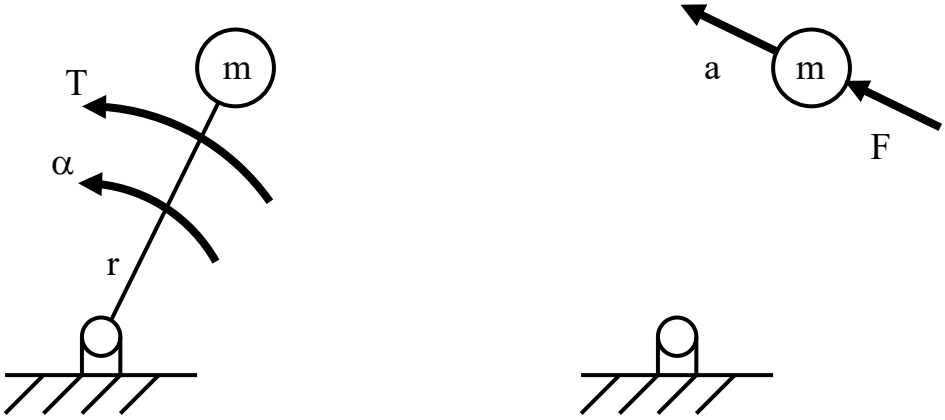
# Mass Moment of Inertia Quiz

| | value | units | answer |
|---|---|---|---|
| mass m | _____ | _____ | _____ |
| length L | _____ | _____ | _____ |
| width w | _____ | _____ | _____ |
| thickness t | _____ | _____ | _____ |
| density ρ | _____ | _____ | _____ |
| mass moment J | _____ | _____ | _____ |

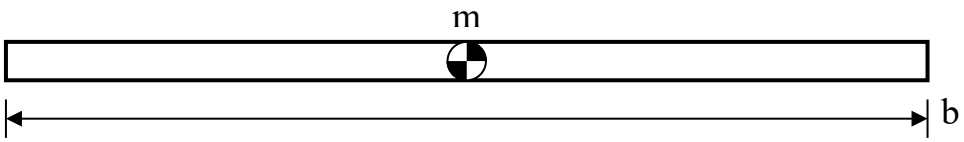
1 slug = 32.174 lbm

# Mass Moment of Inertia Review

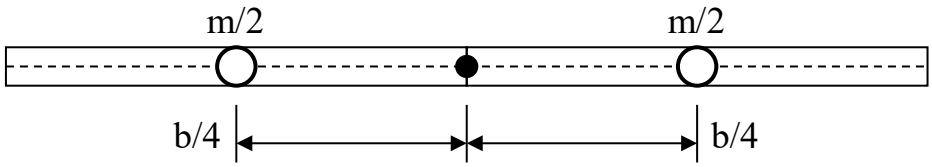



$$T = F\,r \qquad F = m\,a \qquad a = r\,\alpha$$

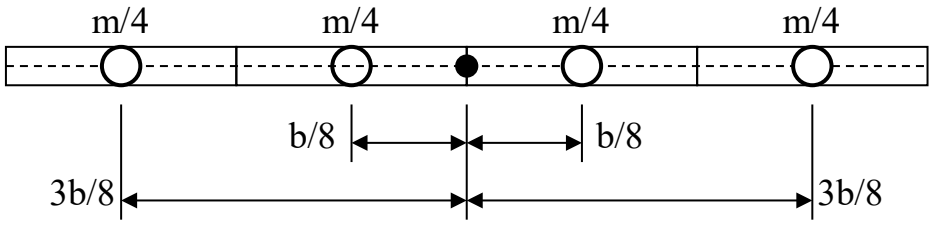
$$F = m\,r\,\alpha \qquad T = m\,r^2\,\alpha \qquad T = J\,\alpha \qquad J = m\,r^2$$



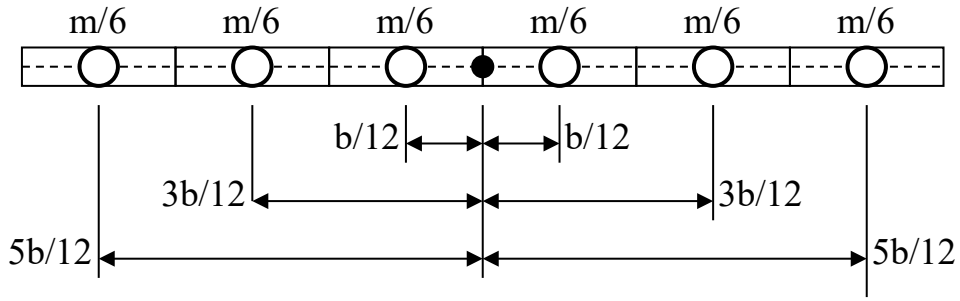

$$J_G = \frac{1}{12}mb^2$$




$$J_G = \frac{m}{2}\left(\frac{b}{4}\right)^2 + \frac{m}{2}\left(\frac{b}{4}\right)^2 = \frac{1}{16}mb^2$$

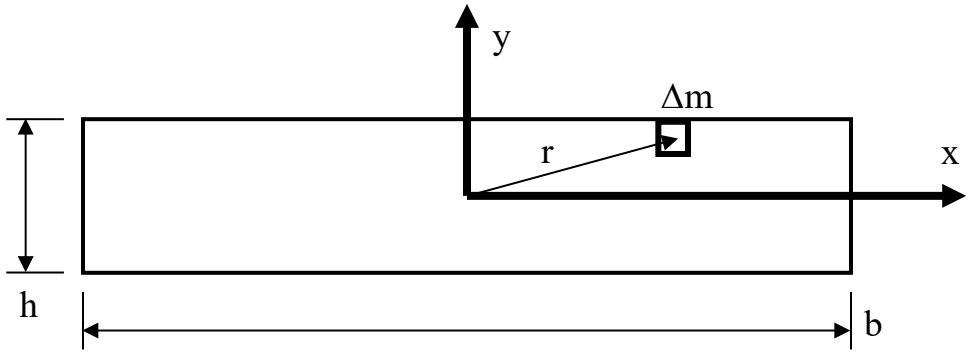



$$J_G = \frac{m}{4}\left(\frac{3b}{8}\right)^2 + \frac{m}{4}\left(\frac{b}{8}\right)^2 + \frac{m}{4}\left(\frac{b}{8}\right)^2 + \frac{m}{4}\left(\frac{3b}{8}\right)^2 = \frac{20}{256}mb^2 = \frac{1}{12.8}mb^2$$




$$J_G = \frac{m}{6}\left(\frac{5b}{12}\right)^2 + \frac{m}{6}\left(\frac{3b}{12}\right)^2 + \frac{m}{6}\left(\frac{b}{12}\right)^2 + \frac{m}{6}\left(\frac{b}{12}\right)^2 + \frac{m}{6}\left(\frac{3b}{12}\right)^2 + \frac{m}{6}\left(\frac{5b}{12}\right)^2 = \frac{70}{864}mb^2 = \frac{1}{12.343}mb^2$$

$$J_G = 2\left(\frac{m}{n}\right)\left(\frac{b^2}{2n}\right)\sum\left(1^2 + 3^2 + 5^2 + \ldots\right) = \frac{mb^2}{2n^3}\sum_{i-1}^{n/2}(2i-1)^2 \qquad \text{for } n = 100, \quad J = \frac{1}{12.0012}mb^2$$




$$J_G = \sum\left(\Delta m\, r^2\right) = \sum\left(\Delta m\left(x^2 + y^2\right)\right) = \rho\, t \sum\left(\Delta x\, \Delta y\left(x^2 + y^2\right)\right)$$

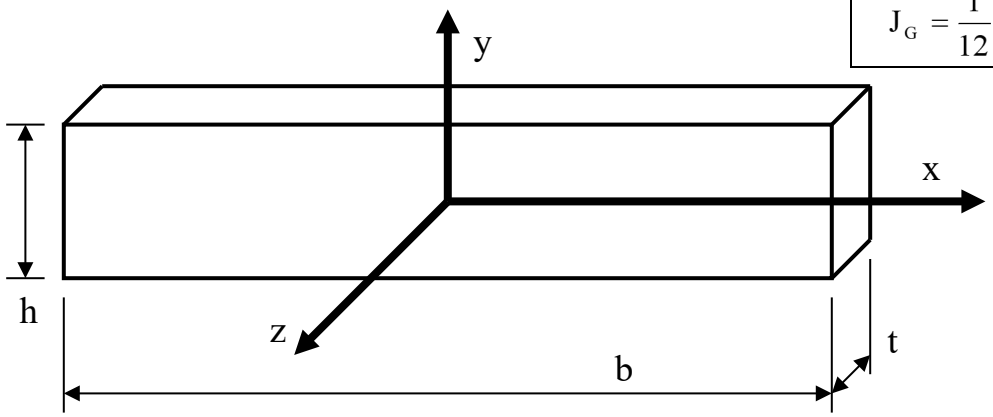
$$J_G = \rho\, t \int_{-h/2}^{h/2} \int_{-b/2}^{b/2} \left(x^2 + y^2\right)\, dx\, dy$$

$$J_G = \rho\, t \int_{-h/2}^{h/2} \left(\left(\frac{1}{3}x^3 + xy^2\right)_{-b/2}^{b/2}\right) dy = \rho\, t \int_{-h/2}^{h/2} \left(\left(\frac{1}{24}b^3 + \frac{1}{2}by^2\right) - \left(-\frac{1}{24}b^3 - \frac{1}{2}by^2\right)\right) dy$$

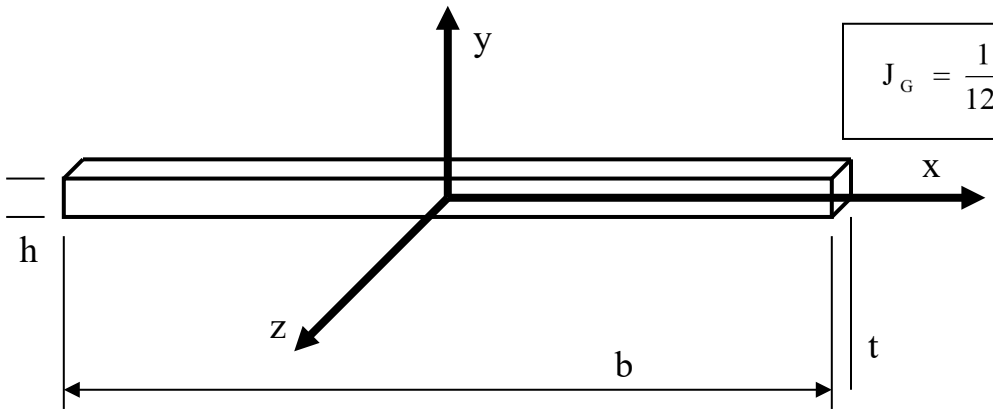
$$J_G = \rho\, t \int_{-h/2}^{h/2} \left(\frac{1}{12}b^3 + by^2\right) dy = \rho\, t \left(\frac{1}{12}b^3 y + \frac{1}{3}by^3\right)_{-h/2}^{h/2} = \rho\, t\left(\frac{1}{12}b^3 h + \frac{1}{12}bh^3\right)$$

$$J_G = \frac{1}{12}\rho\, t\, b\, h\left(b^2 + h^2\right) = \frac{1}{12}m\left(b^2 + h^2\right)$$

$$J_G = \frac{1}{12} m(b^2 + h^2)$$




$$J_G = \frac{1}{12} m b^2 \quad \text{for} \quad b >> h$$




$$J_G = \rho\, t \int_0^{2\pi} \int_0^R r^2 (dr)(r\, d\theta) = \rho\, t \int_0^{2\pi} \int_0^R r^3\, dr\, d\theta$$

$$J_G = \rho\, t \int_0^{2\pi} \left( \frac{1}{4} r^4 \right)_0^R d\theta = \frac{1}{4} \rho\, t\, R^4 \int_0^{2\pi} d\theta$$

$$J_G = \frac{1}{4} \rho\, t\, R^4 (\theta)_0^{2\pi} = \frac{\pi}{2} \rho\, t\, R^4$$

$$J_G = \frac{1}{2} m\, R^2$$

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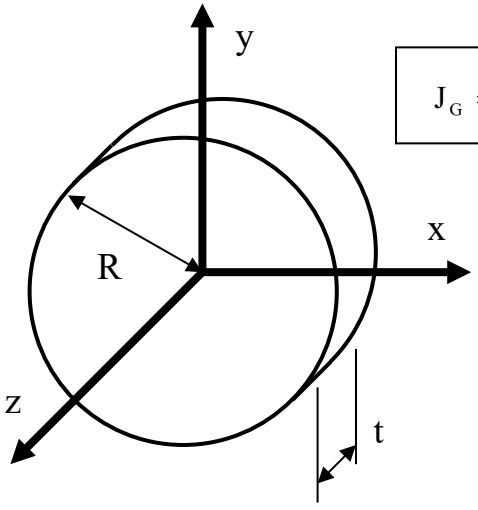


## Radius of Gyration

Radius of gyration k is defined as an equivalent radius at which the entire mass of an object would be concentrated if it were a thin ring. It is a simple way to express mass moment that helps visualize an equivalent size.

$$J_G = m\,k^2 \qquad k = \sqrt{\frac{J_G}{m}}$$

As an example, for a long thin rod $J_G = \frac{1}{12} m\,b^2$ $k = \sqrt{\frac{1}{12}}\,b = 0.2887\,b$

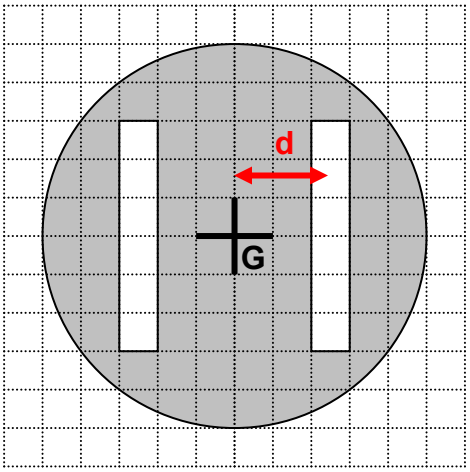
As an example, for a circular disk $J_G = \frac{1}{2} m\,R^2$ $k = \sqrt{\frac{1}{2}}\,R = 0.7071\,R$

# Mass Moment of Inertia for Disk with Slots

The disk shown below has two rectangular slots. The disk is 2.5 cm thick and the slots go completely through the disk. The density is 7.8 g/cm$^3$.

Determine the mass, centroidal mass moment of inertia and the radius of gyration of the disk with slots.

m 1.298 kg $J_G$ 16.21 kg.cm$^2$ $k_G$ 3.534 cm




1 cm

$r_C = 5$ cm
$m_C = \pi\ r^2\ t\ \rho = 1.532$ kg
$J_C = m_C\ r_C^2 / 2 = 19.144$ kg.cm$^2$

$b_S = 6$ cm, $h_S = 1$ cm, $d_S = 2.5$ cm
$m_S = b_S\ h_S\ t\ \rho = 0.117$ kg
$J_S = m_S\ (b_S^2 + h_S^2) / 12 = 0.361$ kg.cm$^2$

$m = m_C - 2\ m_S = 1.298$ kg
$J_G = J_C - 2\ J_S - 2\ m_S\ d_S^2 = 16.96$ kg.cm$^2$

$k_G = \text{sqrt}(\ J_G / m\ ) = 3.534$ cm

What material was used to manufacture this disk? steel

Comment on the manufacturability of this design.

square corners on slots are difficult to machine