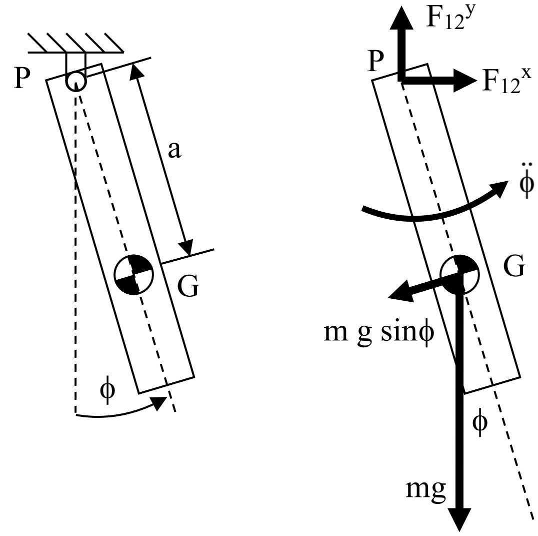
Measuring Mass Moment of Inertia as a Simple Pendulum



m = mass
J_G = mass moment about centroid
a = distance from pivot to centroid
g = acceleration of gravity
τ = time period for one oscillation

$\sum$M about P CCW +

$$-(m g \sin\phi)a = (J_G + m a^2)\ddot{\phi}$$

$$(J_G + m a^2)\ddot{\phi} + m g a \sin\phi = 0$$

for small angles $\phi < 15°$, $\sin\phi \approx \phi$

$$(J_G + m a^2)\ddot{\phi} + m g a \phi = 0$$

assume $\phi = \phi_{MAX} \cos(2\pi f t)$ $\dot{\phi} = -\phi_{MAX} 2\pi f \sin(2\pi f t)$ $\ddot{\phi} = -\phi_{MAX} 4\pi^2 f^2 \cos(2\pi f t)$

$$-(J_G + m a^2)\phi_{MAX} \cos(2\pi f t) + m g a \phi_{MAX} \cos(2\pi f t) = 0$$

$$4\pi^2 f^2 (J_G + m a^2) = m g a$$

using $\tau = 1/f$

$$J_G + m a^2 = \frac{m g a \tau^2}{4\pi^2}$$

2 x 4 x 24 from Notes_07_01 $J_G = 111.9$ lbm.in^2

m = 2.286 lbm a = 12.25 in $20\tau = 25.74$ sec $\tau = 1.2870$ sec

$$\frac{m g a \tau^2}{4\pi^2} = \left(\frac{2.286 \text{ lbm}}{4\pi^2}\right)\left(\frac{386 \text{ in}}{\text{sec}^2}\right)(12.25 \text{ in})(1.2870 \text{ sec})^2 = 453.52 \text{ lbm.in}^2$$

$ma^2 = (2.286 \text{ lbm})(12.25 \text{ in})^2 = 343.04$ lbm.in^2

$J_G = 110.48$ lbm.in^2 (-1.2% difference)

Measuring Mass Moment of Inertia with Torsional Pendulum

Torsional pendulums use a lightweight circular or triangular platform suspended from three light cables as shown below. The platform must be horizontal and the cables must be of equal length. The cables should be attached to the platform equidistant from the center of the platform in an equilateral triangular pattern. Cable attachments need not be at the edge of the platform.

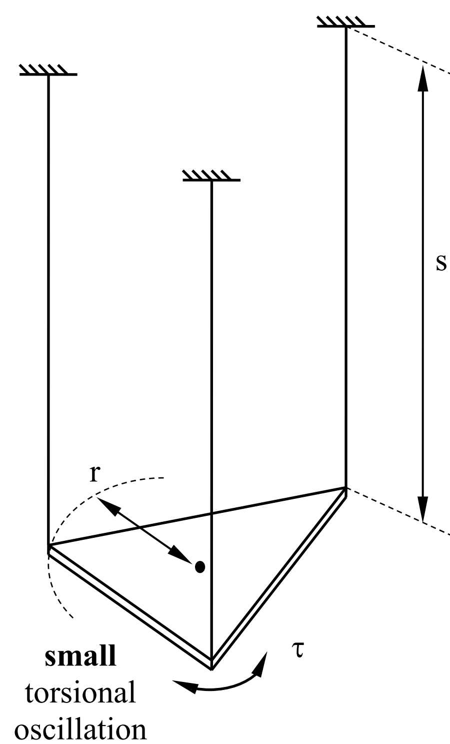
The object should be placed with its center of mass directly over the center of the platform. The centroidal axis of the object must be vertical. The platform must oscillate in pure rotation with small angular displacement about the central vertical axis and must not swing laterally.

This procedure should be used first with an empty platform to determine the mass moment of the platform alone and then repeated with the object centered on the platform to measure the mass moment of the object plus the platform.

Reference: Shigley, J.E. and J.J. Uicker, Theory of Machines and Mechanisms
McGraw-Hill, 1995, second edition

$$J_{GO} + J_{GP} = \frac{g(m_O + m_P) r^2 \tau^2}{4 s \pi^2}$$

J_{GO} = centroidal polar mass moment of inertia of object
J_{GP} = centroidal polar mass moment of inertia of platform
g = acceleration of gravity
m_O = mass of object
m_P = mass of platform
r = radius of cable attachments from center of platform (not platform radius)
τ = time period for one torsional oscillation
s = length of cables



Measuring Mass Moment of Inertia with Platform Pendulum

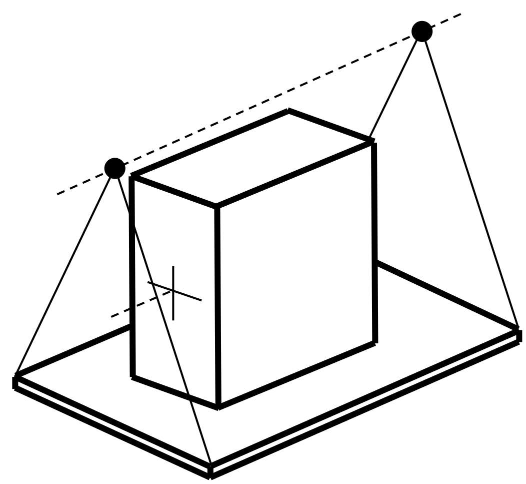
Platform pendulums use a lightweight rectangular platform suspended from four light cables as shown below. The platform must be horizontal and the cables must be of equal length. Cable attachments need not be at the edge of the platform. Pendulum design should minimize a_O to maximize accuracy.

The object should be placed with its center of mass directly over the center of the platform. The centroidal axis of the object must be horizontal. The platform must swing laterally with small displacement about the centerline of the cable pivots and must not twist or swing axially.

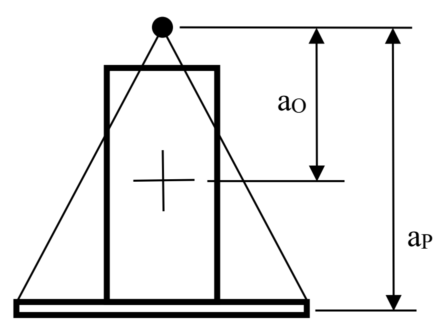
This procedure should be used first with an empty platform to determine the mass moment of the platform itself and then repeated with the object centered on the platform to measure the mass moment of the object plus the platform.

Reference: Mabie, H.H. and C.F. Reinholtz, Mechanisms and Dynamics of Machinery, Wiley, 1987, fourth edition

$$J_{GO} + m_O a_O^2 + J_P = \frac{g(m_O a_O + m_P a_P)\tau^2}{4\pi^2}$$

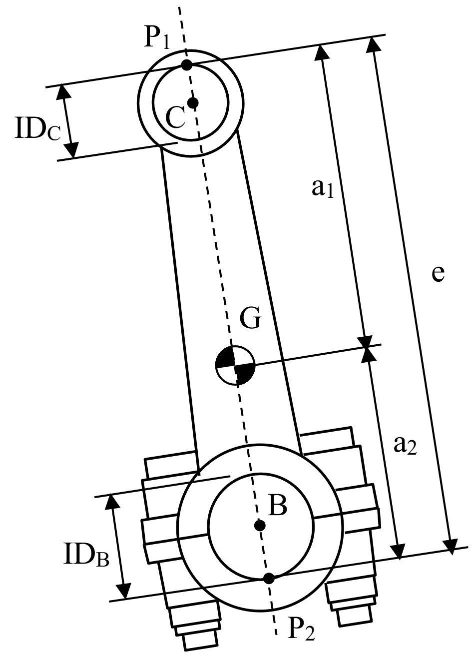


J_{GO} = centroidal polar mass moment of inertia of object
J_P = polar mass moment of inertia of platform
g = acceleration of gravity
m_O = mass of object
m_P = mass of platform
a_O = distance from pivot to centroid of object
a_P = distance from pivot to centroid of platform
τ = time period for one oscillation



Measuring Mass Moment of Inertia and Centroid Location Simultaneously

Small objects



1) Suspend object by point P_1 and measure τ_1
2) Suspend object by point P_2 and measure τ_2
3) Measure m
4) Measure distances e, ID_B, ID_C

$$J_G + m a_1^2 = \frac{m g a_1 \tau_1^2}{4\pi^2}$$

$$J_G + m a_2^2 = \frac{m g a_2 \tau_2^2}{4\pi^2}$$

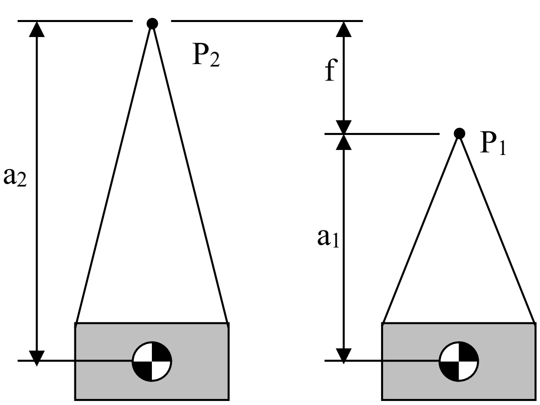
$$4\pi^2\left(a_1^2 - a_2^2\right) = g a_1 \tau_1^2 - g a_2 \tau_2^2$$

$$a_2 = e - a_1 \qquad a_2^2 = e^2 - 2ea_1 + a_1^2$$

$$a_1 = \frac{e\left(g \tau_2^2 - 4\pi^2 e\right)}{g\left(\tau_1^2 + \tau_2^2\right) - 8\pi^2 e}$$

$$BG = a_2 - ID_B/2 \qquad CG = a_1 - ID_C/2$$

Suspended objects

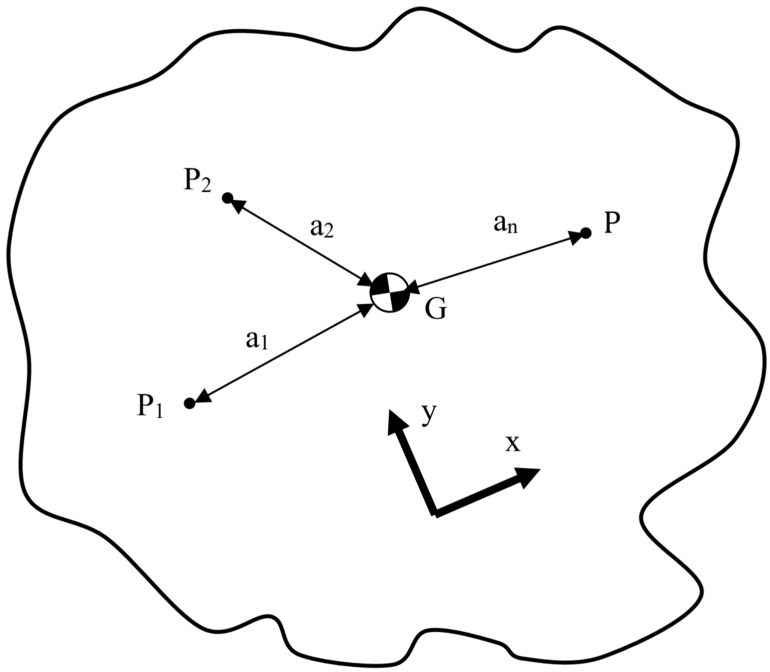


1) Suspend object by point P_1 and measure τ_1
2) Suspend object by point P_2 and measure τ_2
3) Measure m
4) Measure distance f

$$a_2 = f + a_1 \qquad a_2^2 = f^2 + 2 f a_1 + a_1^2$$

$$a_1 = \frac{f\left(g \tau_2^2 - 4\pi^2 f\right)}{g\left(\tau_1^2 - \tau_2^2\right) + 8\pi^2 f}$$

Measuring Mass Moment of Inertia and Centroid using Multiple Pivots



1) Suspend object by points $P_1, P_2, \cdots, P_n$ and measure $\tau_1, \tau_2, \cdots, \tau_n$. All points on same body.

2) Measure local coordinates $\begin{Bmatrix} x \\ y \end{Bmatrix}^{'P1}, \begin{Bmatrix} x \\ y \end{Bmatrix}^{'P2}, \cdots, \begin{Bmatrix} x \\ y \end{Bmatrix}^{'Pn}$

3) Weigh the object to measure m

$$J_G + m\,a_i^{\,2} = \frac{m\,g\,a_i\,\tau_i^{\,2}}{4\pi^2} \qquad a_i^{\,2} = \left(x'^{Pi} - x'^{G}\right)^2 + \left(y'^{Pi} - y'^{G}\right)^2$$

iteratively solve for $\{q\} = \begin{Bmatrix} x'^{G} \\ y'^{G} \\ J_G \end{Bmatrix}$ using $\{\Phi\} = \begin{Bmatrix} J_G + m a_1^{\,2} - m\,g\,a_1\,\tau_1^{\,2}/4\pi^2 \\ J_G + m a_2^{\,2} - m\,g\,a_2\,\tau_2^{\,2}/4\pi^2 \\ \vdots \\ J_G + m a_n^{\,2} - m\,g\,a_n\,\tau_n^{\,2}/4\pi^2 \end{Bmatrix}$

$$\{q\}_{k+1} = \{q\}_k - \left(\left[\Phi_q\right]^T \left[\Phi_q\right]\right)^{-1} \left[\Phi_q\right]^T \{\Phi\}$$

$$\frac{\partial \Phi_i}{\partial x'^{G}} = -m\left(2\,a_i - g\,\tau_1^{\,2}/4\pi^2\right)\frac{(x'^{Pi} - x'^{G})}{a_i} \qquad \frac{\partial \Phi_i}{\partial y'^{G}} = -m\left(2\,a_i - g\,\tau_1^{\,2}/4\pi^2\right)\frac{(y'^{Pi} - y'^{G})}{a_i}$$

$$[\Phi_q] = \begin{bmatrix} \partial\Phi_1 / \partial x'^G & \partial\Phi_1 / \partial y'^G & 1 \\ \partial\Phi_2 / \partial x'^G & \partial\Phi_2 / \partial y'^G & 1 \\ \vdots & \vdots & \vdots \\ \partial\Phi_n / \partial x'^G & \partial\Phi_n / \partial y'^G & 1 \end{bmatrix}$$

```
% t_mult.m - test multiple point measurement of mass moment
% HJSIII, 09.04.11

clear

% constants
m = 96;
g = 1;

% experimental values for xg=8, yg=3, JG=2336
% x, y, t
xyt = [ 12  2  19.894 ;
        10  5  21.244 ;
        14  5  20.039 ];
x = xyt(:,1);
y = xyt(:,2);
t = xyt(:,3);
t2term = t.*t /4/pi/pi;
n = length( t );

% initial estimate
q = [ 0 ; 0 ; 0 ];

% loop
for iter = 1 : 10,

% current estimates
  xg = q(1);
  yg = q(2);
  JG = q(3);

% evaluate residuals
  a2 = (x-xg).*(x-xg) + (y-yg).*(y-yg);
  a = sqrt( a2 );
  PHI = JG + m*a2 - m*g*a.*t2term ;

% Jacobian
  par = -m * (2*a - g*t2term ) ./a;
  JAC(:,1) = par .* (x-xg);
  JAC(:,2) = par .* (y-yg);
  JAC(:,3) = ones(n,1);

% update
  q = q -inv(JAC)*PHI;
  disp( q' )
end
```

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Moment of Inertia

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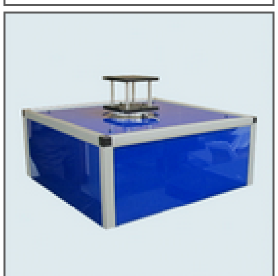
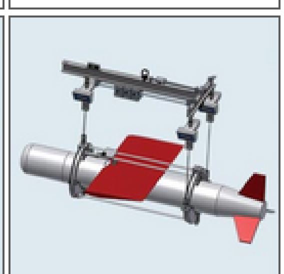


Dynamic Balancing

Gimbal Balancing

Moment Weight Scales

Fixtures & Accessories



Center of Buoyancy