Static Force Analysis for Skid Loader - Scalar

A trunnion mount hydraulic cylinder actuates the arm of a skid steer loader as shown below. At this position, $e = 40$ inches, $\theta = 61.131°$, $\dot{e} = -12$ ips, $\dot{\theta} = -0.3625$ rad/s.

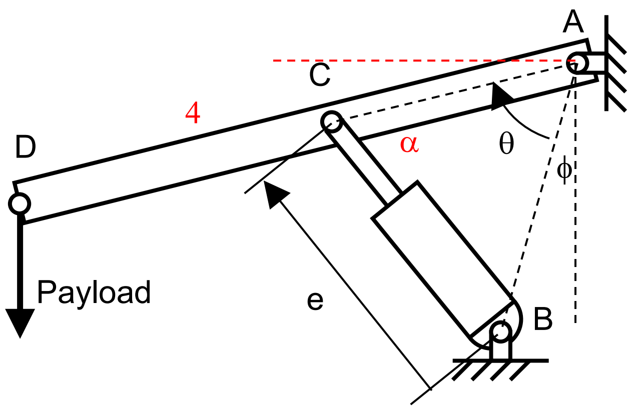
Determine the force on the hydraulic cylinder required to lower an 800 lbf payload attached to point D by a cable. The payload moves with constant velocity at the position shown. You may neglect the effects of friction. The weight of the arm and cylinder are small compared to the payload. Show your work.

$\sin\alpha / AB = \sin\theta / e$

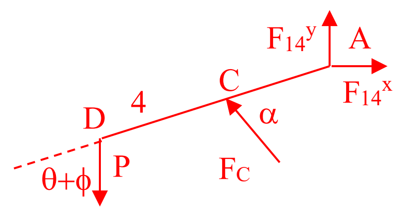
$\alpha = 52.01°$

$\theta+\phi = 77.131°$

$F_{CYLINDER}$ 2261.9 lbf up/left



Not to scale
AB = 36 inches
AC = 42 inches
AD = 96 inches
$\phi = 16°$



$\sum M$ on 4 about A CCW+

$-(F_C \sin\alpha)\,AC + (P\sin(\theta+\phi))\,AD = 0$

$F_C = 2261.9$ lbf

What corresponding hydraulic pressure would be required for a cylinder with a 3 inch DIA bore?

$P_{CYLINDER}$ 320 psi

$A = \pi D^2 / 4 = 7.069 \text{ in}^2$

Is this value reasonable? Why?

OK, industrial hydraulics often go to 3000 psi

If you include friction between the piston and cylinder wall, will it increase or decrease your computation for pressure.

increase **(decrease)** Why? pressure pushes up
friction force will be up opposing piston motion

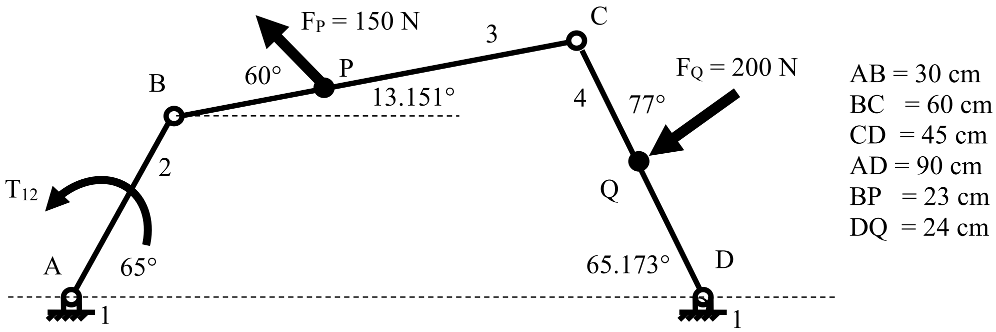
What value would you use for the coefficient of friction between the piston and cylinder wall?

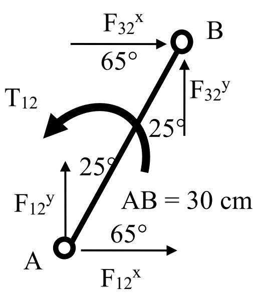
μ 0.1 lubricated Why?

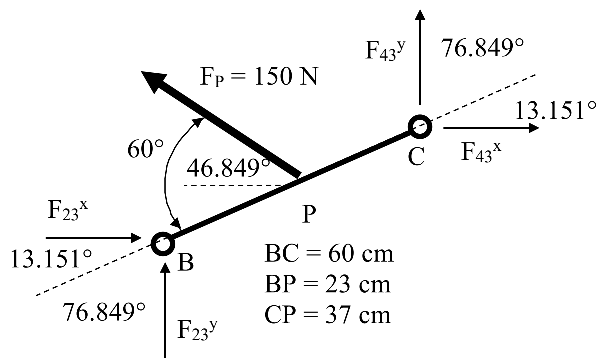
Should your analysis be different if the cylinder were retracting at constant velocity instead of the payload moving at constant velocity?

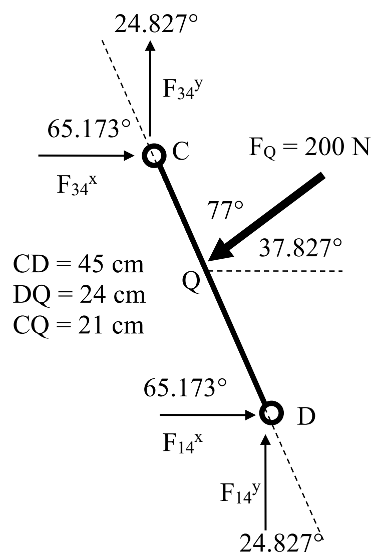
(yes) no Why? constant $\dot{e}$ means $\dot{\theta}$ will not be constant means velocity of the payload will not be constant, therefore must account for acceleration of payload mass

Static Force Analysis for Four Bar - Scalar









ΣM on 4 about D CCW+

$- F_{34}^x (CD) \sin 65.173° - F_{34}^y (CD) \sin 24.827° + F_Q (DQ) \sin 77° = 0$

$40.842\, F_{34}^x + 18.895\, F_{34}^y = 4676.98$ N

ΣM on 3 about B CCW+

$- F_{43}^x (BC) \sin 13.151° + F_{43}^y (BC) \sin 76.849° + F_P (BP) \sin 60° = 0$

$- 13.651\, F_{43}^x + 58.426\, F_{43}^y = -2987.78$ N

$$\begin{bmatrix} 40.842 & 18.895 \\ +13.651 & -58.426 \end{bmatrix} \begin{Bmatrix} F_{34}^{\ x} \\ F_{34}^{\ y} \end{Bmatrix} = \begin{Bmatrix} 4676.98 \text{ N} \\ -2987.78 \text{ N} \end{Bmatrix} \qquad \begin{Bmatrix} F_{34}^{\ x} \\ F_{34}^{\ y} \end{Bmatrix} = \begin{Bmatrix} 81.993 \text{ N} \\ 70.295 \text{ N} \end{Bmatrix}$$

ΣF on 4 right + $F_{34}^x - F_Q \cos 37.827° + F_{14}^x = 0$ $F_{14}^x = 75.980$ N

ΣF on 4 up + $F_{34}^y - F_Q \sin 37.827° + F_{14}^y = 0$ $F_{14}^y = 52.360$ N

ΣF on 3 right + $F_{43}^x - F_P \cos 46.849° + F_{23}^x = 0$ $F_{23}^x = 184.580$ N

ΣF on 3 up + $F_{43}^y + F_P \sin 46.849° + F_{23}^y = 0$ $F_{23}^y = -39.138$ N

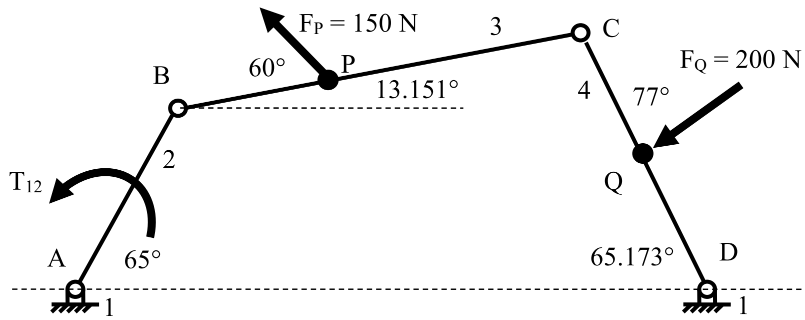
ΣM on 2 about A CCW+ $- F_{32}^x (AB) \sin 65° + F_{32}^y (AB) \sin 25° + T_{12} = 0$

$T_{12} = - 5514.8$ N.cm

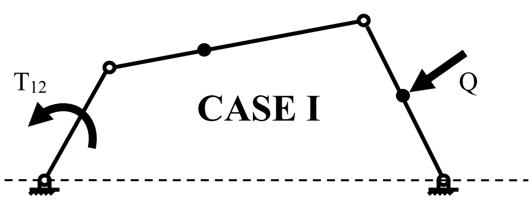
ΣF on 2 right + $F_{12}^x + F_{32}^x = 0$ $F_{12}^x = 184.580$ N

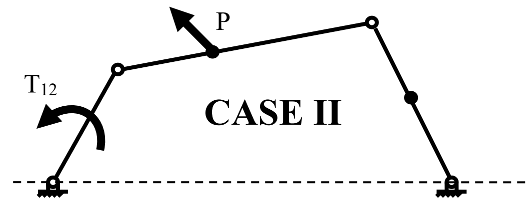
ΣF on 2 up $F_{12}^y + F_{32}^y = 0$ $F_{12}^y = -39.138$ N

Static Force Analysis for Four Bar - Superposition



AB = 30 cm
BC = 60 cm
CD = 45 cm
AD = 90 cm
BP = 23 cm
DQ = 24 cm





CASE I - F_{34} parallel to BC

ΣM on 4 about D CCW +

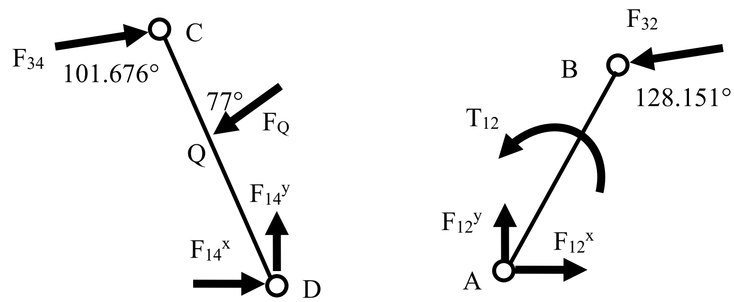
$- F_{34} \sin 101.676^o \, CD + F_Q \sin 77^o \, DQ = 0$

$F_{34} = 106.129 \text{ N} = F_{32}$

ΣM on 2 about A CCW +

$+ F_{32} \sin 128.151^o \, AB + T_{12} = 0$

$T_{12} = -2503.75 \text{ N.cm}$

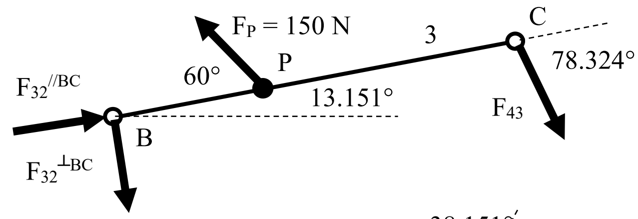


CASE II – F_{43} parallel to CD

ΣM on 3 about B CCW +

$- F_{43} \sin 101.676^o \, BC + F_P \sin 60^o \, BP = 0$

$F_{43} = 50.849 \text{ N}$



ΣF on 3 //BC right +

$+ F_{43} \cos 78.324^o - F_P \cos 60^o + F_{32}^{//BC} = 0$

$F_{32}^{//BC} = 64.709 \text{ N}$

ΣF on 3 ⊥BC up +

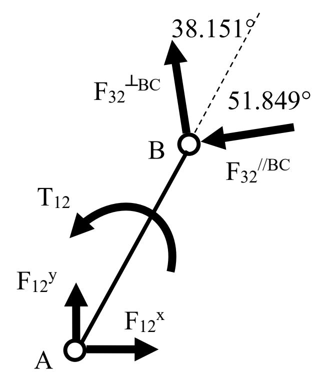
$- F_{43} \sin 78.324^o + F_P \cos 60^o - F_{32}^{\perp BC} = 0$

$F_{32}^{\perp BC} = 80.107 \text{ N}$

ΣM on 2 about A CCW +

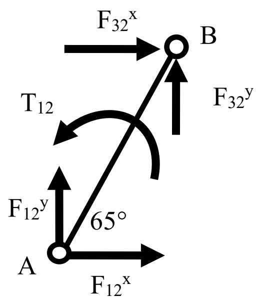
$+ F_{32}^{//BC} \sin 51.849^o \, AB + F_{32}^{\perp BC} \sin 38.151^o \, AB + T_{12} = 0$

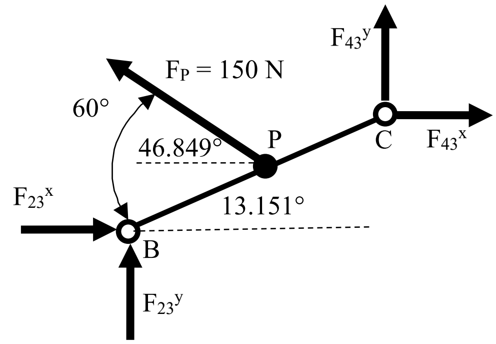
$T_{12} = -3011.14 \text{ N.cm}$

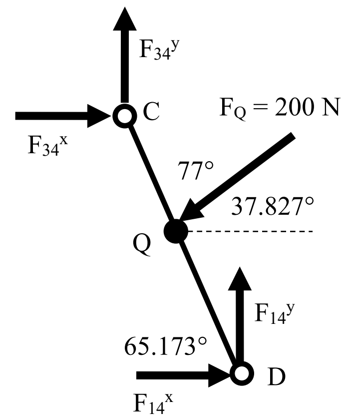


$T_{12} = T_{12}(\text{Case I}) + T_{12}(\text{Case II}) = -5514.89 \text{ N.cm}$

Static Force Analysis for Four Bar - Matrix







ΣF on 2 right +	$F_{12}^x + F_{32}^x = 0$
ΣF on 2 up +	$F_{12}^y + F_{32}^y = 0$
ΣM on 2 about A CCW +	$-F_{32}^x\, r_{B/A}^y + F_{32}^y\, r_{B/A}^x + T_{12} = 0$
ΣF on 3 right +	$F_{23}^x + F_{43}^x + F_P^x = 0$
ΣF on 3 up +	$F_{23}^y + F_{43}^y + F_P^y = 0$
ΣM on 3 about P CCW +	$F_{23}^x\, r_{B/P}^y - F_{23}^y\, r_{B/P}^x - F_{43}^x\, r_{C/P}^y + F_{43}^y\, r_{C/P}^x = 0$
ΣF on 4 right +	$F_{34}^x + F_{14}^x + F_Q^x = 0$
ΣF on 4 up +	$F_{34}^y + F_{14}^y + F_Q^x = 0$
ΣM on 4 about Q CCW +	$-F_{34}^x\, r_{C/Q}^y - F_{34}^y\, r_{C/Q}^x + F_{14}^x\, r_{D/Q}^y + F_{14}^y\, r_{D/Q}^x = 0$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & r_{B/A}^y & -r_{B/A}^x & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & r_{B/P}^y & -r_{B/P}^x & r_{C/P}^y & -r_{C/P}^x & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -r_{C/Q}^y & -r_{C/Q}^x & r_{D/Q}^y & r_{D/Q}^x & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ -F_P^x \\ -F_P^y \\ 0 \\ -F_Q^x \\ -F_Q^y \\ 0 \end{Bmatrix}$$

$\vec{F}_P = 150$ N @ 133.151° = - 102.589 + j 109.433 N

$\vec{F}_Q = 200$ N @ 217.827° = - 157.973 - j 122.656 N

B/A = 30 cm @ 65° = 12.678 + j 27.189 cm

B/P = 23 cm @ 193.151° = -22.396 - j 5.233 cm

C/P = 37 cm @ 13.151° = 36.030 + j 8.418 cm

C/Q = 21 cm @ 114.827° = -8.817 + j 19.059 cm

D/Q = 24 cm @ -65.173° = 10.077 - j 21.782 cm

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 27.189 & -12.678 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -5.233 & 22.396 & 8.418 & -36.030 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -19.059 & 8.817 & -21.782 & 10.077 & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{y} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 102.589 \\ -109.433 \\ 0 \\ 157.973 \\ 122.656 \\ 0 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 184.59 \text{ N} \\ -39.14 \text{ N} \\ 184.59 \text{ N} \\ -39.14 \text{ N} \\ 82.00 \text{ N} \\ 70.29 \text{ N} \\ 75.98 \text{ N} \\ 52.36 \text{ N} \\ -5514.89 \text{ N.cm} \end{Bmatrix}$$

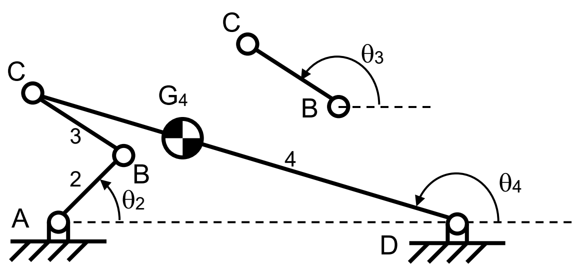
Static Force Analysis for Pushups - Matrix

A person doing pushups is represented with wrists A, elbows B, shoulders C, and toes D. Mass of the torso/legs is 180 lbm.

Use the additional assumptions:

a) All muscular effort is provided by triceps as torque T_{32} across the elbows.
b) Angular velocity of link 4 is constant at this position
c) Weight of the arms is negligible compared to weight of the torso/legs.
d) Friction is negligible at A, B, C and D.
e) No muscular torque is generated at A, C and D.

AD = 52 inch	$\theta_2 = 45°$
AB = 12 in	$\theta_3 = 149.14°$
BC = 14 in	$\theta_4 = 164.24°$
CD = 57.7 in	$\omega_2 = 0.5$ rad/sec
DG_4 = 39 in	$\omega_3 = -1.435$ rad/sec
	$\omega_4 = -0.387$ rad/sec



Determine angular velocity across the elbows $\omega_{2/3}$ for the position and velocity provided above.

$\omega_{2/3} = \omega_2 - \omega_3 = +1.935$ rad/sec

Determine elbow torque T_{32} for the position and velocity provided above.

$T_{32} = 1351.1$ in.lbf CCW

Do the magnitude and direction for your answer seem reasonable? Why?

direction seems OK approximate solution $W_4 (DG_4) \sim F_{34}^y (CD)$, $F_{34}^y \sim 122$ lbf $\sim F_{12}^y$
$F_{12}^y (AB) \sim T_{32} \sim 1464$ in.lbf

Rate the last four assumptions and state your reasoning.

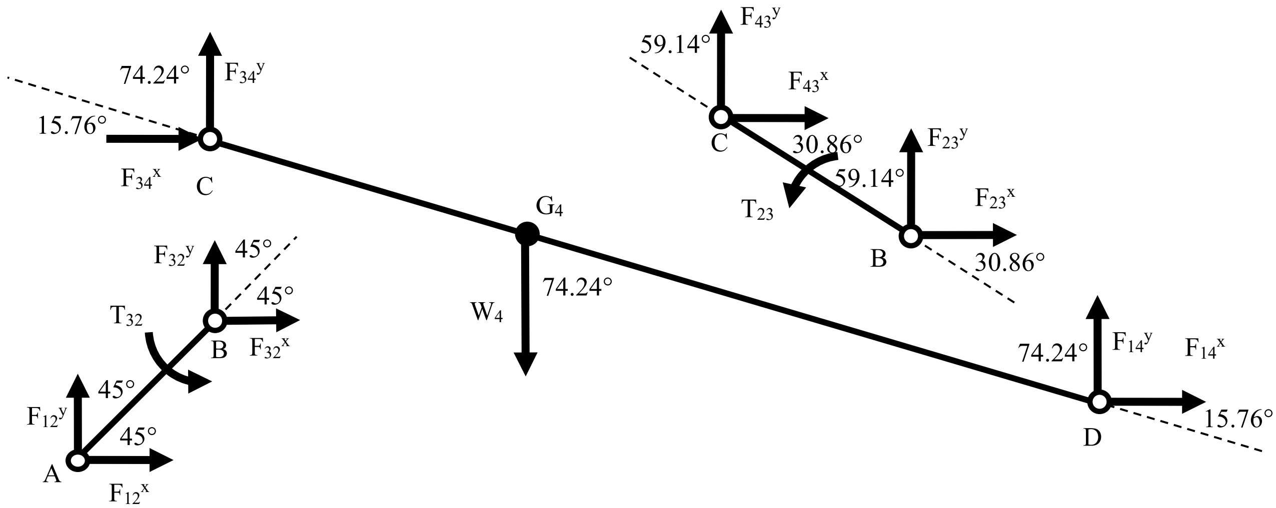
b) constant ω_4 1=poor **(2=acceptable for an approximation)** 3=very good

c) weight of arms is negligible 1=poor **(2=acceptable for an approximation)** 3=very good

d) friction is negligible 1=poor 2=acceptable for an approximation **(3=very good)**

e) no muscle force at A, C, D **(1=poor)** 2=acceptable for an approximation 3=very good

Determine ω_4 of the torso/legs when the forearm is aligned with the upper arms ($\theta_2 = \theta_3$). $\omega_4 = 0$



ΣF on 2 right + $F_{12}^x + F_{32}^x = 0$

ΣF on 2 up + $F_{12}^y + F_{32}^y = 0$

ΣM on 2 about A CCW + $-(F_{32}^x \sin 45°)\ AB + (F_{32}^y \sin 45°)\ AB + T_{32} = 0$

ΣF on 3 right + $F_{23}^x + F_{43}^x = 0$

ΣF on 3 up + $F_{23}^y + F_{43}^y = 0$

ΣM on 3 about B CCW + $-(F_{43}^x \sin 30.86°)\ BC - (F_{43}^y \sin 59.14°)\ BC + T_{23} = 0$

ΣF on 4 right + $F_{34}^x + F_{14}^x = 0$

ΣF on 4 up + $F_{34}^y + F_{14}^y - W_4 = 0$

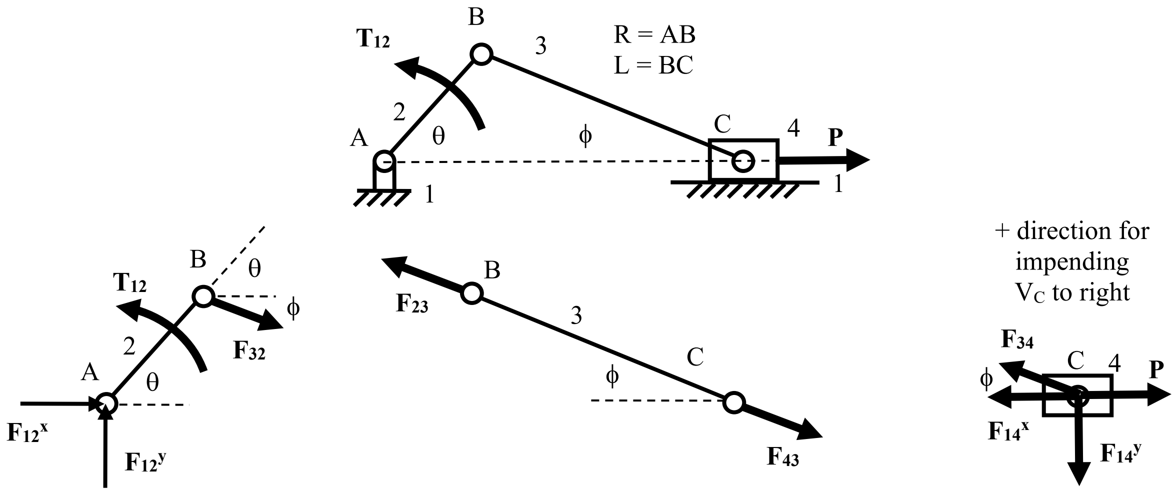
ΣM on 4 about D CCW + $-(F_{34}^x \sin 15.76°)\ CD - (F_{34}^y \sin 74.24°)\ CD + (W_4 \sin 74.24°)\ DG_4 = 0$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & AB\sin 45° & -AB\sin 45° & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & BC\sin 30.86° & BC\sin 59.14° & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -CD\sin 15.76° & -CD\sin 74.24° & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{32} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ +W_4 \\ -(W_4 \sin 74.24°)DG_4 \end{Bmatrix}$$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8.4853 & -8.4853 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 7.1812 & 12.0179 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -15.6718 & -55.5309 & 0 & 0 & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{32} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 180 \\ -6756.1 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{32} \end{Bmatrix} = \begin{Bmatrix} -29.3 \text{ lbf} \\ 129.9 \text{ lbf} \\ -29.3 \text{ lbf} \\ 129.9 \text{ lbf} \\ -29.3 \text{ lbf} \\ 129.9 \text{ lbf} \\ 29.3 \text{ lbf} \\ 50.1 \text{ lbf} \\ 1351.1 \text{ in.lbf} \end{Bmatrix}$$

Static Force Analysis for Slider Crank - Scalar



ΣF on 4 right + $+ P - F_{14}^{x}\,\mathrm{sign}(V_C) - F_{34}\cos\phi = 0$

ΣF on 4 up + $+ F_{34}\sin\phi - F_{14}^{y} = 0$

ΣM on 3 $F_{23} = F_{43}$

ΣM on 2 about A CCW + $+ T_{12} - (F_{32}\sin(\theta+\phi))\, R = 0$

friction $F_{14}^{x} = \mu\, F_{14}^{y}$

$$F_{34} = \frac{P}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)}$$

$$F_{14}^{\ y} = \frac{P\sin\phi}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)} = P\tan\phi \text{ for } \mu = 0$$

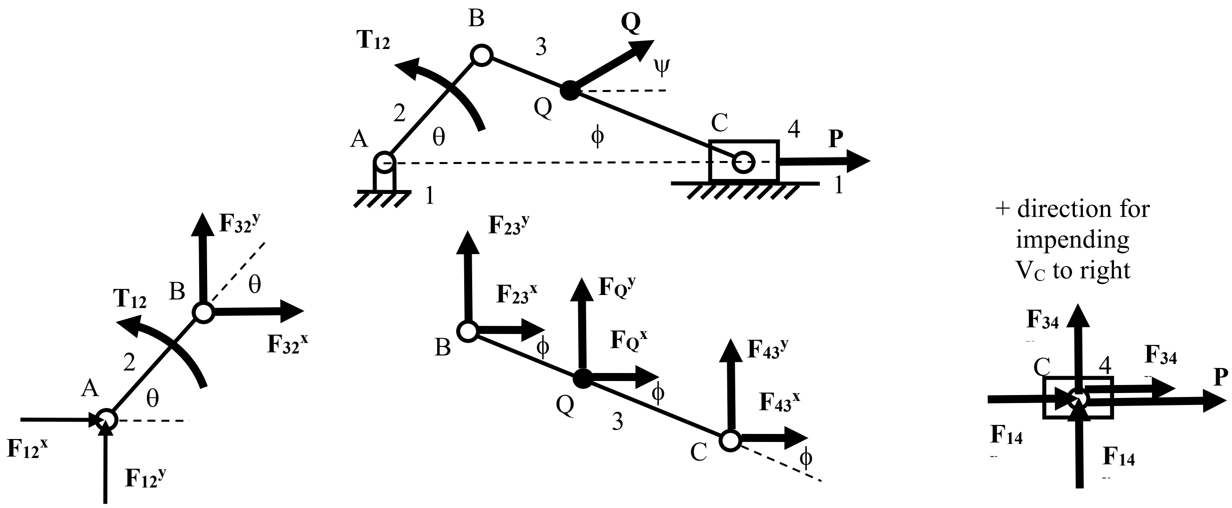
$$F_{14}^{\ x} = \frac{\mu P\sin\phi}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)}$$

$$F_{12}^{\ x} = -\frac{P\cos\phi}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)} = -P \text{ for } \mu = 0$$

$$F_{12}^{\ y} = \frac{P\sin\phi}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)} = P\tan\phi \text{ for } \mu = 0$$

$$T_{12} = \frac{P\ R\ \sin(\theta+\phi)}{(\cos\phi + \mathrm{sign}(V_C)\,\mu\sin\phi)}$$

Static Force Analysis for Slider Crank - Matrix



ΣF on 2 right +	$F_{12}^x + F_{32}^x = 0$
ΣF on 2 up +	$F_{12}^y + F_{32}^y = 0$
ΣM on 2 about A CCW +	$-(F_{32}^x \sin\theta)\,AB + (F_{32}^y \cos\theta)\,AB + T_{12} = 0$
ΣF on 3 right +	$F_{23}^x + F_{43}^x + F_Q^x = 0$
ΣF on 3 up +	$F_{23}^y + F_{43}^y + F_Q^y = 0$
ΣM on 3 about Q CCW +	$-(F_{23}^x \sin\phi)\,BQ - (F_{23}^y \cos\phi)\,BQ + (F_{43}^x \sin\phi)\,CQ + (F_{43}^y \cos\phi)\,CQ = 0$
ΣF on 4 right +	$F_{14}^x + F_{34}^x + P = 0$
ΣF on 4 up +	$F_{14}^y + F_{34}^y = 0$
friction	$F_{14}^x = -\mu\,\mathrm{abs}(F_{14}^y)\,\mathrm{sign}(V_C)$

$$
\begin{bmatrix}
1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & +AB\sin\theta & -AB\cos\theta & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & -BQ\sin\phi & -BQ\cos\phi & +CQ\sin\phi & +CQ\cos\phi & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & +\mu\,\mathrm{sign}(V_C) & 0
\end{bmatrix}
\begin{Bmatrix}
F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{12}
\end{Bmatrix}
=
\begin{Bmatrix}
0 \\ 0 \\ 0 \\ -F_Q^x \\ -F_Q^y \\ 0 \\ -P \\ 0 \\ 0
\end{Bmatrix}
$$

$$\begin{bmatrix} & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \\ & & & & & & & & \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{y} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \\ \_\_\_\_\_\_ \end{Bmatrix}$$