**Coulomb Friction in Prismatic Joint**

4

C

 ≠ 0





4

C





x









**pseudo-code**





else







end

end

**ODE**



**Coulomb Friction in Revolute Joint**

**2**

**3**

**2**

**3**

**simple friction model**



**Free Vibration with Viscous Damping**

spring-mass-damper with viscous damping has exponential envelope





use x0 at a positive peak and xn at a positive peak that occurs n cycles later

log decrement  for exponential envelope

damping ratio 

**Free Vibration with Coulomb Friction**

spring-mass with Coulomb friction has linear envelope



use x0 at a positive peak and xn at a positive peak that occurs n cycles later

slope  for linear envelope

friction force 

rotary motion  

simple pendulum (small angle approx)  

**Pacejka Magic Formula**

Pacejka, H.B., 2006, Tire and Vehicle Dynamics, 2nd edition, SAE International

works well for tire friction

does not work well for stick-slip

yA

yP

xM

slope

x

y

**values from curve**

yP = peak value for y

xM = x value at peak

yA = asymptotic value for y

**coefficients**

B = stiffness factor

C = shape factor

D = peak value

E = curvature factor



D = yP











% try\_pacejka.m - test Pacejka magic formula

% HJSIII, 11.04.25

clear

% constants

% define curve

xm = 0.01;

yp = 0.9;

ya = 0.6;

% Pacejka coefficients

D = yp;

C = 1 + (1 - 2 \* asin( ya/D ) / pi);

slope = yp / (xm/2);

B = slope /C /D;

E = (B\*xm - tan(pi/2/C)) / (B\*xm - atan(B\*xm));

% plot

x = ( -1 : 0.001 : 1 )';

y = D\*sin( C\*atan( B\*x - E\*(B\*x-atan(B\*x)) ) );

figure( 1 )

clf

plot( x,y )

% bottom - try\_pacejka

**Stick-Slip using Pseudo-code**

****

**Stick-Slip using Pacejka Magic Formula with xm = 0.01**

****

|  |  |  |  |
| --- | --- | --- | --- |
| friction model | CPU time [sec] | time stpes | ave time step [msec] |
| pseudo-code | 1.7690 | 10972 | 1.5145 |
| Pacejka xm=0.01 mps | 0.7313 | 6667 | 2.4004 |
| Pacejka xm=0.001 mps | 7.3105 | 68107 | 0.2349 |

+ faster execution

- velocity is not exactly zero during stick phase

- asymmetric acceleration

see ME 481 H14

function yd = ode\_stickslip\_pacejka\_yd( t, y )

% provides yd for integration

% stick slip drag sled

% m\*xdd = Fext + Fspr + Ff Pacejka friction model from Notes\_08\_06

% HJSIII, 20.04.26

global m k mu\_s mu\_d g v\_driver eps

% free motion

Fext = 0;

% individual terms

x = y(1);

xd = y(2);

% spring force

x\_driver = v\_driver \* t;

Fspr = k \* ( x\_driver - x );

% Pacejka constants

xm\_pacj = 0.01;

yp\_pacj = mu\_s;

ya\_pacj = mu\_d;

% Pacejka coefficients

D = yp\_pacj;

C = 1 + (1 - 2 \* asin( ya\_pacj/D ) / pi);

slope = yp\_pacj / (xm\_pacj/2);

B = slope /C /D;

E = (B\*xm\_pacj - tan(pi/2/C)) / (B\*xm\_pacj - atan(B\*xm\_pacj));

% Pacejka variables

x\_pacj = xd;

mu = D\*sin( C\*atan( B\*x\_pacj - E\*(B\*x\_pacj-atan(B\*x\_pacj)) ) );

% Pacejka friction model

Ff = -mu \* m \* g;

% ordinary differential equation (ODE)

xdd = ( Fext + Fspr + Ff ) / m;

% return values

yd(1,1) = xd;

yd(2,1) = xdd;

return

% bottom - ode\_stickslip\_pacejka\_yd