Forward Versus Inverse Dynamics

Inverse dynamics

1) kinematically driven

2) know desired position, velocity and acceleration kinematics

3) know external forces (torques) on system

4) find driver forces (torques) and joint reactions to cause desired motion

5) statics is a subset of inverse dynamics
   true statics - velocities = 0, accelerations = 0
   quasi-statics - velocities = constant, accelerations = 0

Forward dynamics

1) dynamically driven - no knowledge of exact motion trajectory

2) know current state of system - positions and velocities of links

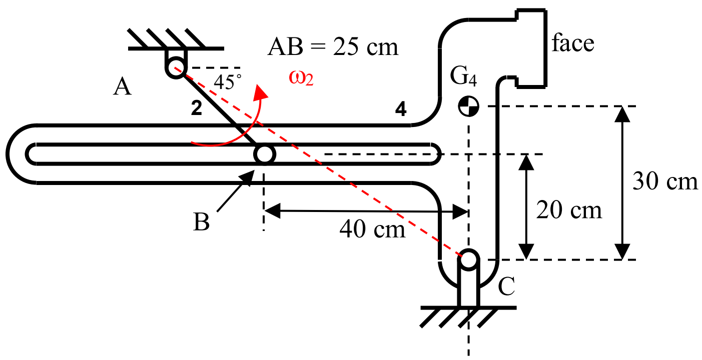
3) know external forces (torques) on system

4) compute accelerations (translational and rotational) of links using differential equations

5) forward time integration of accelerations to get new positions and velocities

Inverse Dynamics

Pin B at the end of crank link 2 forms a pin-in-slot joint with the horizontal slot in hammer link 4 as shown below. The mechanism is drawn approximately to scale. The weight of crank link 2 is very small compared to the weight of hammer link 4. You may neglect the effects of friction at A, B and C. The hammer face is not yet in contact with its platen. Show your work.



$m_4 = 2.3$ kg
$J_{G4} = 30$ N.cm.sec^2
$\omega_2 = 50$ rpm CCW constant
$\omega_4 = 2.32$ rad/sec CW
$\alpha_4 = 20.13$ rad/sec^2 CW

a) Determine the mass moment of inertia of link 4 about C. 50.7 N.cm.sec^2

b) Determine the magnitude and direction of $\bar{F}_{2\_on\_4}$ required to cause this motion at this position.

$\bar{F}_{2\_on\_4} =$ 25.5 N @ 90°

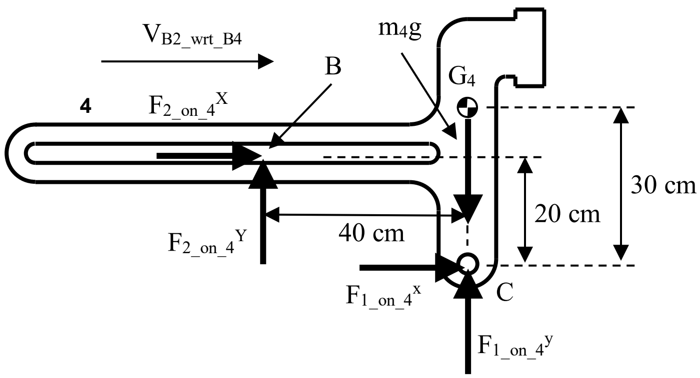
c) Determine the magnitude and direction of motor torque $T_{1\_on\_2}$ on crank 2 required to cause this motion at this position.

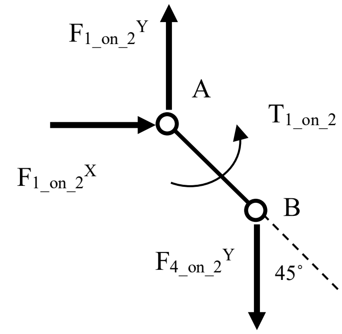
$T_{1\_on\_2} =$ 451 N.cm CCW

d) Determine the magnitude and direction of motor torque $T_{1\_on\_2}$ on crank 2 required to cause $\omega_2 = 50$ rpm CW constant at this position.

$T_{1\_on\_2} =$ same as part c)

$$J_C = J_{G4} + m_4(CG_4)^2 = 30 \text{ N.cm.sec}^2 + (2.3 \text{ kg})(30 \text{ cm})^2\left(\frac{\text{N sec}^2}{\text{kg m}}\right)\left(\frac{\text{m}}{100 \text{ cm}}\right) = 50.7 \text{ N.cm.sec}^2$$





assume no friction $F_{2\_on\_4}^X = 0$

ΣM on 4 about C CCW + $- F_{2\_on\_4}^Y (40 \text{ cm}) = J_C \alpha_4$

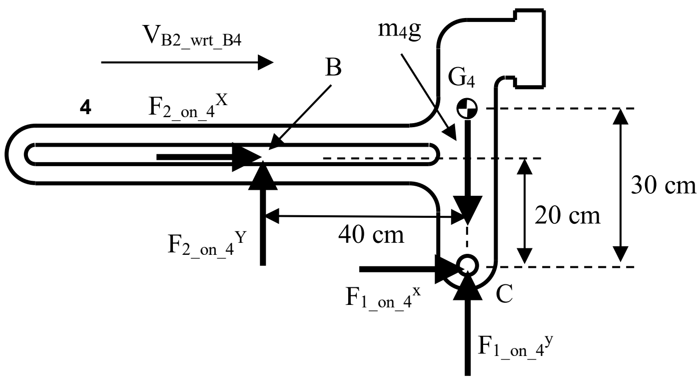
$F_{2\_on\_4}^Y = - (50.7 \text{ N.cm.sec}^2) (-20.13 \text{ rad/sec}^2) / (40 \text{ cm}) = 25.5 \text{ N}$ up

$F_{4\_on\_2}^Y = 25.5 \text{ N}$ down

ΣM on 2 about A CCW + $T_{1\_on\_2} - F_{4\_on\_2} (AB \sin 45°) = 0$

$T_{1\_on\_2} = 451 \text{ N.cm}$ CCW

part d) all normal and Coriolis accelerations will be the same magnitude AND the same direction as part c), which causes α_4 to have the same magnitude and direction



ΣF on 4 right +	$+ F_{2\_on\_4}^X + F_{1\_on\_4}^X = m_4 A_{G4}^X$
ΣF on 4 up +	$+ F_{2\_on\_4}^Y + F_{1\_on\_4}^Y - m_4 g = m_4 A_{G4}^Y$
ΣM on 4 about G_4 CCW +	$+ F_{2\_on\_4}^X (10\text{ cm}) - F_{2\_on\_4}^Y (40\text{ cm}) + F_{14}^x (30\text{ cm}) = J_{G4}\,\alpha_4$
friction	$F_{2\_on\_4}^X = \mu\, F_{2\_on\_4}^Y$

$A_{G4}^T = (CG_4)\,\alpha_4 = 603.9\text{ cps}^2$ right

$m_4\, A_{G4}^X = (2.3\text{ kg})\,(603.9\text{ cm/sec}^2)\,(1\text{ m} / 100\text{ cm}) = +13.89\text{ N}$

$A_{G4}^N = (CG_4)\,\omega_4^2 = 161.5\text{ cps}^2$ down

$m_4\, A_{G4}^y = -3.71\text{ N}$

$m_4 g = 22.56\text{ N}$

$J_{G4}\,\alpha_4 = (30\text{ N.cm.sec}^2)\,(-20.13\text{ rad/sec}^2) = -603.9\text{ N.cm}$

$\mu = 0$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ +30\text{ cm} & 0 & +10\text{ cm} & -40\text{ cm} \\ 0 & 0 & 1 & -\mu \end{bmatrix} \begin{Bmatrix} F_{1\_on\_4}^X \\ F_{1\_on\_4}^Y \\ F_{2\_on\_4}^X \\ F_{2\_on\_4}^Y \end{Bmatrix} = \begin{Bmatrix} m_4 A_{G4}^X \\ m_4 A_{G4}^X + m_4 g \\ J_{G4}\alpha_4 \\ 0 \end{Bmatrix} = \begin{Bmatrix} +13.89\text{ N} \\ -3.71 + 22.56\text{ N} \\ -603.9\text{ N.cm} \\ 0 \end{Bmatrix} =$$

$$\begin{Bmatrix} F_{1\_on\_4}^X \\ F_{1\_on\_4}^Y \\ F_{2\_on\_4}^X \\ F_{2\_on\_4}^Y \end{Bmatrix} = \begin{Bmatrix} 13.8000 \\ -6.5975 \\ 0 \\ 25.4475 \end{Bmatrix} \text{N}$$