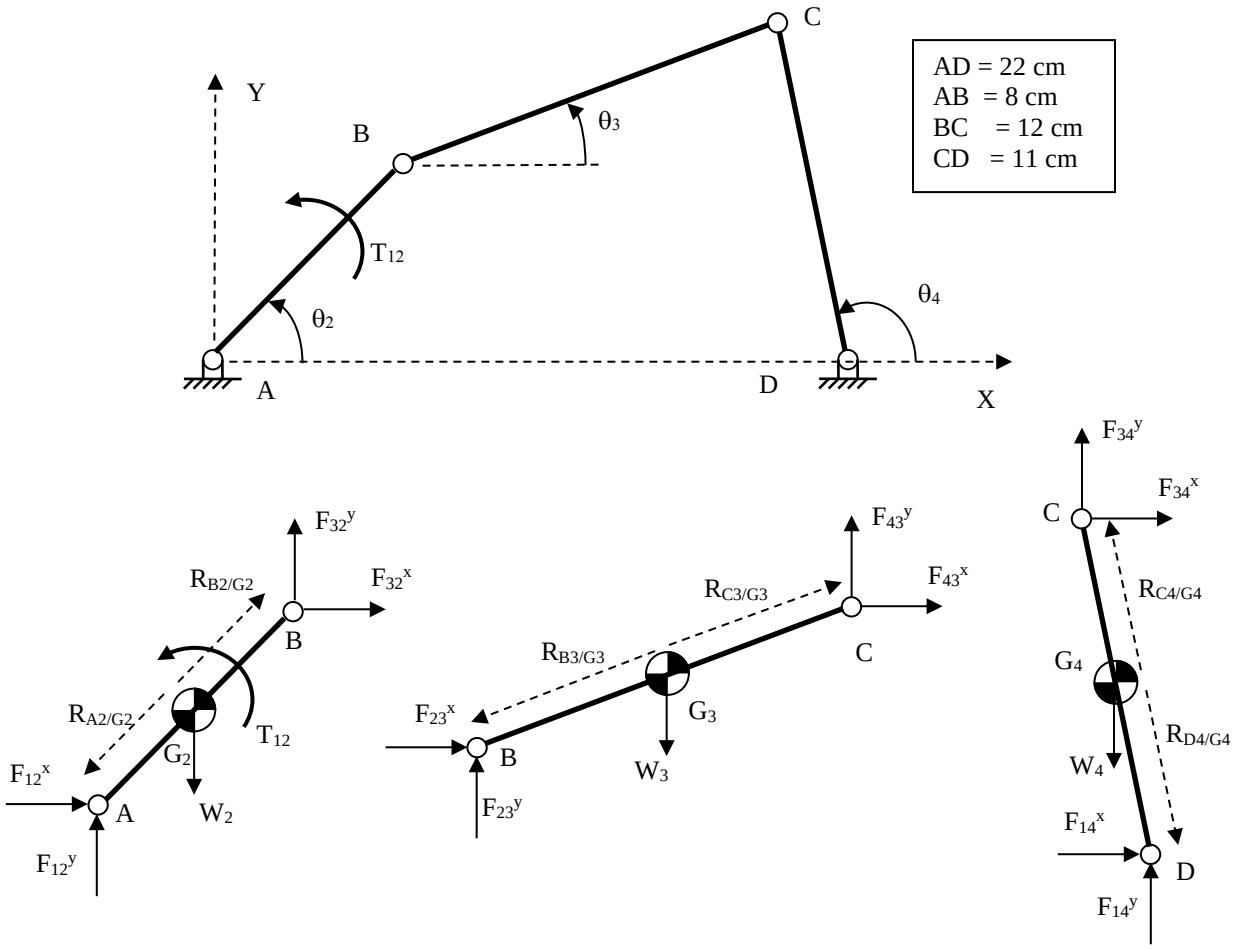
Matrix Dynamic Analysis for Four Bar

The four bar linkage shown below operates in a vertical plane. Each link is a uniform bar with 2 cm by 2 cm square cross-section stainless steel. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity.

θ_2 = 45 deg	ω_2 = 20 rad/s CW	α_2 = 100 rad/s/s CCW
θ_3 = 20 deg	ω_3 = 12.82 rad/s CCW	α_3 = 39.6 rad/s/s CW
θ_4 = 117.4 deg	ω_4 = 6.20 rad/s CW	α_4 = 482.5 rad/s/s CCW

m_2 = 0.248 kg	J_{G2}' = 1.405 kg.cm^2	ρ = 7.75 g/cm^3
m_3 = 0.372 kg	J_{G3}' = 4.588 kg.cm^2	
m_4 = 0.341 kg	J_{G4}' = 3.552 kg.cm^2	

V_{G2} = 56.57 - j 56.57 cps	A_{G2} = -1414.2 - j 848.5 cps^2
V_{G3} = 86.83 - j 40.86 cps	A_{G3} = -3672.5 - j 2260.9 cps^2
V_{G4} = 30.27 - j 15.71 cps	A_{G4} = -2262.4 - j 1412.4 cps^2



ΣF on 2 right + $F_{12}^{x} + F_{32}^{x} = m_2 A_{G2}^{x}$

ΣF on 2 up + $F_{12}^{y} + F_{32}^{y} + W_2 = m_2 A_{G2}^{y}$

ΣM on 2 about G2 CCW + $- F_{12}^{x} r_{A2/G2}^{y} + F_{12}^{y} r_{A2/G2}^{x} - F_{32}^{x} r_{B2/G2}^{y} + F_{32}^{y} r_{B2/G2}^{x} + T_{12} = J_{G2}' \alpha_2$

ΣF on 3 right + $F_{23}^{x} + F_{43}^{x} = m_3 A_{G3}^{x}$

ΣF on 3 up + $F_{23}^{y} + F_{43}^{y} + W_3 = m_2 A_{G3}^{y}$

ΣM on 3 about G3 CCW + $- F_{23}^{x} r_{B3/G3}^{y} + F_{23}^{y} r_{B3/G3}^{x} - F_{43}^{x} r_{C3/G3}^{y} + F_{43}^{y} r_{C3/G3}^{x} = J_{G3}' \alpha_3$

ΣF on 4 right + $F_{34}^{x} + F_{14}^{x} = m_4 A_{G4}^{x}$

ΣF on 4 up + $F_{34}^{y} + F_{14}^{y} + W_4 = m_4 A_{G4}^{y}$

ΣM on 4 about G4 CCW + $- F_{34}^{x} r_{C4/G4}^{y} + F_{34}^{y} r_{C4/G4}^{x} - F_{14}^{x} r_{D4/G4}^{y} + F_{14}^{y} r_{D4/G4}^{x} = J_{G4}' \alpha_4$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ +r_{A2/G2}^{y} & -r_{A2/G2}^{x} & r_{B2/G2}^{y} & -r_{B2/G2}^{x} & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -r_{B3/G3}^{y} & r_{B3/G3}^{x} & r_{C3/G3}^{y} & -r_{C3/G3}^{x} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -r_{C4/G4}^{y} & r_{C4/G4}^{x} & -r_{D4/G4}^{y} & r_{D4/G4}^{x} & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{y} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} m_2 A_{G2}^{x} \\ m_2 A_{G2}^{y} - W_2 \\ J_{G2}' \alpha_2 \\ m_3 A_{G3}^{x} \\ m_3 A_{G3}^{y} - W_3 \\ J_{G3}' \alpha_3 \\ m_4 A_{G4}^{x} \\ m_4 A_{G4}^{y} - W_4 \\ J_{G4}' \alpha_4 \end{Bmatrix}$$

$W_2 = -j\ 2.433$ N $\quad m_2 A_{G2} = -3.507 - j\ 2.104$ N $\quad J_{G2}' \alpha_2 = 1.405$ N.cm

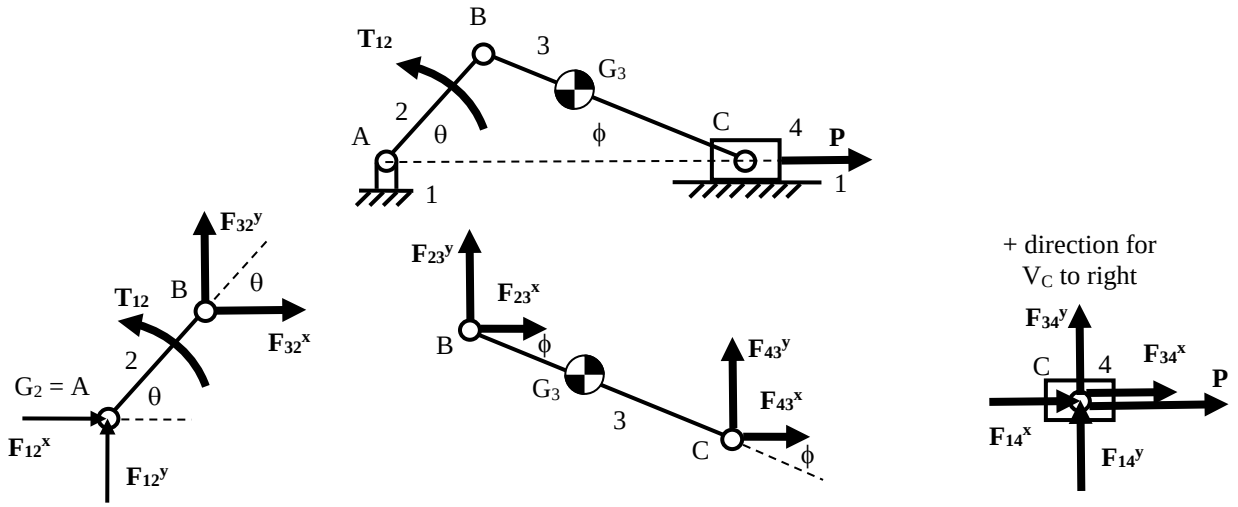
$W_3 = -j\ 3.649$ N $\quad m_3 A_{G3} = -13.662 - j\ 8.411$ N $\quad J_{G3}' \alpha_3 = -1.817$ N.cm

$W_4 = -j\ 3.345$ N $\quad m_4 A_{G4} = -7.715 - j\ 4.816$ N $\quad J_{G4}' \alpha_4 = 17.138$ N.cm

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 2.828 & -2.828 & 2.828 & -2.828 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 2.052 & -5.638 & 2.052 & -5.638 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -4.883 & -2.531 & 4.883 & 2.531 & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} -3.507 \\ 0.329 \\ 1.405 \\ -13.662 \\ -4.762 \\ -1.817 \\ -7.715 \\ -1.471 \\ 17.138 \end{Bmatrix}$$

$$\begin{Bmatrix} F_{12}^{x} \\ F_{12}^{y} \\ F_{23}^{x} \\ F_{23}^{y} \\ F_{34}^{x} \\ F_{34}^{x} \\ F_{14}^{x} \\ F_{14}^{y} \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} -22.236\ \text{N} \\ -6.221\ \text{N} \\ -18.729\ \text{N} \\ -6.550\ \text{N} \\ -5.067\ \text{N} \\ -1.788\ \text{N} \\ -2.648\ \text{N} \\ +0.317\ \text{N} \\ +81.135\ \text{N.cm} \end{Bmatrix}$$

Matrix Dynamic Analysis for Slider Crank



ΣF on 2 right + $F_{12}^x + F_{32}^x = m_2 A_{G2}^x = 0$

ΣF on 2 up + $F_{12}^y + F_{32}^y = m_2 A_{G2}^y = 0$

ΣM on 2 about A CCW + $-(F_{32}^x \sin\theta)\, AB + (F_{32}^y \cos\theta)\, AB + T_{12} = J_{G2}\, \alpha_2$

ΣF on 3 right + $F_{23}^x + F_{43}^x = m_3 A_{G3}^x$

ΣF on 3 up + $F_{23}^y + F_{43}^y = m_3 A_{G3}^y$

ΣM on 3 about G3 CCW + $-(F_{23}^x \sin\phi)\, BG_3 - (F_{23}^y \cos\phi)\, BG_3 + (F_{43}^x \sin\phi)\, CG_3 + (F_{43}^y \cos\phi)\, CG_3 = J_{G3}\, \alpha_3$

ΣF on 4 right + $F_{14}^x + F_{34}^x + P = m_4 A_{G4}^x$

ΣF on 4 up + $F_{14}^y + F_{34}^y = m_4 A_{G4}^y = 0$

friction $F_{14}^x = -\mu\, \mathrm{abs}(F_{14}^y)\, \mathrm{sign}(V_C)$

$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & +AB\sin\theta & -AB\cos\theta & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & -BG_3\sin\phi & -BG_3\cos\phi & +CG_3\sin\phi & +CG_3\cos\phi & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & +\mu\,\mathrm{sign}(V_C) & 0 \end{bmatrix} \begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ J_{G2}\alpha_2 \\ m_3 A_{G3}^x \\ m_3 A_{G3}^y \\ J_{G3}\alpha_3 \\ m_4 A_{G4}^x - P \\ 0 \\ 0 \end{Bmatrix}$$