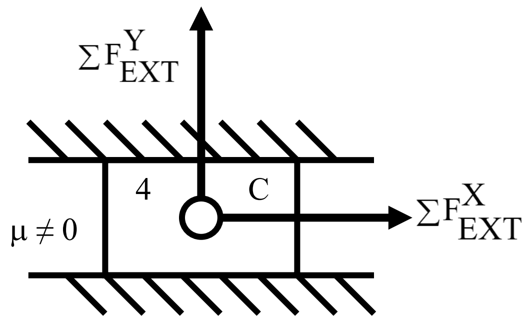
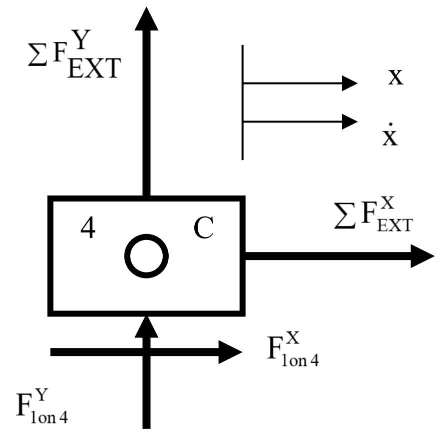
Coulomb Friction in Prismatic Joint





$\sum F \text{ on } 4 \uparrow +$ $\qquad \sum F^{Y}_{EXT} + F^{Y}_{1on4} = 0 \qquad F^{Y}_{1on4} = -\sum F^{Y}_{EXT}$

pseudo-code

if $|\dot{x}| > \varepsilon$

$\quad F^{X}_{1on4} = -\mu_D \left|F^{Y}_{1on4}\right| \operatorname{sign}(\dot{x})$

else

$\quad F^{X}_{1on4} = -\sum F^{X}_{EXT}$

$\quad$ if $\left|F^{X}_{1on4}\right| > \mu_S \left|F^{Y}_{1on4}\right|$

$\quad\quad F^{X}_{1on4} = -\mu_S \left|F^{Y}_{1on4}\right| \operatorname{sign}\left(\sum F^{X}_{EXT}\right)$

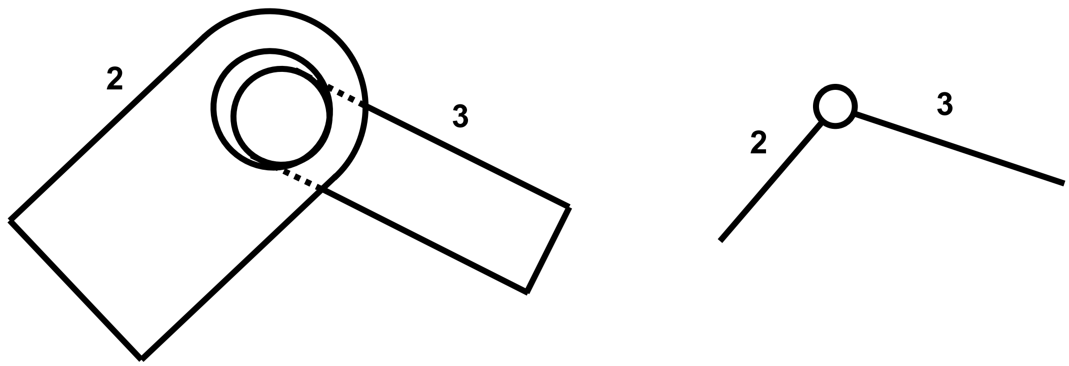
$\quad$ end

end

ODE

$$m_4 \ddot{x} = +\sum F^{X}_{EXT} + F^{X}_{1on4}$$

Coulomb Friction in Revolute Joint



simple friction model

$$T_{2on3} = -\mu_D \; r_{BRG} \; |F_{2on3}| \; \mathrm{sign}(\dot{\theta}_3 - \dot{\theta}_2) \qquad \text{for } \dot{\theta}_3 - \dot{\theta}_2 \neq 0$$

Free Vibration with Viscous Damping

spring-mass-damper with viscous damping has exponential envelope

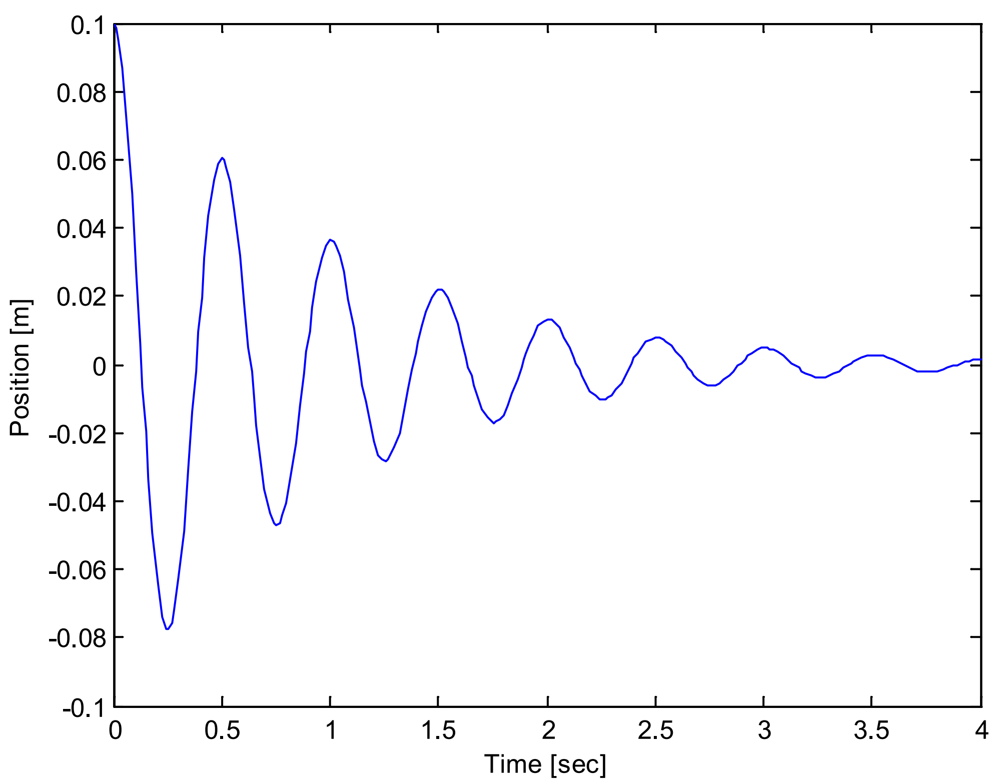
$$m\ddot{x} + c\dot{x} + kx = 0$$

$$\omega_n = \sqrt{\frac{k}{m}} = 2\pi f_n \qquad \zeta = \frac{c}{c_{CR}} = \frac{c}{2\,m\,\omega_n} \qquad \omega_d = \omega_n\sqrt{1-\zeta^2} = 2\pi f_d \qquad \tau = 1/f_d$$

use x_0 at a positive peak and x_n at a positive peak that occurs n cycles later

log decrement $\quad \delta = \frac{1}{n}\ln\left(\frac{x_0}{x_n}\right) = \frac{2\pi\zeta}{\sqrt{1-\zeta^2}} \quad$ for exponential envelope

damping ratio $\quad \zeta = \frac{\delta}{\sqrt{4\pi^2 + \delta^2}}$



Free Vibration with Coulomb Friction

spring-mass with Coulomb friction has linear envelope

$$m\ddot{x} + kx = -f\ \mathrm{sign}(\dot{x})$$

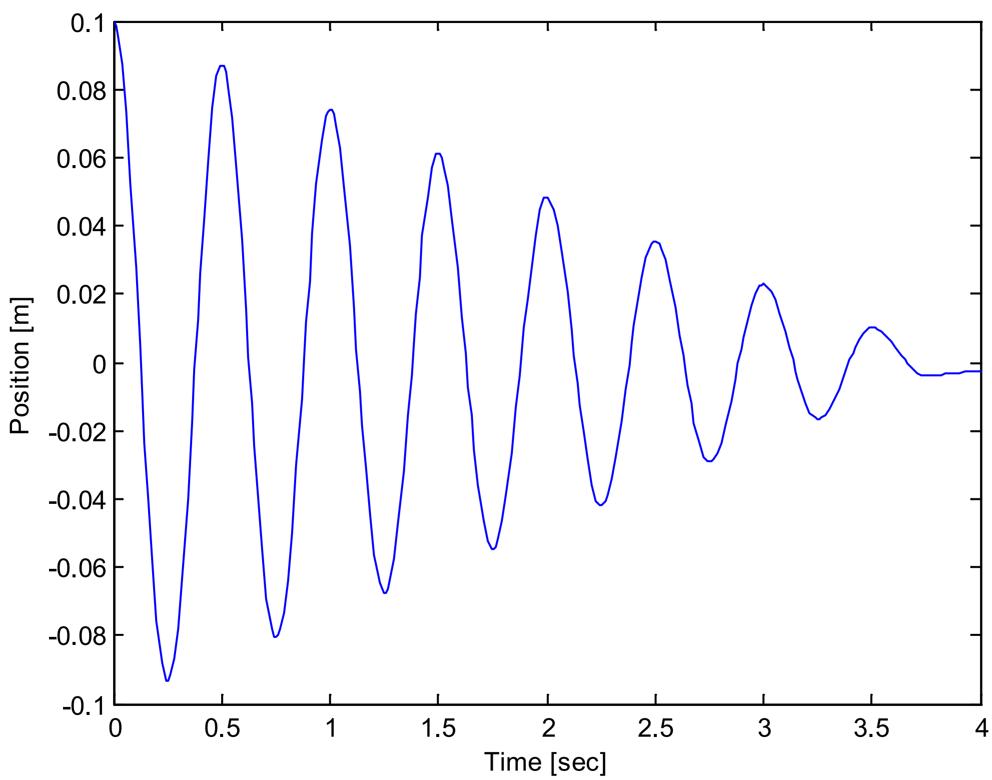
use x_0 at a positive peak and x_n at a positive peak that occurs n cycles later

slope $\quad s = \dfrac{x_0 - x_n}{nT} = \dfrac{2\ f\ \omega_n}{\pi\ k}$ $\quad$ for linear envelope

friction force $\quad f = \dfrac{\pi\ k\ s}{2\ \omega_n} = \dfrac{k(x_0 - x_n)}{4\ n}$

rotary motion $\quad s = \dfrac{\theta_0 - \theta_n}{n\ \tau} \qquad T_f = \dfrac{\pi\ k_\theta\ s}{2\ \omega_n} = \dfrac{k_\theta(\theta_0 - \theta_n)}{4\ n}$

simple pendulum (small angle approx) $\quad s = \dfrac{\theta_0 - \theta_n}{n\ \tau} \qquad T_f = \dfrac{\pi\ m\ g\ a\ s}{2\ \omega_n} = \dfrac{m\ g\ a\ (\theta_0 - \theta_n)}{4\ n}$

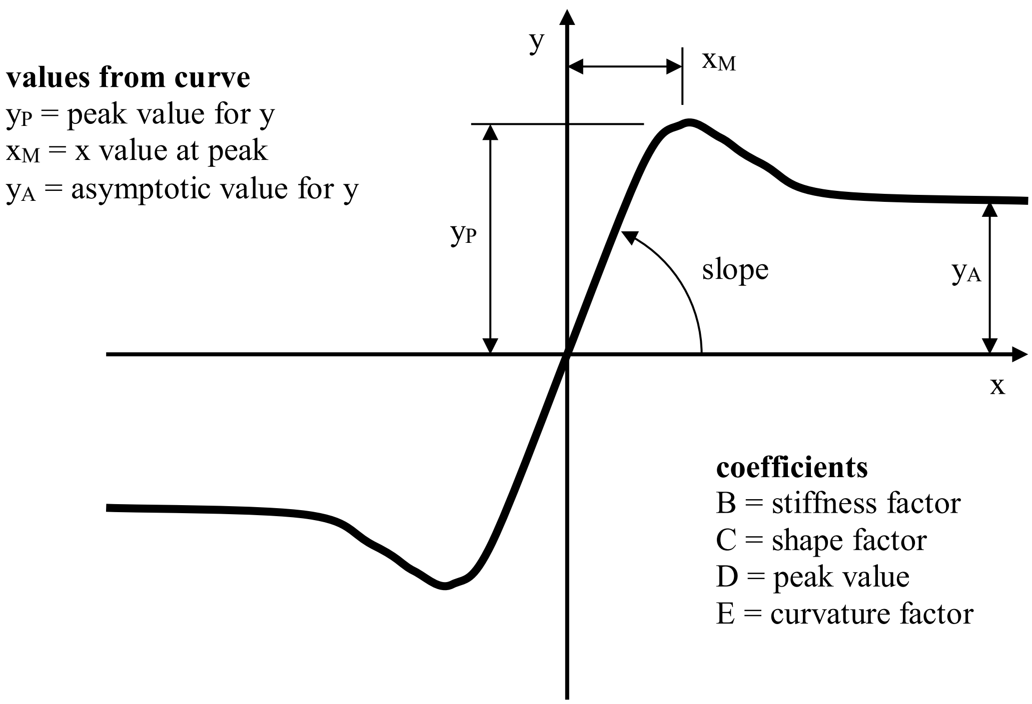


Pacejka Magic Formula

Pacejka, H.B., 2006, Tire and Vehicle Dynamics, 2nd edition, SAE International

works well for tire friction

does not work well for stick-slip



$$y = D \sin\left(C \operatorname{a\,tan}\left(Bx - E\left(Bx - \operatorname{a\,tan} Bx\right)\right)\right)$$

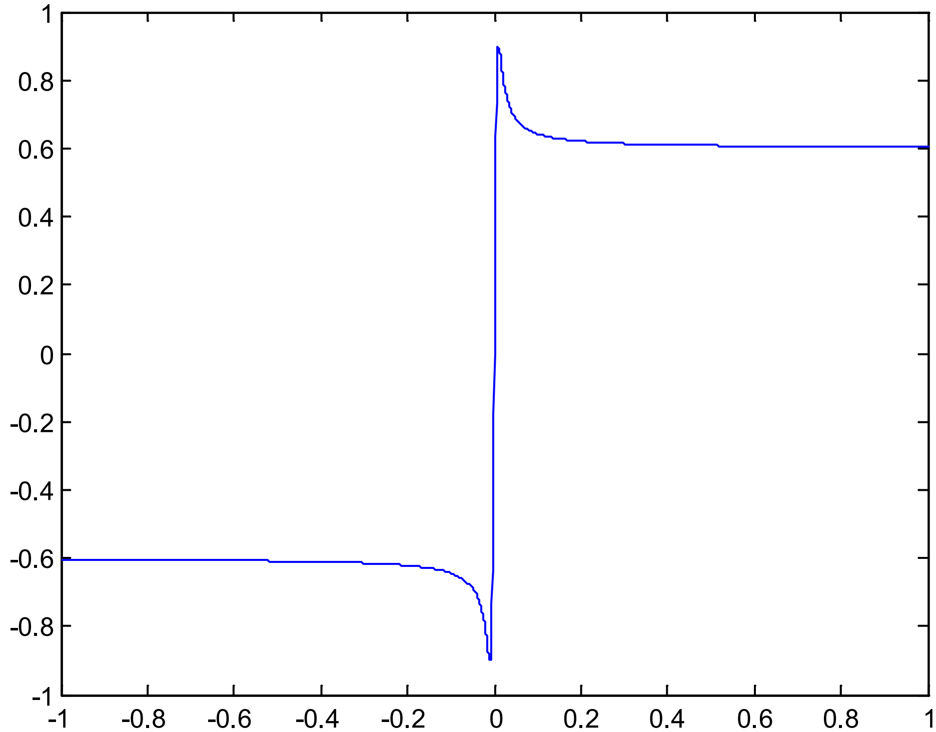
$$D = y_P$$

$$\text{slope} = \frac{D}{x_M / 2} = B\,C\,D$$

$$C = 1 \pm \left(1 - \frac{2}{\pi} \operatorname{a\,sin} \frac{y_A}{D}\right)$$

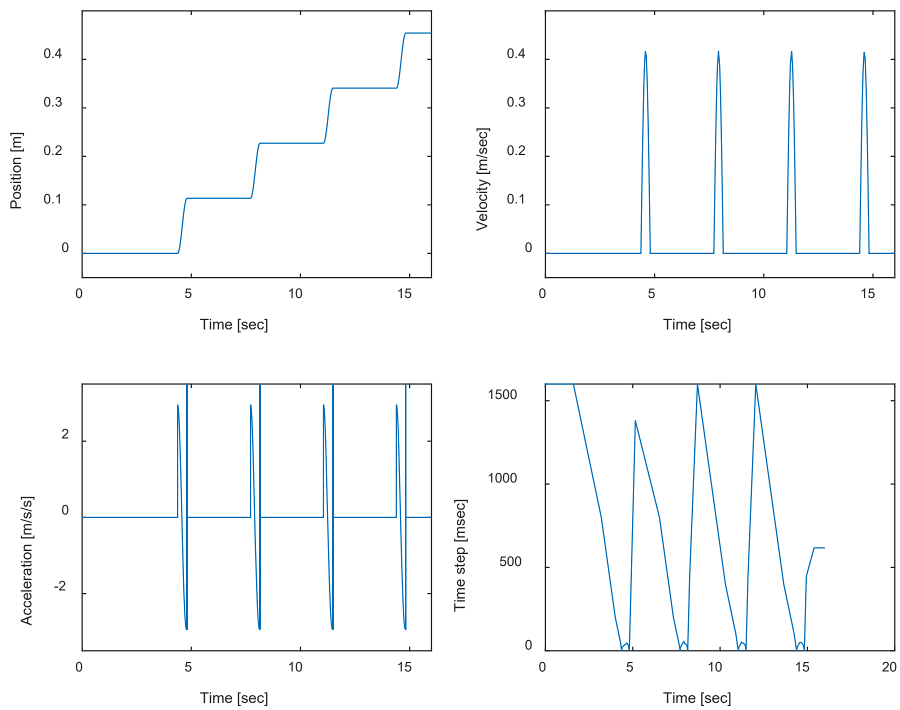
$$B = \frac{\text{slope}}{CD}$$

$$E = \frac{B\,x_M - \tan\left(\pi / (2C)\right)}{B\,x_M - \operatorname{a\,tan}\left(B\,x_M\right)} \qquad \text{for} \quad C > 1$$

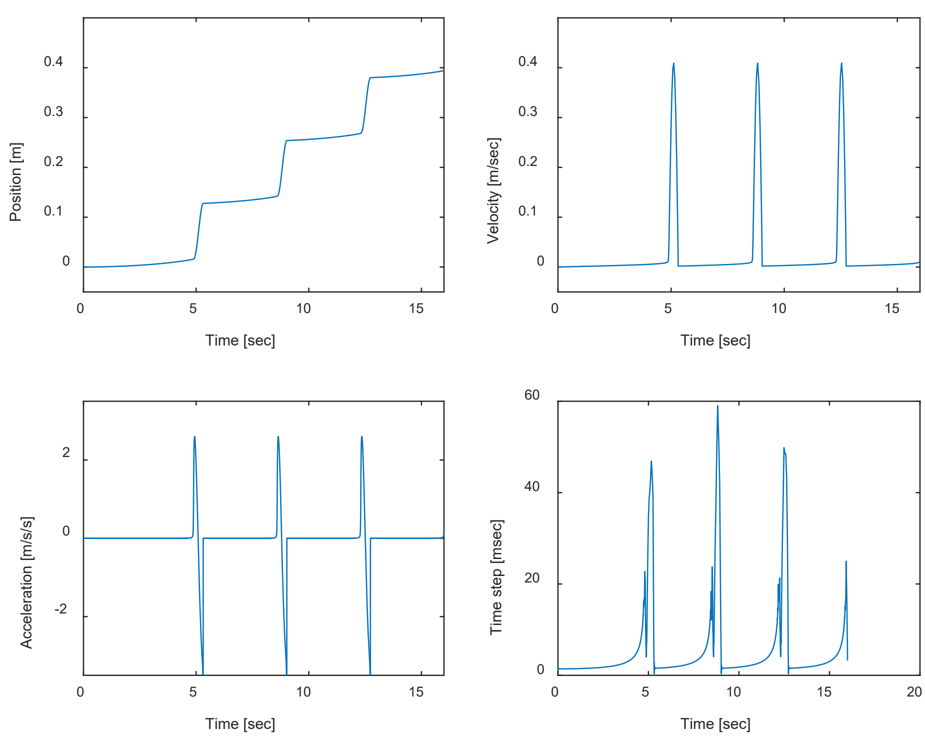



```
% try_pacejka.m - test Pacejka magic formula
% HJSIII, 11.04.25

clear

% constants

% define curve
xm = 0.01;
yp = 0.9;
ya = 0.6;

% Pacejka coefficients
D = yp;
C = 1 + (1 - 2 * asin( ya/D ) / pi);
slope = yp / (xm/2);
B = slope /C /D;
E = (B*xm - tan(pi/2/C)) / (B*xm - atan(B*xm));

% plot
x = ( -1 : 0.001 : 1 )';
y = D*sin( C*atan( B*x - E*(B*x-atan(B*x)) ) );
figure( 1 )
  clf
  plot( x,y )

% bottom - try_pacejka
```

Stick-Slip using Pseudo-code



Stick-Slip using Pacejka Magic Formula with xm = 0.01



friction model	CPU time [sec]	time stpes	ave time step [msec]
pseudo-code	1.7690	10972	1.5145
Pacejka xm=0.01 mps	0.7313	6667	2.4004
Pacejka xm=0.001 mps	7.3105	68107	0.2349

+ faster execution
- velocity is not exactly zero during stick phase
- asymmetric acceleration

see ME 481 H14

```
function yd = ode_stickslip_pacejka_yd( t, y )
% provides yd for integration
% stick slip drag sled
%   m*xdd = Fext + Fspr + Ff      Pacejka friction model from Notes_08_06
% HJSIII, 20.04.26

global  m  k  mu_s  mu_d  g  v_driver  eps

% free motion
Fext = 0;

% individual terms
x = y(1);
xd = y(2);

% spring force
x_driver = v_driver * t;
Fspr = k * ( x_driver - x );

% Pacejka constants
xm_pacj = 0.01;
yp_pacj = mu_s;
ya_pacj = mu_d;

% Pacejka coefficients
D = yp_pacj;
C = 1 + (1 - 2 * asin( ya_pacj/D ) / pi);
slope = yp_pacj / (xm_pacj/2);
B = slope /C /D;
E = (B*xm_pacj - tan(pi/2/C)) / (B*xm_pacj - atan(B*xm_pacj));

% Pacejka variables
x_pacj = xd;
mu = D*sin( C*atan( B*x_pacj - E*(B*x_pacj-atan(B*x_pacj)) ) );

% Pacejka friction model
Ff = -mu * m * g;

% ordinary differential equation (ODE)
xdd = ( Fext + Fspr + Ff ) / m;

% return values
yd(1,1) = xd;
yd(2,1) = xdd;

return
% bottom - ode_stickslip_pacejka_yd
```