**d’Alembert’s Principle**

An accelerating rigid body can be transformed into an equivalent quasi-static system by adding a fictitious imaginary "inertial force" and "inertial moment". The inertial force and inertial moment may be treated as an external force and external moment. This is particularly attractive for inverse dynamic problems.

Newton 

d’Alembert 

Unfortunately this concept promotes the incorrect causality that acceleration causes force.

Acceleration does not cause force. Force causes acceleration.

Inertial forces and inertial moments DO NOT EXIST. They are only fictitious algebraic constructs.

Pin B at the end of crank link 2 forms a pin-in-slot joint with the horizontal slot in hammer link 4 as shown below. The mechanism is drawn approximately to scale. The weight of crank link 2 is very small compared to the weight of hammer link 4. You may neglect the effects of friction at A, B and C. The hammer face is not yet in contact with its platen. Show your work.

m4 = 2.3 kg

JG4 = 30 N.cm.sec2

2 = 50 rpm CCW constant

4 = 2.32 rad/sec CW

4 = 20.13 rad/sec2 CW

45˚

A

B

C

G4

20 cm

30 cm

40 cm

AB = 25 cm

**2**

**4**

face

Determine the magnitude and direction of motor torque T1\_on\_2 on crank 2 required to cause this motion at this position.

451 N.cm CCW

T1\_on\_2 = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_

using virtual work and d'Alembert's Principle



assume T1\_on\_2 CCW

2 = 50 rev /min = 5.236 rad/sec CCW

AG4T = (CG4) 4 = 603.9 cps2 right

FI\_on\_4X = - m4 AG4T = (2.3 kg) (603.9 cm/sec2) (1 m / 100 cm) = 13.89 N left

VG4X = (CG4) 4 = 69.6 cps right

AG4N = (CG4) 42 = 161.5 cps2 down

FI\_on\_4Y = - m4 AG4N = 3.71 N up

VG4Y = 0

m4g = 22.56 N down

MI\_on\_4 = - JG4 4 = (30 N.cm.sec2) (20.13 rad/sec2) = 603.9 N.cm CCW

4 = 2.32 rad/sec CW



+ T1\_on\_2 (5.236 rad/sec) - (13.89 N)(69.6 cm/sec) - (603.9 N.cm) (2.32 rad/sec) = 0

T1\_on\_2 = +452.2 N.cm matches Notes\_08\_01

**Dynamic Force Analysis for Four Bar**

The four bar linkage shown below operates in a vertical plane. Each link is a uniform bar with 2 cm by 2 cm square cross-section stainless steel. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity.

 = 45 deg  = 20 rad/s CW  = 100 rad/s/s CCW

= 20 deg  = 12.82 rad/s CCW  = 39.6 rad/s/s CW

 = 117.4 deg  = 6.20 rad/s CW  = 482.5 rad/s/s CCW

m2 = 0.248 kg JG2’ = 1.405 kg.cm2  = 7.75 g/cm3

m3 = 0.372 kg JG3’ = 4.588 kg.cm2

m4 = 0.341 kg JG4’ = 3.552 kg.cm2

 = 56.57 - j 56.57 cps  = -1414.2 - j 848.5 cps2

 = 86.83 - j 40.86 cps  = -3672.5 - j 2260.9 cps2

 = 30.27 - j 15.71 cps  = -2262.4 - j 1412.4 cps2







A

D

X

C

B

Y

T

AD = 22 cm

AB = 8 cm

BC = 12 cm

CD = 11 cm

D

C

F14y

F14x

F34y

F34x

G4

W4

RD4/G4

RC4/G4

A

B

F32y

F32x

F12y

F12x

G2

W2

RA2/G2

RB2/G2

T12

C

B

F43y

F43x

F23y

F23x

G3

W3

RB3/G3

RC3/G3

Weights are true external forces caused by gravity. d'Alembert inertial forces and moments may be treated as if they were external forces/moments acting on the links.

 = -j 2.433 N  = +3.507 + j 2.104 N MI2 = - JG2’ 2 = -1.405 N.cm

 = -j 3.649 N  = +13.662 + j 8.411 N MI3 = - JG3’ 3 = +1.817 N.cm

 = -j 3.345 N = +7.715 + j 4.816 N MI4 = - JG4’ 4 = 17.138 N.cm

Actual power at A and D is zero 

Actual power at B is zero 

Actual power at C is zero 

Must include actual power at G 

Must include d'Alembert power at G 

Must include actual power into crank 



 = +79.368 N.cm/s

 = +842.598 N.cm/s

 = +309.192 N.cm/s

 = +28.100 N.cm/s

 = +23.292 N.cm/s

 = +106.258 N.cm/s

 = +137.635 N.cm/s

 = +149.098 N.cm/s

 = -52.550 N.cm/s

 = -20 T12 rad/s

1622.89 N.cm/s - 20 T12 rad/s = 0 T12 = +81.14 N.cm matches Notes\_08\_02