d'Alembert's Principle

An accelerating rigid body can be transformed into an equivalent quasi-static system by adding a fictitious imaginary "inertial force" and "inertial moment". The inertial force and inertial moment may be treated as an external force and external moment. This is particularly attractive for inverse dynamic problems.

Newton $\sum\{F\} = m\{\ddot{r}_G\}$ $\sum M = J_G \dot{\omega}$

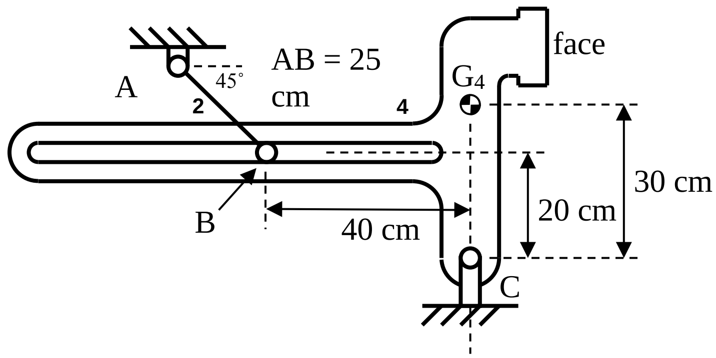
d'Alembert $\{F_I\} = -m\{\ddot{r}_G\}$ $\sum\{F\} + \{F_I\} = 0$ $M_I = -J_G \dot{\omega}$ $\sum M + M_i = 0$

Unfortunately this concept promotes the incorrect causality that acceleration causes force.

Acceleration does not cause force. Force causes acceleration.

Inertial forces and inertial moments DO NOT EXIST. They are only fictitious algebraic constructs.

Pin B at the end of crank link 2 forms a pin-in-slot joint with the horizontal slot in hammer link 4 as shown below. The mechanism is drawn approximately to scale. The weight of crank link 2 is very small compared to the weight of hammer link 4. You may neglect the effects of friction at A, B and C. The hammer face is not yet in contact with its platen. Show your work.



$m_4 = 2.3$ kg
$J_{G4} = 30$ N.cm.sec^2
$\omega_2 = 50$ rpm CCW constant
$\omega_4 = 2.32$ rad/sec CW
$\alpha_4 = 20.13$ rad/sec^2 CW

Determine the magnitude and direction of motor torque $T_{1\_on\_2}$ on crank 2 required to cause this motion at this position.

$T_{1\_on\_2}$ = 451 N.cm CCW

using virtual work and d'Alembert's Principle

$$T_{1\_on\_2} \circ \omega_2 + \overline{F}_{I\_on\_4} \circ \overline{V}_{G4} + M_{I\_on\_4} \circ \omega_4 = 0$$

assume $T_{1\_on\_2}$ CCW

$\omega_2 = 50$ rev /min = 5.236 rad/sec CCW

$A_{G4}^T = (CG_4)\,\alpha_4 = 603.9$ cps^2 right

$F_{I\_on\_4}^X = -\,m_4 A_{G4}^T = (2.3$ kg$)(603.9$ cm/sec$^2)(1$ m / 100 cm$) = 13.89$ N left

$V_{G4}^X = (CG_4)\,\omega_4 = 69.6$ cps right

$A_{G4}^N = (CG_4)\,\omega_4^2 = 161.5$ cps^2 down

$F_{I\_on\_4}^Y = -\,m_4 A_{G4}^N = 3.71$ N up

$V_{G4}^Y = 0$

$m_4 g = 22.56$ N down

$M_{I\_on\_4} = -\,J_{G4}\,\alpha_4 = (30$ N.cm.sec$^2)(20.13$ rad/sec$^2) = 603.9$ N.cm CCW

ω_4 = 2.32 rad/sec CW

$$T_{1\_on\_2} \circ \omega_2 + \overline{F}_{I\_on\_4} \circ \overline{V}_{G4} + M_{I\_on\_4} \circ \omega_4 = 0$$

+ $T_{1\_on\_2}$ (5.236 rad/sec) - (13.89 N)(69.6 cm/sec) - (603.9 N.cm) (2.32 rad/sec) = 0

$T_{1\_on\_2}$ = +452.2 N.cm matches Notes_08_01

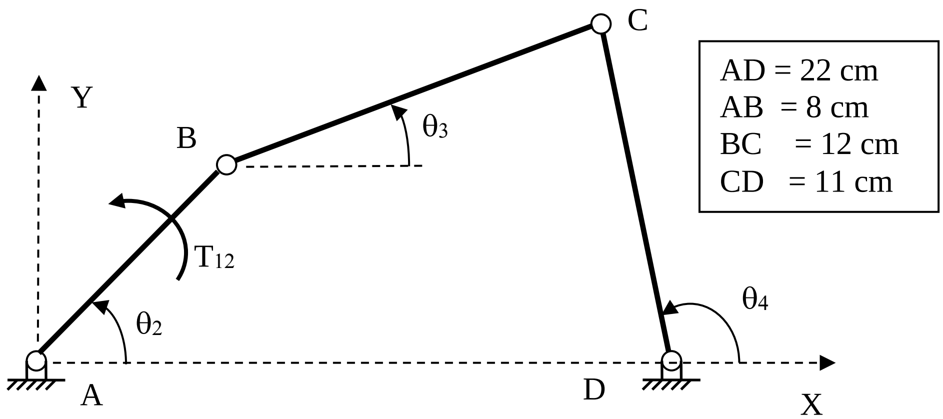
Dynamic Force Analysis for Four Bar

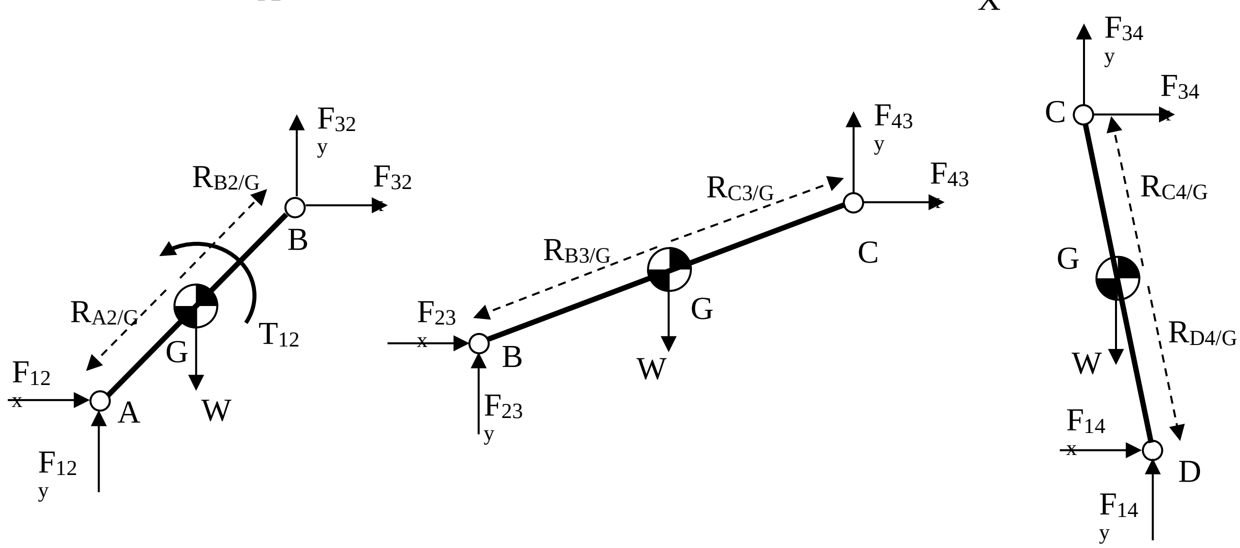
The four bar linkage shown below operates in a vertical plane. Each link is a uniform bar with 2 cm by 2 cm square cross-section stainless steel. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity.

θ_2 = 45 deg	ω_2 = 20 rad/s CW	$\dot{\omega}_2$ = 100 rad/s/s CCW
θ_3 = 20 deg	ω_3 = 12.82 rad/s CCW	$\dot{\omega}_3$ = 39.6 rad/s/s CW
θ_4 = 117.4 deg	ω_4 = 6.20 rad/s CW	$\dot{\omega}_4$ = 482.5 rad/s/s CCW

m_2 = 0.248 kg	J_{G2}' = 1.405 kg.cm^2	ρ = 7.75 g/cm^3
m_3 = 0.372 kg	J_{G3}' = 4.588 kg.cm^2	
m_4 = 0.341 kg	J_{G4}' = 3.552 kg.cm^2	

$\{\dot{r}_2\}^G$ = 56.57 - j 56.57 cps	$\{\ddot{r}_2\}^G$ = -1414.2 - j 848.5 cps^2
$\{\dot{r}_3\}^G$ = 86.83 - j 40.86 cps	$\{\ddot{r}_3\}^G$ = -3672.5 - j 2260.9 cps^2
$\{\dot{r}_4\}^G$ = 30.27 - j 15.71 cps	$\{\ddot{r}_4\}^G$ = -2262.4 - j 1412.4 cps^2





Weights are true external forces caused by gravity. d'Alembert inertial forces and moments may be treated as if they were external forces/moments acting on the links.

$\{W_2\}$ = -j 2.433 N $\{F_{I2}\} = -m_2\{\ddot{r}_2\}^G$ = +3.507 + j 2.104 N $M_{I2} = - J_{G2}'\ \alpha_2 = -1.405$ N.cm
$\{W_3\}$ = -j 3.649 N $\{F_{I3}\} = -m_3\{\ddot{r}_3\}^G$ = +13.662 + j 8.411 N $M_{I3} = - J_{G3}'\ \alpha_3 = +1.817$ N.cm
$\{W_4\}$ = -j 3.345 N $\{F_{I4}\} = -m_4\{\ddot{r}_4\}^G$ = +7.715 + j 4.816 N $M_{I4} = - J_{G4}'\ \alpha_4 = 17.138$ N.cm

Actual power at A and D is zero $\{F_{12}\}\circ\{\dot{r}_2\}^A = 0$ $\{F_{14}\}\circ\{\dot{r}_4\}^D = 0$

Actual power at B is zero $\{F_{32}\}\circ\{\dot{r}_2\}^B + \{F_{23}\}\circ\{\dot{r}_3\}^B = 0$

Actual power at C is zero $\{F_{43}\}\circ\{\dot{r}_3\}^C + \{F_{34}\}\circ\{\dot{r}_4\}^C = 0$

Must include actual power at G $\{W_i\}\circ\{\dot{r}_i\}^G$

Must include d'Alembert power at G $\{F_{Ii}\}\circ\{\dot{r}_i\}^G + M_{Ii}\omega_i$

Must include actual power into crank $T_{12}\circ\omega_2$

$$+\{F_{I2}\}\circ\{\dot{r}_2\}^G + \{F_{I3}\}\circ\{\dot{r}_3\}^G + \{F_{I4}\}\circ\{\dot{r}_4\}^G + M_{I2}\omega_2 + M_{I3}\omega_3 + M_{I4}\omega_4$$
$$+\{W_2\}\circ\{\dot{r}_2\}^G + \{W_3\}\circ\{\dot{r}_3\}^G + \{W_4\}\circ\{\dot{r}_4\}^G + T_{12}\omega_2 = 0$$

$\{F_{I2}\}\circ\{\dot{r}_2\}^G$ = +79.368 N.cm/s
$\{F_{I3}\}\circ\{\dot{r}_3\}^G$ = +842.598 N.cm/s
$\{F_{I4}\}\circ\{\dot{r}_4\}^G$ = +309.192 N.cm/s
$M_{I2}\omega_2$ = +28.100 N.cm/s
$M_{I3}\omega_3$ = +23.292 N.cm/s
$M_{I4}\omega_4$ = +106.258 N.cm/s
$\{W_2\}\circ\{\dot{r}_2\}^G$ = +137.635 N.cm/s
$\{W_3\}\circ\{\dot{r}_3\}^G$ = +149.098 N.cm/s
$\{W_4\}\circ\{\dot{r}_4\}^G$ = -52.550 N.cm/s
$T_{12}\omega_2$ = -20 T_{12} rad/s

1622.89 N.cm/s - 20 T_{12} rad/s = 0 T_{12} = +81.14 N.cm matches Notes_08_02