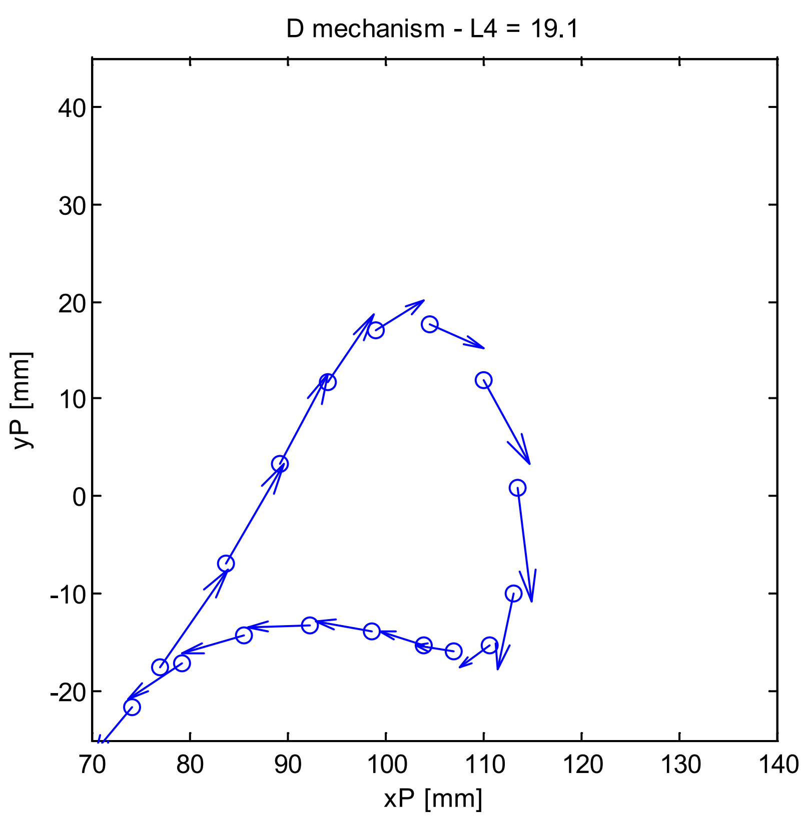
$$\{\Phi\} = \begin{Bmatrix} \{r_2\}^A - \{r_1\}^A \\ \{r_3\}^B - \{r_2\}^B \\ \{r_4\}^C - \{r_3\}^C \\ \{r_4\}^D - \{r_1\}^D \\ \phi_2 - \phi_{2\_START} - \omega_2 t \end{Bmatrix} = \{0\}$$

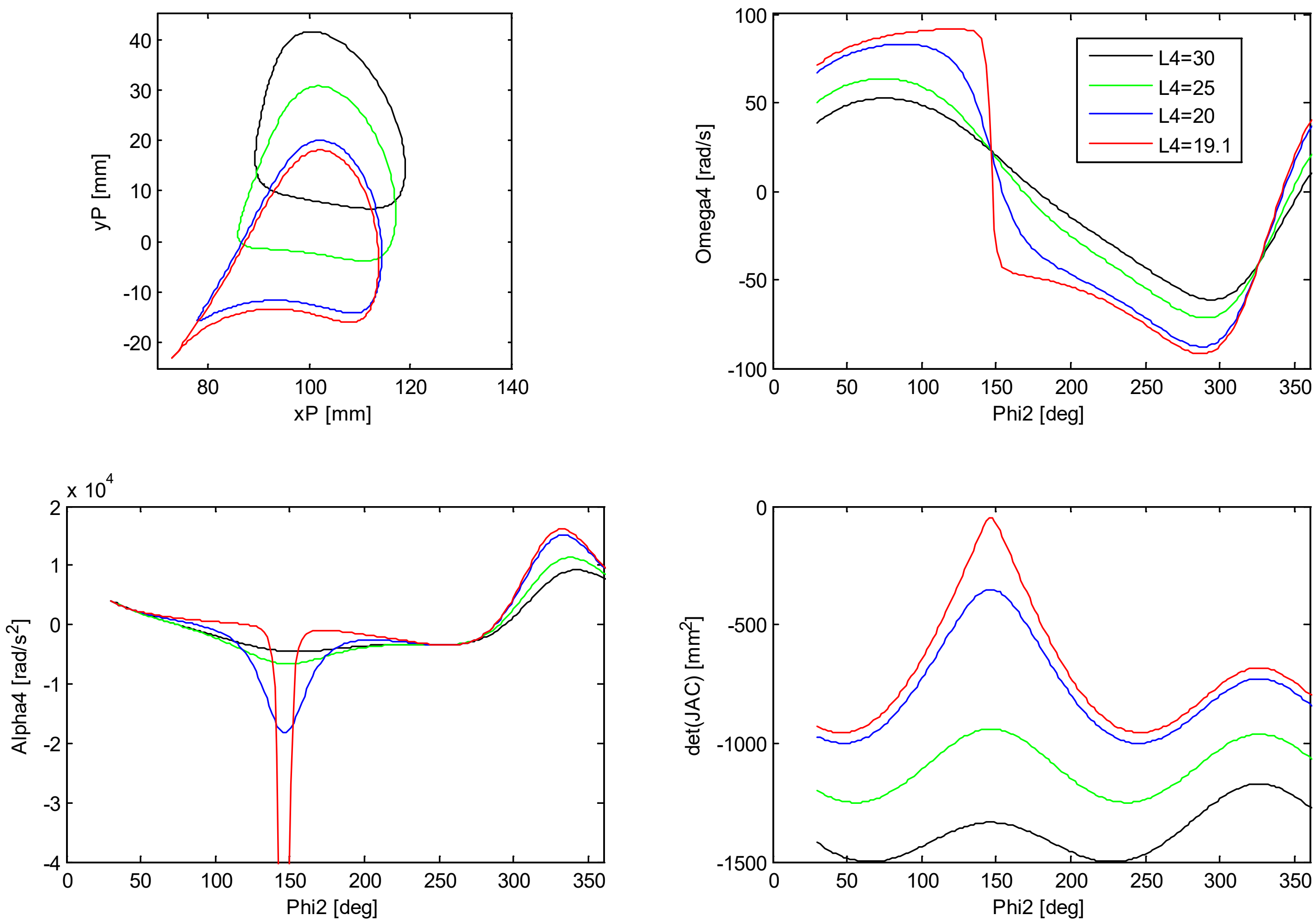
$$[\Phi_q] = \begin{bmatrix} +[I_2] & +[B_2]\{s_2\}'^A & [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] \\ -[I_2] & -[B_2]\{s_2\}'^B & +[I_2] & +[B_3]\{s_3\}'^B & [0_{2x2}] & [0_{2x1}] \\ [0_{2x2}] & [0_{2x1}] & -[I_2] & -[B_3]\{s_3\}'^C & +[I_2] & +[B_4]\{s_4\}'^C \\ [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] & +[I_2] & +[B_4]\{s_4\}'^D \\ [0_{1x2}] & 1 & [0_{1x2}] & 0 & [0_{1x2}] & 0 \end{bmatrix}$$






D mechanism - L4 = 30
yP [mm]
xP [mm]
40
30
20
10
0
-10
-20
70
80
90
100
110
120
130
140


D mechanism - L4 = 19.1
yP [mm]
xP [mm]
40
30
20
10
0
-10
-20
70
80
90
100
110
120
130
140


yP [mm]
xP [mm]
Omega4 [rad/s]
Phi2 [deg]
L4=30
L4=25
L4=20
L4=19.1
x 10^4
Alpha4 [rad/s^2]
Phi2 [deg]
det(JAC) [mm^2]
Phi2 [deg]

```
% dm_main.m - D-mechanism four-bar - ref: Haug, Fig P5.1, page 196
%   main
% HJSIII - 22.02.13

% initialize
dm_ini

% starting position
phi2_start = 0 * d2r;  % start at zero

% time loop
tpr = 2 * pi / w2;              % one revolution at constant speed
t_start = 0;                    % start
t_end = tpr;                    % end
nt = 180;                       % number of time steps
dt = (t_end - t_start) / nt;

keep_q = [];
keep = [];
for t = t_start : dt : t_end;

% kinematics
  dm_kin

% save kinematics
  detJAC = det(JAC);

  x3P = r3P(1);
  y3P = r3P(2);
  x3Pd = r3Pd(1);
  y3Pd = r3Pd(2);

  keep_q = [ keep_q ; q'  qd'  qdd' ];
  keep = [ keep ; detJAC x3P y3P x3Pd y3Pd ];

% bottom - for t
end

% data for plotting
ang2 = keep_q(:,3) /d2r;
phi4d = keep_q(:,18);
phi4dd = keep_q(:,27);
detJAC = keep(:,1);
x3P = keep(:,2);
y3P = keep(:,3);
x3Pd = keep(:,4);
y3Pd = keep(:,5);

% four figures per page
figure( 1 )

% locus of coupler point
subplot( 2,2,1 )
  plot( x3P,y3P,plot_str )
  hold on
  axis( [70 140 -25 45] )
  axis square
  xlabel( 'xP [mm]' )
  ylabel( 'yP [mm]' )

% follower velocity
subplot( 2,2,2 )
  plot( ang2, phi4d, plot_str )
  hold on
  axis( [ 0 360 -100 100 ] )
  xlabel( 'Phi2 [deg]' )
  ylabel( 'Omega4 [rad/s]' )
  legend( [ 'L4=', num2str(len4) ] )

% follower acceleration
subplot( 2,2,3 )
```

```
  plot( ang2, phi4dd, plot_str )
  hold on
  axis( [ 0 360 -40000 20000 ] )
  xlabel( 'Phi2 [deg]' )
  ylabel( 'Alpha4 [rad/s^2]' )

% Jacobian
subplot( 2,2,4 )
  plot( ang2, detJAC, plot_str )
  hold on
  axis( [ 0 360 -1500 0 ] )
  xlabel( 'Phi2 [deg]' )
  ylabel( 'det(JAC) [mm^2]' )

% coupler point velocity - quiver plot
npos = length( x3P );
iskip = (1:12:npos);
figure( 2 )
  quiver( x3P(iskip),y3P(iskip), x3Pd(iskip),y3Pd(iskip) )
  axis( [70 140 -25 45] )
  axis square
  hold on
  plot( x3P(iskip),y3P(iskip),'bo' )
  xlabel( 'xP [mm]' )
  ylabel( 'yP [mm]' )
  title( [ 'D mechanism - L4 = ' num2str(len4) ] )

% bottom - dm_main
```

```
% dm_ini.m - D-mechanism four-bar - ref: Haug, Fig P5.1, page 196
%   initialize constants and assembly guesses
% HJSIII - 22.02.13

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% mechanism constants
len2 = 15;
len3 = 50;
len4 = 30;

% possible lengths for parameter study of link
len4 = 19.1;     plot_str = 'r';
len4 = 20;       plot_str = 'b';
len4 = 25;       plot_str = 'g';
len4 = 30;       plot_str = 'k';

% local body-fixed locations
s1pA = [ 0  30 ]';
s1pD = [ 45  0 ]';

s2pA = [ 0  0 ]';
s2pB = [ len2  0 ]';

s3pB = [ 0  0 ]';
s3pC = [ len3  0 ]';
s3pP = [ 105  -5.00 ]';

s4pC = [ 0  0 ]';
s4pD = [ len4  0 ]';

% global locations
r1A = [  0  30 ]';
r1D = [ 45   0 ]';

% initial guesses - angles measured by protractor
phi2 = 60*d2r;
phi3 = -15*d2r;
phi4 = -110*d2r;

q(1,1) = 0;
q(2,1) = 30;
q(3,1) = phi2;

q(4,1) = 7.5;
q(5,1) = 43;
q(6,1) = phi3;

q(7,1) = 55;
q(8,1) = 28;
q(9,1) = phi4;

% driver for crank - phi2 = phi2_start + w2*t
phi2_start = 60 * d2r;
w2 = +1000 * 2 * pi / 60;   % 1000 rpm CCW, convert to rad/sec

% start time
t = 0;

% bottom - dm_ini
```

```
% dm_phi.m - D-mechanism four-bar - ref: Haug, Fig P5.1, page 196
%   evaluate constraints and Jacobian for crank driving constraint
% HJSIII - 22.02.13

% global location of local frames and rotation matrices
% Eq 2.4.4, page 33 - Eq 2.6.5, page 42
r2 = q(1:2);
r3 = q(4:5);
r4 = q(7:8);
phi2 = q(3);
phi3 = q(6);
phi4 = q(9);
A2 = [ cos(phi2) -sin(phi2); sin(phi2) cos(phi2) ];
A3 = [ cos(phi3) -sin(phi3); sin(phi3) cos(phi3) ];
A4 = [ cos(phi4) -sin(phi4); sin(phi4) cos(phi4) ];
B2 = R * A2;
B3 = R * A3;
B4 = R * A4;

% global locations - Eq 2.4.8, page 33
r2A = r2 + A2*s2pA;
r2B = r2 + A2*s2pB;
r3B = r3 + A3*s3pB;
r3C = r3 + A3*s3pC;
r4C = r4 + A4*s4pC;
r4D = r4 + A4*s4pD;
r3P = r3 + A3*s3pP;

% revolute constraints - A,B,C,D - Eq 3.3.10, page 65
PHI(1:2,1) = r2A - r1A;
PHI(3:4,1) = r3B - r2B;
PHI(5:6,1) = r4C - r3C;
PHI(7:8,1) = r4D - r1D;

% crank driving constraint
PHI(9,1) = phi2 - phi2_start - w2*t;

% Jacobian by rows - Eq 3.3.12, page 66 for revolutes
JAC = zeros(9,9);
JAC(1:2,1:3) = [  eye(2)   B2*s2pA ];

JAC(3:4,1:3) = [ -eye(2)  -B2*s2pB ];
JAC(3:4,4:6) = [  eye(2)   B3*s3pB ];

JAC(5:6,4:6) = [ -eye(2)  -B3*s3pC ];
JAC(5:6,7:9) = [  eye(2)   B4*s4pC ];

JAC(7:8,7:9) = [  eye(2)   B4*s4pD ];

% driving constraint in Jacobian - Eq 3.1.9, page 52
JAC(9,3) = 1;

% current results
current_crank = phi2 / d2r;

% bottom - dm_phi
```

```
% dm_kin.m - D-mechanism four-bar - ref: Haug, Fig P5.1, page 196
%   positon, velocity, and acceleration at desired_crank
% HJSIII - 20.03.04

% Newton-Raphson position solution - Eq 3.6.7 and 3.6.8, page 100
assy_tol = 0.00001;
dm_phi
while max(abs(PHI)) > assy_tol,
  q = q - inv(JAC) * PHI;
  dm_phi
end

% velocity - Eq 3.1.9, page 52 - also page 66 for revolutes
velrhs(9,1) = w2;
qd = inv(JAC) * velrhs;

% global velocities - Eq 2.6.4, page 41
r2d = qd(1:2);
r3d = qd(4:5);
r4d = qd(7:8);
phi2d = qd(3);
phi3d = qd(6);
phi4d = qd(9);
r3Pd = r3d + phi3d * B3 * s3pP;

% acceleration - Eq 3.1.10, page 53 - also page 66 for revolutes
accrhs(1:2,1) = A2*s2pA*phi2d*phi2d;
accrhs(3:4,1) = A3*s3pB*phi3d*phi3d - A2*s2pB*phi2d*phi2d;
accrhs(5:6,1) = A4*s4pC*phi4d*phi4d - A3*s3pC*phi3d*phi3d;
accrhs(7:8,1) = A4*s4pD*phi4d*phi4d;
accrhs(9,1) = 0;
qdd = inv(JAC) * accrhs;

% global accelerations
r2dd = qdd(1:2);
r3dd = qdd(4:5);
r4dd = qdd(7:8);
phi2dd = qdd(3);
phi3dd = qdd(6);
phi4dd = qdd(9);
r3Pdd = r3dd + phi3dd*B3*s3pP - phi3d*phi3d*A3*s3pP;

% bottom - dm_kin
```