Two-Dimensional Generalized Forces

Moment about origin

$$T = \left(\{r_i\}^P - \{r_i\}\right) \times \{F_{on\ i}\}^P$$

$$\left(\{r_i\}^P - \{r_i\}\right) = \{s_i\}^P = [A_i]\{s_i\}'^P$$

$$\{s_i\}^P \times \{F_{on\ i}\}^P = \left([A_i]\{s_i\}'^P\right) \times \{F_{on\ i}\}^P = \{s_i\}'^P \times \{F_{on\ i}\}'^P \quad \textbf{OK}$$

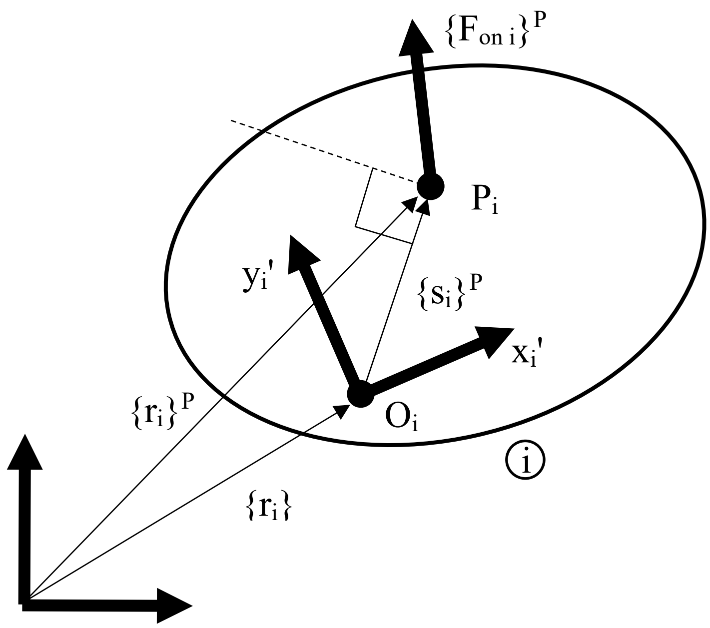
$$\{s_i\}'^P \times \{F_{on\ i}\}^P = \{s_i\}^P \times \{F_{on\ i}\}'^P \quad \textbf{NO!}$$

$$[R][A_i]\{s_i\}'^P = [B_i]\{s_i\}'^P \quad \perp \quad \{s_i\}^P$$

$[B_i]\{s_i\}'^P \circ \{F_{on\ i}\}^P$ component of $\{F_{on\ i}\}^P$ perpendicular to $\{s_i\}^P$ multiplied by $\left(\text{norm}\{s_i\}^P\right)$

$$T = \left(\{r_i\}^P - \{r_i\}\right) \times \{F_{on\ i}\}^P$$

$$T = \left([B_i]\{s_i\}'^P\right)^T \{F_{on\ i}\}^P = \left(\{F_{on\ i}\}^P\right)^T [B_i]\{s_i\}'^P$$



Generalized force on body i about origin

$$\{q_i\} = \begin{Bmatrix} \{r_i\} \\ \phi_i \end{Bmatrix}$$

$$\{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\} \\ T_{on\ i} \end{Bmatrix}$$

Pure force

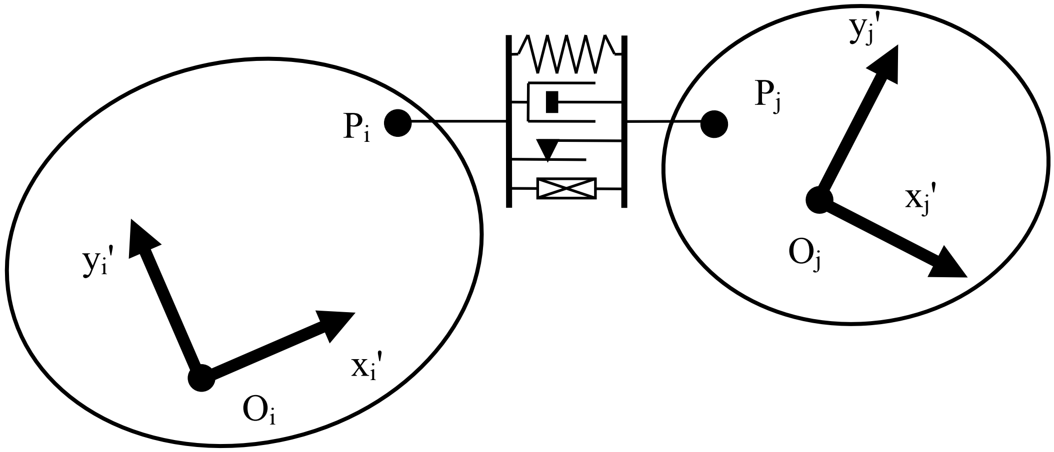
$$\{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\}^P \\ \left([B_i]\{s_i\}^{'P}\right)^T \{F_{on\ i}\}^P \end{Bmatrix}$$

Pure moment

$$\{Q_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ T_{on\ i} \end{Bmatrix}$$

Translational spring-damper-actuator

$\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$ Note similarity to double revolute constraint



$\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$

$\{\ddot{d}_{ij}\} = \{\ddot{r}_j\}^P - \{\ddot{r}_i\}^P$

$\ell^2 = \{d_{ij}\}^T\{d_{ij}\}$

$2\ell\dot{\ell} = 2\{d_{ij}\}^T\{\dot{d}_{ij}\}$

$\dot{\ell} = \{d_{ij}\}^T\{\dot{d}_{ij}\}/\ell$

$f = k(\ell - \ell_o) + c\,\dot{\ell} + f_F \text{sign}(\dot{\ell}) + f_{ACT}(\ell, \dot{\ell}, t)$

$\{F_{on\,i}\} = \frac{f}{\ell}\{d_{ij}\}$ $\{Q_{on\,i}\} = \begin{Bmatrix} \{F_{on\,i}\} \\ \left([B_i]\{s_i\}'^P\right)^T\{F_{on\,i}\} \end{Bmatrix}$

$\{F_{on\,j}\} = -\frac{f}{\ell}\{d_{ij}\}$ $\{Q_{on\,j}\} = \begin{Bmatrix} \{F_{on\,j}\} \\ \left([B_j]\{s_j\}'^P\right)^T\{F_{on\,j}\} \end{Bmatrix}$

$\ddot{\ell} = \left(\{d_{ij}\}^T\{\ddot{d}_{ij}\} + \{\dot{d}_{ij}\}^T\{\dot{d}_{ij}\} - \dot{\ell}^2\right)/\ell$

$\dot{f} = k\dot{\ell} + c\,\ddot{\ell} + \dot{f}_{ACT}(\ell, \dot{\ell}, \ddot{\ell}, t)$

$\{\dot{F}_{on\,i}\} = \left(\dot{f}\,\ell\{d_{ij}\} + f\,\ell\{\dot{d}_{ij}\} - f\,\dot{\ell}\{d_{ij}\}\right)/\ell^2$

$\{\dot{Q}_{on\,i}\} = \begin{Bmatrix} \{\dot{F}_{on\,i}\} \\ \left([B_i]\{s_i\}'^P\right)^T\{\dot{F}_{on\,i}\} - \dot{\phi}_i\left([A_i]\{s_i\}'^P\right)^T\{F_{on\,i}\} \end{Bmatrix}$

$\{\dot{F}_{on\,j}\} = -\left(\dot{f}\,\ell\{d_{ij}\} + f\,\ell\{\dot{d}_{ij}\} - f\,\dot{\ell}\{d_{ij}\}\right)/\ell^2$

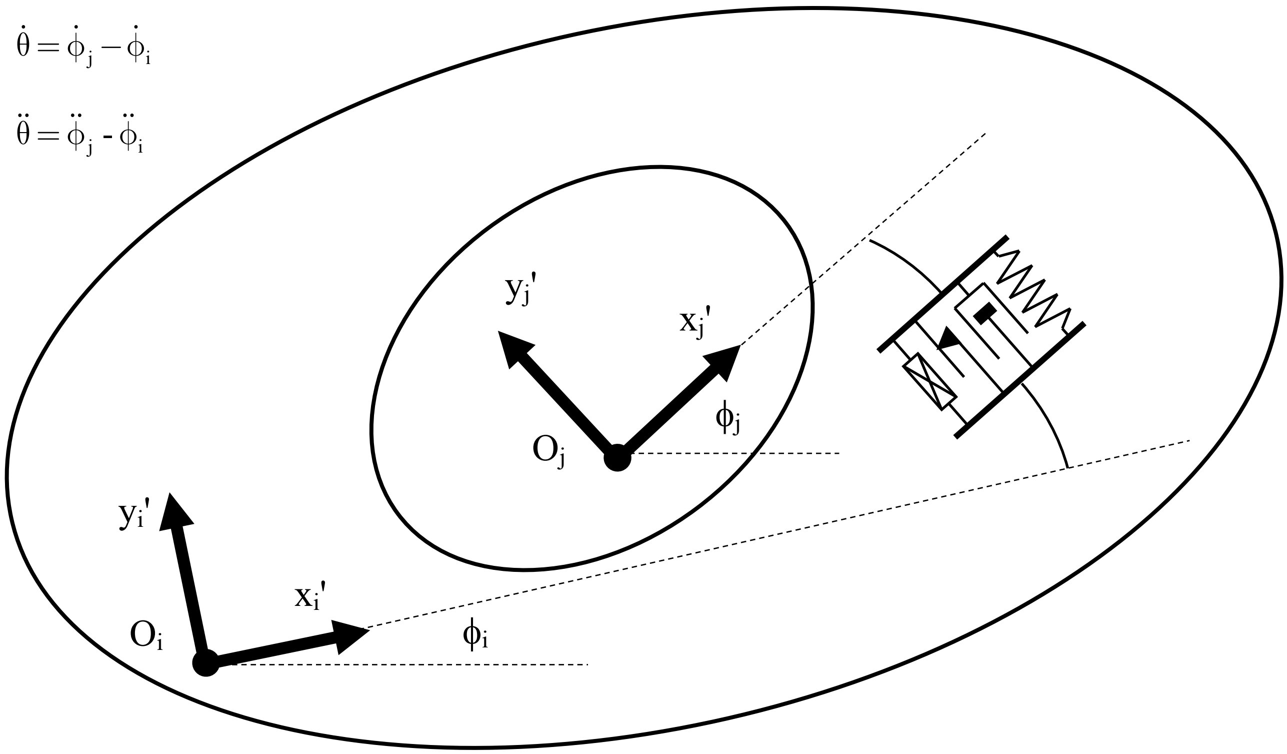
$$\left\{\dot{Q}_{on\ j}\right\} = \begin{Bmatrix} \left\{\dot{F}_{on\ j}\right\} \\ \left(\left[B_j\right]\left\{s_j\right\}'^{P}\right)^T \left\{\dot{F}_{on\ j}\right\} - \dot{\phi}_j \left(\left[A_j\right]\left\{s_j\right\}'^{P}\right)^T \left\{F_{on\ j}\right\} \end{Bmatrix}$$

Rotational spring-damper-actuator

$\theta = \phi_j - \phi_i$ Note similarity to relative angle constraint

$\dot{\theta} = \dot{\phi}_j - \dot{\phi}_i$

$\ddot{\theta} = \ddot{\phi}_j - \ddot{\phi}_i$



$$T = k_\theta(\theta - \theta_o) + c_\theta\ \dot{\theta} + T_F \text{sign}(\dot{\theta}) + T_{ACT}(\theta, \dot{\theta}, t)$$

$$\{Q_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ T \end{Bmatrix}$$

$$\{Q_{on\ j}\} = -\begin{Bmatrix} 0 \\ 0 \\ T \end{Bmatrix}$$

$$\dot{T} = k_\theta \dot{\theta} + c_\theta\ \ddot{\theta} + T_{ACT}(\theta, \dot{\theta}, \ddot{\theta}, t)$$

$$\{\dot{Q}_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ \dot{T} \end{Bmatrix}$$

$$\left\{\dot{Q}_{\text{on j}}\right\} = -\begin{Bmatrix} 0 \\ 0 \\ \dot{T} \end{Bmatrix}$$