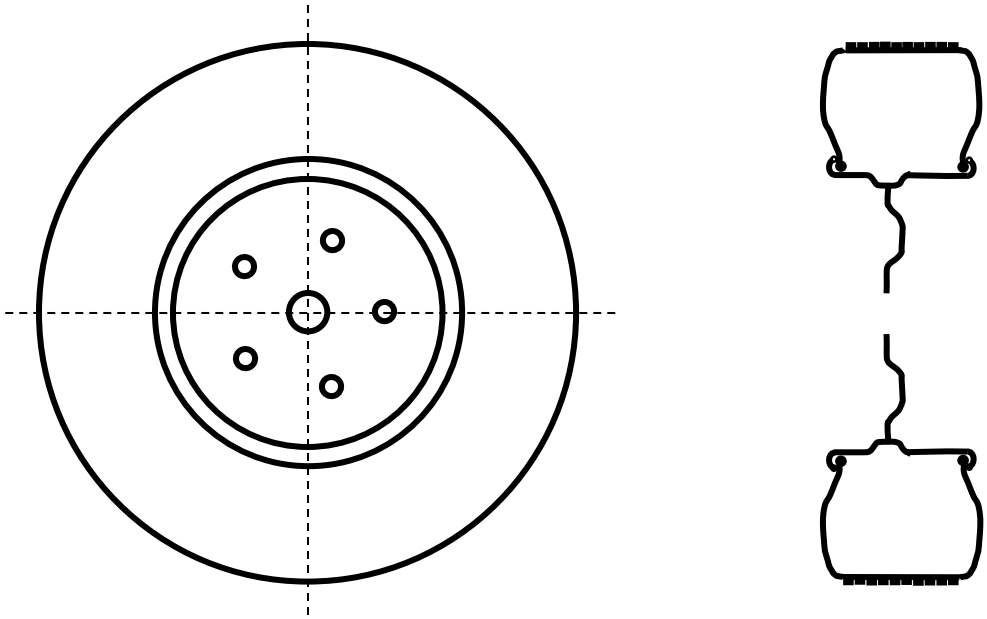
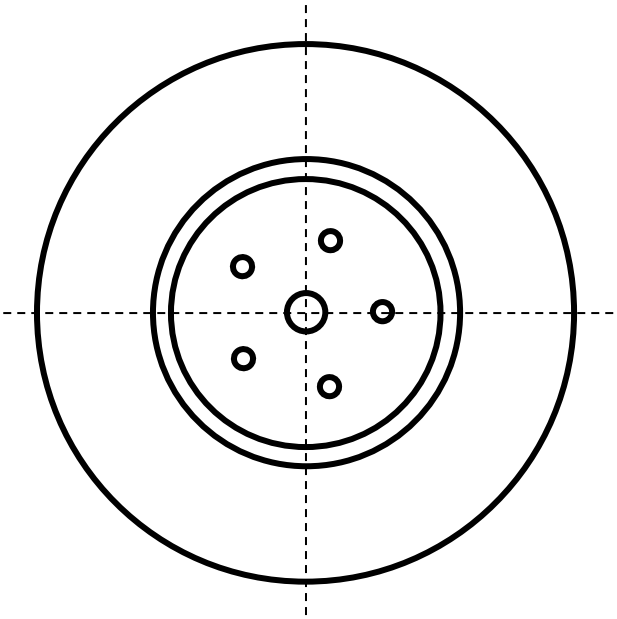
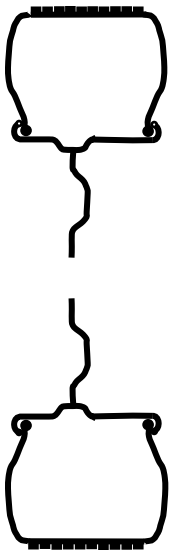
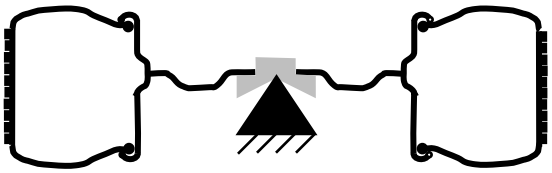
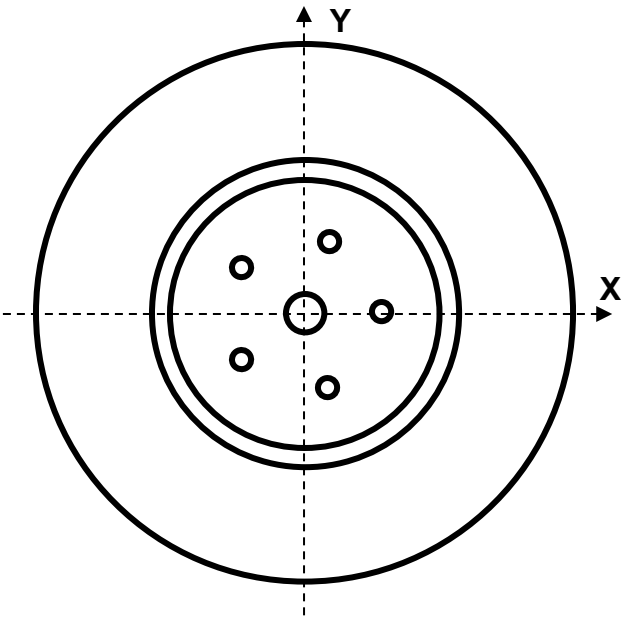
Multiplanar Balancing

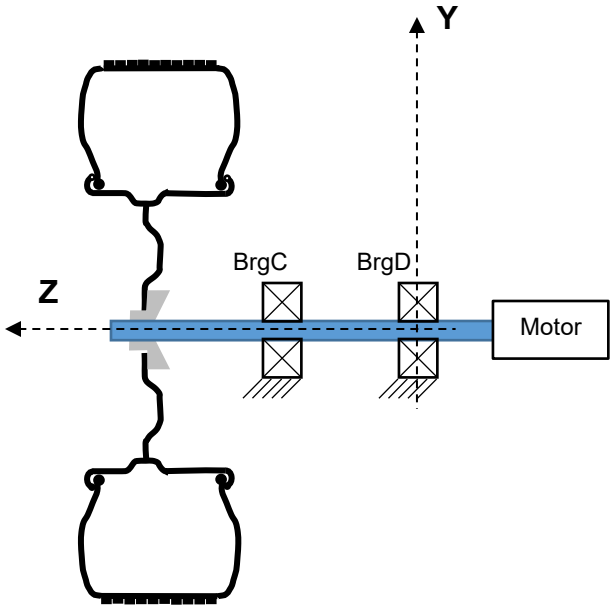


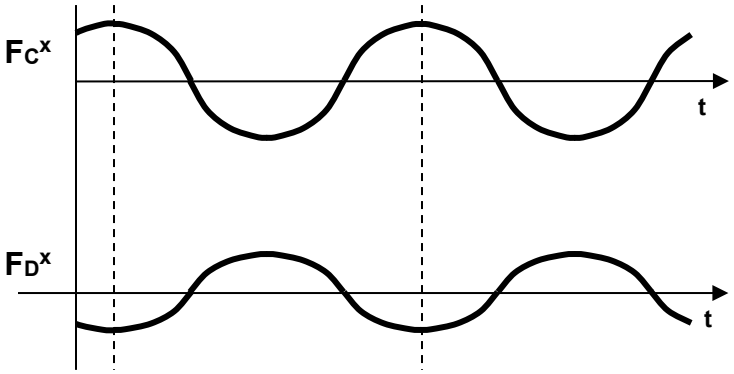






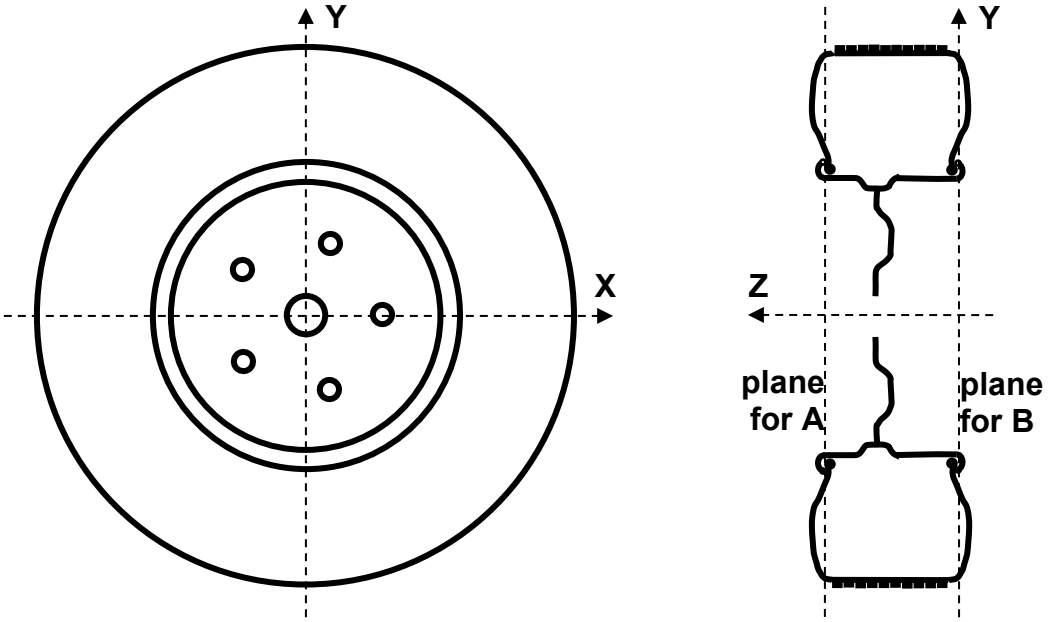






$$F_C{}^X + F_D{}^X = m_i\, r_i\, \omega^2 \qquad m_i\, r_i = \frac{F_C{}^X + F_D{}^X}{\omega^2}$$

$$F_C{}^X\, z_C = z_i\, m_i\, r_i\, \omega^2 \qquad z_i = z_C \frac{F_C{}^X}{F_C{}^X + F_D{}^X}$$



in-plane primary balancing (static balancing)

$$F_{IMB\_i}{}^{X} = m_i\ r_i\ \omega^2\ \cos\theta_i \qquad F_{IMB\_i}{}^{Y} = m_i\ r_i\ \omega^2\ \sin\theta_i$$

$$F_{BAL\_A}{}^{X} = m_A\ r_A\ \omega^2\ \cos\theta_A \qquad F_{BAL\_A}{}^{Y} = m_A\ r_A\ \omega^2\ \sin\theta_A$$

$$F_{BAL\_B}{}^{X} = m_B\ r_B\ \omega^2\ \cos\theta_B \qquad F_{IMB\_B}{}^{Y} = m_B\ r_{Bi}\ \omega^2\ \sin\theta_B$$

forces in X $\quad \sum\left(m_i\ r_i\ \cos\theta_i\right) + m_A\ r_A\ \cos\theta_A + m_B\ r_B\ \cos\theta_B = 0$ Eq. 1

forces in Y $\quad \sum\left(m_i\ r_i\ \sin\theta_i\right) + m_A\ r_A\ \sin\theta_A + m_B\ r_B\ \sin\theta_B = 0$ Eq. 2

secondary dynamic balancing to prevent shaking moments

$$M_{IMB\_i}{}^{Y} = +F_{IMB\_i}{}^{X}\left(z_i\right) = m_i\ r_i\ z_i\ \omega^2\ \cos\theta_i$$

$$M_{IMB\_i}{}^{X} = -F_{IMB\_i}{}^{Y}\left(z_i\right) = -m_i\ r_i\ z_i\omega^2\ \sin\theta_i$$

moments in Y $\quad \sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right) + m_A\ r_A\ z_A\ \cos\theta_A = 0 \qquad z_B = 0$ Eq. 3

moments in X $\quad \sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right) + m_A\ r_A\ z_A\ \sin\theta_A = 0 \qquad z_B = 0$ Eq. 4

$$m_A\ r_A\ z_A\ \sin\theta_A = -\sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right)$$
$$m_A\ r_A\ z_A\ \cos\theta_A = -\sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right)$$

$$\tan\theta_A = \frac{\sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right)}{\sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right)}$$

for only one imbalance $\tan\theta_A = \tan\theta_i$

$$\left(m_A\ r_A\ z_A\ \sin\theta_A\right)^2 + \left(m_A\ r_A\ z_A\ \cos\theta_A\right)^2 = \left(\sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right)\right)^2 + \left(\sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right)\right)^2$$

$$\left(m_A\ r_A\ z_A\right)^2 = \left(\sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right)\right)^2 + \left(\sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right)\right)^2$$

$$\left(m_A\ r_A\right) = \frac{\sqrt{\left(\sum\left(m_i\ r_i\ z_i\ \sin\theta_i\right)\right)^2 + \left(\sum\left(m_i\ r_i\ z_i\ \cos\theta_i\right)\right)^2}}{z_A}$$

for only one imbalance $\left(m_A\ r_A\right) = \left(m_i\ r_i\right)\frac{z_i}{z_A}$

$$m_B\ r_B\ \sin\theta_B = -\sum\left(m_i\ r_i\ \sin\theta_i\right) - m_A\ r_A\ \sin\theta_A$$
$$m_B\ r_B\ \cos\theta_B = -\sum\left(m_i\ r_i\ \cos\theta_i\right) - m_A\ r_A\ \cos\theta_A$$

$$\tan\theta_B = \frac{\sum\left(m_i\ r_i\ \sin\theta_i\right) + m_A\ r_A\ \sin\theta_A}{\sum\left(m_i\ r_i\ \cos\theta_i\right) + m_A\ r_A\ \cos\theta_A}$$

$$\left(m_B\ r_B\right) = \sqrt{\left(\sum\left(m_i\ r_i\ \sin\theta_i\right) + m_A\ r_A\ \sin\theta_A\right)^2 + \left(\sum\left(m_i\ r_i\ \cos\theta_i\right) + m_A\ r_A\ \cos\theta_A\right)^2}$$