**Differential-Algebraic Equations (DAE) for Anthropomorphic Manipulator**

3

2=2

y1

A

B

x1

C

**T1on2**

**T2on3**

G2

G3

x2

y2

x3

y3

3

r2 = AG2

d2 = AB

r3 = BG3

d3 = BC

Two solid rigid bars with revolute joints A and B

Tool center point (TCP) at C (endpoint)

Centroids G2 and G3

Masses m2 and m3

Centroidal mass moments of inertia JG2 and JG3

T1on2 is torque of ground on bar 2 about revolute A measured CCW positive

T2on3 is torque of bar 2 on bar 3 about revolute B measured CCW positive

Gravity g acts along negative y axis

nL = 3 nJ1=2 m = 3 (nL-1) – 2 nJ1 = 2

Lagrangian method (from Notes\_09\_02)

 Note: 3 measured relative to 2













**inverse dynamics**



know driver motion **at any time** t - find 

compute driver torques  from Lagrangian equations

compute joint forces  from Newtonian equations

may arbitrarily choose **any other time** t

**forward dynamics**



know current state  at current t

compute  from Lagrangian equations

compute from Newtonian equations



must integrate  to get new  at the **next time step**

Newtonian method (from free body diagrams)



**inverse dynamics**

know driver motion **at any time** t - find positions, velocities and accelerations

compute joint forces 

may arbitrarily choose **any other time** t

**forward dynamics**

know current positions and velocities at current t

very cumbersome to compute joint forces **and** accelerations **simultaneously**

must integrate accelerations to get new positions and velocities at the **next time step**

DAE dynamics





**inverse dynamics**



know driver motion **at any time** t, find 

compute and  **simultaneously**



may arbitrarily choose **any other time** t

**forward dynamics**



know current state at current t, find 



compute and **simultaneously**



must integrate to get new at the **next time step**



**Inverse Dynamics – joint interpolated motion**

A

B

2

3

C

(independent position controllers on each joint)







**Inverse Dynamics – straight-line TCP interpolated motion**

(interpolated position controllers on each joint)

A

B

2

3

C















**Forward Dynamics – double pendulum**

(no actuators)







**Forward Dynamics – proximal link kinematically driven, distal link pendulum**

(position controller **only** on proximal joint)









**Forward Dynamics – computed torque control**

(torque controllers on each joint)

T1 on 2

T2 on 3





