**Lagrangian Dynamics for Simple Pendulum**



a

Y

A

B

X

**T**

m, JG

G

gravity



















second order dynamics using Gibbs-Appell

















third order dynamics using direct time derivative



third-order Lagrange ?????















 **Lagrangian Dynamics for Spring-Mass**

**k**

**m**

**FEXT**

**x**

**x**

q = x  Q = FEXT

  L = K - P = 

 

  



**Lagrangian Dynamics for Cylindrical Coordinate Manipulator**

**T**

**X**

**Y**

****

**X**

**Y**

****

**X**

**Y**

****

**r**

**r3**

**a**

**Link 2**

**Link 3**

**Both links**

**F**

Main body link 2 - Shaft and end-effector link 3

Mass centers at a and r3 from waist rotation axis, a=constant, r3 = variable

Masses m2 and m3 - centroidal mass moments of inertia J2 and J3

 CCW from positive x axis – a and r3 radial from rotation axis

T is rotary actuator torque of ground on body 2 about waist measured CCW positive

F is radial actuator force of body 2 on body 3 measured positive outward

Gravity g acts along negative y axis

q2 =  q3 = r   Q2 = T Q3 = F

x2 = a cos 

y2 = a sin 

x3 = r3 cos 

y3 = r3 sin 





P = m2 y2 g + m3 y3 g P = m2 g asin + m3 g r3 sin

L = K - P



  

















inverse dynamics



forward dynamics



third order dynamics using direct time derivatives



**Lagrangian Dynamics for a Bar Balanced on a Cylinder**

A steel bar is initially balanced on top of a fixed cylinder as shown below left. When the left end of the bar is depressed and then released, it will oscillate in a rocking fashion as shown below right. The radius of the cylinder is 1.906 inches. The bar is 19.5 inches long and weighs 0.45 pounds. Assume thickness of the bar is negligible and that it rolls without slipping.

a) Determine the natural frequency of oscillation for an initial release angle 0 = 10º.

b) Determine the natural frequency of oscillation if an aluminum bar with identical geometry were substituted.



G

P

C

bar

cylinder

motion

G

P

C









r

G

P

C



X

Y























using r = 1.906 inch, L = 19.5 inch, m = 0.45 lbm, g = 386 ips2

 = 4.818 rad/sec = 0.767 Hz









Steel and aluminum bars with the same geometry should have the same frequency because mass cancels out of N.

**Lagrangian Dynamics for Two Link Anthropomorphic Manipulator (Double Pendulum)**

3

2

a3

d3

d2

a2

Y

A

B

X

C

**T2**

**T3**

m2, J2

m3, J3

Two solid rigid bars with revolute joints A and B

Lengths d2 and d3 - mass centers at a2 and a3 from proximal ends

Masses m2 and m3 - centroidal mass moments of inertia J2 and J3

2 CCW from positive x axis T2 is torque of ground on bar 2 about pin A, CCW positive

3 CCW from centerline of bar 2 T3 is torque of bar 2 on bar 3 about pin B, CCW positive

Gravity g acts along negative y axis















































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inverse dynamics





forward dynamics

 

third order dynamics using direct time derivatives











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**Lagrangian Dynamics for Three Link Anthropomorphic Manipulator**

**T2**

3

2

a3

d3

d2

a2

Y

A

B

X

C

**T3**

m2, J2

m3, J3

4

a4

d4

D

m4, J4

**T4**

Three solid rigid bars with revolute joints A, B and C

Lengths d2 d3 d4 - mass centers at a2 a3 a4 from proximal ends

Masses m2 m3 m4 - centroidal mass moments of inertia J2 J3 J4

2 CCW from positive x axis T2 is torque of ground on bar 2 about pin A, CCW positive

3 CCW from centerline of bar 2 T3 is torque of bar 2 on bar 3 about pin B, CCW positive

4 CCW from centerline of bar 3 T4 is torque of bar 3 on bar 4 about pin C, CCW positive

Gravity g acts along negative y axis







































































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Validated with harmonic drivers using Haug's inverse dynamics and then calculate torques per above



**Linear State Space Model for Two Link Manipulator**



linearize about nominal values of ****

 































