## Static Force Analysis for Skid Loader - Virtual Work

A trunnion mount hydraulic cylinder actuates the arm of a skid steer loader as shown below. At this position, $\mathrm{e}=40$ inches, $\theta=61.131^{\circ}$, $\dot{\mathrm{e}}=-12 \mathrm{ips}, \dot{\theta}=-0.3625 \mathrm{rad} / \mathrm{s}$.

Determine the force on the hydraulic cylinder required to lower an 800 lbf payload attached to point D by a cable. The payload moves with constant velocity at the position shown. You may neglect the effects of friction. The weight of the arm and cylinder are small compared to the
payload. Show your work.
from Newtonian solution
FCylinder 2261.7 lbf up/left $\quad \mathrm{F}_{\mathrm{C}}=2261.9 \mathrm{lbf}$

Not to scale
$A B=36$ inches
$A C=42$ inches $A D=96$ inches $\phi=16^{\circ}$

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\begin{aligned}
& \theta+\phi=77.131^{\circ} \\
& \beta=90^{\circ}-\theta-\phi=12.869^{\circ}
\end{aligned}
$$



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\mathrm{V}_{\mathrm{D}}=\mathrm{AD} \dot{\theta}=34.8 \mathrm{ips}
$$

power
$\overline{\mathrm{F}}_{\mathrm{C}} \circ \overline{\mathrm{V}}_{\mathrm{C}}+\overline{\mathrm{P}} \circ \overline{\mathrm{V}}_{\mathrm{D}}=0$


What corresponding hydraulic pressure would be required for a cylinder with a 3 inch DIA bore?
PCylinder $320 \mathrm{psi} \quad \mathrm{A}=\pi \mathrm{D}^{2} / 4=7.069 \mathrm{in}^{2}$
Is this value reasonable? Why?
OK, industrial hydraulics often go to 3000 psi
If you include friction between the piston and cylinder wall, will it increase or decrease your computation for pressure.

pressure pushes up
friction force will be up opposing piston motion

What value would you use for the coefficient of friction between the piston and cylinder wall?
$\mu$ $\qquad$ Why?

Should your analysis be different if the cylinder were retracting at constant velocity instead of the payload moving at constant velocity?

yes no Why? | constant $\dot{\mathrm{e}}$ means $\dot{\theta}$ will not be constant means velocity of the payload will |
| :--- |
| not be constant, therefore must account for acceleration of payload mass |

## Static Force Analysis for Sewing Machine - Virtual Work

Determine crank torque $\mathrm{T}_{12}$ required to maintain this sewing machine linkage in static equilibrium as shown below for applied load $\mathrm{P}=10 \mathrm{~N}$. Assume that friction and the weight of the links are negligible.


> from velocity solution $\omega_{2}=+8 \pi \mathrm{rad} / \mathrm{sec}$ $\overline{\mathrm{V}}_{\mathrm{F}}=64.71 \mathrm{cps}$ down actual power, no friction, no springs $\mathrm{T}_{12} \circ \omega_{2}+\overline{\mathrm{V}}_{\mathrm{F}} \circ \overline{\mathrm{P}}=0$ assume $\mathrm{T}_{12}$ is CCW $\left(\begin{array}{r}\left.+\mathrm{T}_{12}\right)(+8 \pi \mathrm{rad} / \mathrm{sec}) \\ \\ \quad+(-64.72 \mathrm{~cm} / \mathrm{sec})(+10 \mathrm{~N})=0\end{array}\right.$ $\mathrm{T}_{12}=+25.75 \mathrm{~N} . \mathrm{cm}$

Static Force Analysis for Four Bar - Virtual Work

$\left[\begin{array}{cc}-r_{3} \sin \theta_{3} & r_{4} \sin \theta_{4} \\ r_{3} \cos \theta_{3} & -r_{4} \cos \theta_{4}\end{array}\right]\left\{\begin{array}{l}\dot{\theta}_{3} \\ \dot{\theta}_{4}\end{array}\right\}=\left\{\begin{array}{c}r_{2} \dot{\theta}_{2} \sin \theta_{2} \\ -r_{2} \dot{\theta}_{2} \cos \theta_{2}\end{array}\right\} \quad$ Assume $\dot{\theta}_{2}=+1$ rad/sec virtual velocity
$\left[\begin{array}{cc}-13.651 & 40.841 \\ 58.426 & -18.895\end{array}\right]\left\{\begin{array}{c}\dot{\theta}_{3} \\ \dot{\theta}_{4}\end{array}\right\}=\left\{\begin{array}{c}27.189 \\ -12.678\end{array}\right\} \quad\left\{\begin{array}{c}\dot{\theta}_{3} \\ \dot{\theta}_{4}\end{array}\right\}=\left\{\begin{array}{c}-0.390 \\ 0.535\end{array}\right\} \mathrm{rad} / \mathrm{sec}$
$\vec{V}_{P}=j r_{2} \dot{\theta}_{2} \mathrm{e}^{\mathrm{j} \theta 2}+\mathrm{j}(B P) \dot{\theta}_{3} \mathrm{e}^{\mathrm{j} \theta 3}=-\mathrm{r}_{2} \dot{\theta}_{2} \sin \theta_{2}+\mathrm{j} \mathrm{r}_{2} \dot{\theta}_{2} \cos \theta_{2}-(B P) \dot{\theta}_{3} \sin \theta_{3}+\mathrm{j}(B P) \dot{\theta}_{3} \cos \theta_{3}$
$\vec{V}_{P}=-25.148+j 3.944 \mathrm{~cm} / \mathrm{sec}$
$\vec{V}_{Q}=j(D Q) \dot{\theta}_{4} e^{j \theta 4}=(D Q) \dot{\theta}_{4} \sin \theta_{4}+j(D Q) \dot{\theta}_{4} \cos \theta_{4}=-11.653-j 5.391 \mathrm{~cm} / \mathrm{sec}$
$\overrightarrow{\mathrm{F}}_{\mathrm{P}}=150 \mathrm{~N} @ 133.151^{\circ}=-102.589+\mathrm{j} 109.433 \mathrm{~N}$
$\overrightarrow{\mathrm{F}}_{\mathrm{Q}}=200 \mathrm{~N} @ 217.827^{\circ}=-157.973-\mathrm{j} 122.656 \mathrm{~N}$
$\overrightarrow{\mathrm{T}}_{12} \circ \overrightarrow{\dot{\theta}}_{2}+\overrightarrow{\mathrm{F}}_{\mathrm{P}} \circ \overrightarrow{\mathrm{V}}_{\mathrm{P}}+\overrightarrow{\mathrm{F}}_{\mathrm{Q}} \circ \overrightarrow{\mathrm{V}}_{\mathrm{Q}}=0$
$\mathrm{T}_{12} \dot{\theta}_{2}+(2579.91+431.60) \mathrm{N} . \mathrm{cm} / \mathrm{sec}+(1840.86+661.24) \mathrm{N} . \mathrm{cm} / \mathrm{sec}=0$
$\mathrm{T}_{12}=-5513.6 \mathrm{~N} . \mathrm{cm}$
from Newtonian matrix solution $\quad \mathrm{T}_{12}=-5514.89 \mathrm{~N} . \mathrm{cm}$

## Static Force Analysis for Pushups - Virtual Work

A person doing pushups can be modeled as a four bar linkage. The ground is the base link, the forearms are link 2, the upper arms are link 3, and the torso and legs are link 4 as shown below. The wrists are revolute $\mathrm{O}_{2}$, the elbows are revolute A , the shoulders are revolute B , and the toes are revolute $\mathrm{O}_{4}$. Mass of the torso/legs is 180 lbm and the mass center is located at $\mathrm{G}_{4}$. Assume that all muscular effort is provided by the triceps as torque $\mathrm{T}_{32}$ across the elbows.

For an initial estimate, use the additional assumptions:
a) $\omega_{4}$ is constant at this position
b) Weight of the arms is negligible compared to weight of the torso/legs.
c) Friction is negligible at A, B, C and D.
d) No muscular torque is generated at $\mathrm{A}, \mathrm{C}$ and D .

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\begin{array}{ll}
\mathrm{AD}=52 \text { inch } & \theta_{2}=45^{\circ} \\
\mathrm{AB}=12 \text { in } & \theta_{3}=149.14^{\circ} \\
\mathrm{BC}=14 \text { in } & \theta_{4}=164.24^{\circ} \\
\mathrm{CD}=57.7 \mathrm{in} & \omega_{2}=0.5 \mathrm{rad} / \mathrm{sec} \\
\mathrm{DG}_{4}=39 \mathrm{in} & \omega_{3}=-1.435 \mathrm{rad} / \mathrm{sec} \\
& \omega_{4}=-0.387 \mathrm{rad} / \mathrm{sec}
\end{array}
$$



Determine angular velocity across the elbows $\omega_{2 / 3}$ for the position and velocity provided above.
$\omega_{2 / 3}=\omega_{2}-\omega_{3}=+1.935 \mathrm{rad} / \mathrm{sec}$
Determine elbow torque $\mathrm{T}_{32}$ for the position and velocity provided above.
$\mathrm{V}_{\mathrm{G} 4}=\mathrm{DG}_{4} \omega_{4}=15.093 \mathrm{ips}$ UP, $\mathrm{A}_{\mathrm{G} 4}{ }^{\mathrm{T}}=\mathrm{DG}_{4} \alpha_{4}=0, \mathrm{~A}_{\mathrm{G} 4}{ }^{\mathrm{N}}=\mathrm{DG}_{4} \omega_{4}{ }^{2}=5.84 \mathrm{ips}^{2}=0.015 \mathrm{G}$ negligible
$\overline{\mathrm{W}}_{4} \circ \overline{\mathrm{~V}}_{\mathrm{G} 4}+\overline{\mathrm{T}}_{32} \circ \bar{\omega}_{2 / 3}=0 \quad-\mathrm{W}_{4} \cos \left(180^{\circ}-\theta_{4}\right) \mathrm{V}_{\mathrm{G} 4}+\mathrm{T}_{32} \omega_{2 / 3}=0 \quad \mathrm{~T}_{32}=+1351.2$ in.lbf
Do the magnitude and direction for your answer seem reasonable? Why?
from Newtonian solution $\quad \mathrm{T}_{32}=1351.2$ in.lbf CCW
Rate the last four assumptions and state your reasoning.
b) constant $\omega_{4} \quad 1=$ poor $2=$ acceptable for an approximation $3=$ very good
c) weight of arms is negligible 1=poor 2=acceptable for an approximation 3=very good
d) friction is negligible $1=$ poor $2=$ acceptable for an approximation $3=$ very good
e) no muscle force at A, C, D 1=poor 2=acceptable for an approximation 3=very good

Determine $\omega_{4}$ of the torso/legs when the forearm is aligned with the upper arms $\left(\theta_{2}=\theta_{3}\right) . \quad \omega_{4}=0$

