**Measuring Mass Moment of Inertia as a Simple Pendulum**



mg

m g sin

F12x

F12y

P



G

a



P

G

m = mass

JG = mass moment about centroid

a = distance from pivot to centroid

g = acceleration of gravity

 = time period for one oscillation







for small angles 



assume   





using 



2 x 4 x 24 from Notes\_07\_01 JG = 111.9 lbm.in2

m = 2.286 lbm a = 12.25 in 20  = 25.74 sec  = 1.2870 sec

 = 453.52 lbm.in2

ma2 = (2.286 lbm)(12.25 in)2 = 343.04 lbm.in2

JG = 110.48 lbm.in2 (-1.2% difference)

**Measuring Mass Moment of Inertia with Torsional Pendulum**

Torsional pendulums use a lightweight circular or triangular platform suspended from three light cables as shown below. The platform must be horizontal and the cables must be of equal length. The cables should be attached to the platform equidistant from the center of the platform in an equilateral triangular pattern. Cable attachments need not be at the edge of the platform.

The object should be placed with its center of mass directly over the center of the platform. The centroidal axis of the object must be vertical. The platform must oscillate in pure rotation with small angular displacement about the central vertical axis and must not swing laterally.

This procedure should be used first with an empty platform to determine the mass moment of the platform alone and then repeated with the object centered on the platform to measure the mass moment of the object plus the platform.

**Reference**: Shigley, J.E. and J.J. Uicker, Theory of Machines and Mechanisms

 McGraw-Hill, 1995, second edition

s

**small**

torsional

oscillation

r





 JGO = centroidal polar mass moment of inertia of object

 JGP = centroidal polar mass moment of inertia of platform

 g = acceleration of gravity

 mO = mass of object

 mP = mass of platform

 r = radius of cable attachments from center of platform

 (not platform radius)

  = time period for one torsional oscillation

 s = length of cables

**Measuring Mass Moment of Inertia with Platform Pendulum**

Platform pendulums use a lightweight rectangular platform suspended from four light cables as shown below. The platform must be horizontal and the cables must be of equal length. Cable attachments need not be at the edge of the platform. Pendulum design should minimize aO to maximize accuracy.

The object should be placed with its center of mass directly over the center of the platform. The centroidal axis of the object must be horizontal. The platform must swing laterally with small displacement about the centerline of the cable pivots and must not twist or swing axially.

This procedure should be used first with an empty platform to determine the mass moment of the platform itself and then repeated with the object centered on the platform to measure the mass moment of the object plus the platform.

**Reference**: Mabie, H.H. and C.F. Reinholtz, Mechanisms and Dynamics of Machinery,

 Wiley, 1987, fourth edition



 JGO = centroidal polar mass moment of inertia of object

 JP = polar mass moment of inertia of platform

 g = acceleration of gravity

 mO = mass of object

 mP = mass of platform

 aO = distance from pivot to centroid of object

 aP = distance from pivot to centroid of platform

  = time period for one oscillation

**Measuring Mass Moment of Inertia and Centroid Location Simultaneously**

aO

aP

**Small objects**

a1

a2

e

P2

P1

G

B

C

IDB

IDC

1) Suspend object by point P1 and measure 

2) Suspend object by point P2 and measure 

3) Measure m

4) Measure distances e, IDB, IDC













**Suspended objects**

a1

a2

f

P1

P2

1) Suspend object by point P1 and measure 

2) Suspend object by point P2 and measure 

3) Measure m

4) Measure distance f





**Measuring Mass Moment of Inertia and Centroid using Multiple Pivots**

a1

a2

P2

P1

x

y

G

an

Pn

1) Suspend object by points  and measure . All points on same body.

2) Measure local coordinates 

3) Weigh the object to measure m

 

iteratively solve for  using 



 



% t\_mult.m - test multiple point measurement of mass moment

% HJSIII, 09.04.11

clear

% constants

m = 96;

g = 1;

% experimental values for xg=8, yg=3, JG=2336

% x, y, t

xyt = [ 12 2 19.894 ;

 10 5 21.244 ;

 14 5 20.039 ];

x = xyt(:,1);

y = xyt(:,2);

t = xyt(:,3);

t2term = t.\*t /4/pi/pi;

n = length( t );

% initial estimate

q = [ 0 ; 0 ; 0 ];

% loop

for iter = 1 : 10,

% current estimates

 xg = q(1);

 yg = q(2);

 JG = q(3);

% evaluate residuals

 a2 = (x-xg).\*(x-xg) + (y-yg).\*(y-yg);

 a = sqrt( a2 );

 PHI = JG + m\*a2 - m\*g\*a.\*t2term ;

% Jacobian

 par = -m \* (2\*a - g\*t2term ) ./a;

 JAC(:,1) = par .\* (x-xg);

 JAC(:,2) = par .\* (y-yg);

 JAC(:,3) = ones(n,1);

% update

 q = q -inv(JAC)\*PHI;

 disp( q' )

end

 