Three-Dimensional Vector and Matrix Notation

$\{r_i\}$ global position of the origin of reference frame attached to body i

$\{r_i\}^P$ global position of point P attached to body i

example $\{r_4\}^B = \begin{Bmatrix} x_4^{\ B} \\ y_4^{\ B} \\ z_4^{\ B} \end{Bmatrix}$ global position of point B attached to body 4

$\{\dot{r}_i\}$ global velocity of the origin of the reference attached to body i

$\{\dot{r}_i\}^P$ global velocity of point P attached to body i

$\{\ddot{r}_i\}$ global acceleration of the origin of the reference attached to body i

$\{\ddot{r}_i\}^P$ global acceleration of point P attached to body i

$\{\dddot{r}_i\}$ global jerk of the origin of the reference attached to body i

$\{\dddot{r}_i\}^P$ global jerk of point P attached to body i

$\{s_i\}'^{\,P}$ position of point P on body i relative to the reference frame for body i measured in local body-fixed directions

example $\{s_4\}'^{\,B} = \begin{Bmatrix} x_4'^{\,B} \\ y_4'^{\,B} \\ z_4'^{\,B} \end{Bmatrix}$ location of point B on body 4 relative to the reference frame for body 4 measured in local body-fixed directions for body 4

$\{s_i\}^P$ position of point P on body i relative to the reference frame for body i but measured in global directions

$\{d_{ij}\}$ relative location between two points on bodies i and j measured in global directions

example $\{d_{ij}\} = \{r_4\}^Q - \{r_3\}^P$ relative location of point Q on body 4 with respect to point P on body 3 measured in global directions

$\{p_i\}$ Euler parameters to describe attitude for body i

$\{\omega_i\}$ angular velocity of body i measured in global directions

$\{\omega_i\}'$ angular velocity of body i measured in local body-fixed directions

$\{\dot{\omega}_i\}$ angular acceleration of body i measured in global directions

$\{\dot{\omega}_i\}'$ angular acceleration of body i measured in local body-fixed directions

$\{\ddot{\omega}_i\}$ angular jerk of body i measured in global directions

$\{\ddot{\omega}_i\}'$ angular jerk of body i measured in local body-fixed directions

$[A_i]$ orthonormal rotation matrix that describes global attitude of body i

example $\{s_i\}^P = \lfloor A_i \rfloor \{s_i\}'^P$ rotation matrix converts information in local body-fixed directions into global directions

$\{\hat{f}_i\}$ global direction of unit vector along local x axis attached to body i

$\{\hat{g}_i\}$ global direction of unit vector along local y axis attached to body i

$\{\hat{h}_i\}$ global direction of unit vector along local z axis attached to body i

$\{\hat{f}_i\}'$ local direction of unit vector along local x axis attached to body i $\{\hat{f}_i\}' = [1 \quad 0 \quad 0]^T$

$\{\hat{g}_i\}'$ local direction of unit vector along local y axis attached to body i $\{\hat{g}_i\}' = [0 \quad 1 \quad 0]^T$

$\{\hat{h}_i\}'$ local direction of unit vector along local z axis attached to body i $\{\hat{h}_i\}' = [0 \quad 0 \quad 1]^T$

example $\lfloor A_i \rfloor = \left[\{\hat{f}_i\} \quad \{\hat{g}_i\} \quad \{\hat{h}_i\} \right]$ global unit directions for local axes attached to body i

$[C_i]'^P$ orthonormal rotation matrix that describes relative attitude of joint frame at point P on body i measured in local body-fixed directions

$[C_i]^P$ orthonormal rotation matrix that describes global attitude of joint frame at point P on body i

example $[C_i]^P = [A_i][C_i]'^P$ rotation matrix converts information in local body-fixed directions into global directions

$\{\hat{f}_i\}^P$ global direction of unit vector along x axis at joint frame for P on body i

$\{\hat{g}_i\}^P$ global direction of unit vector along y axis at joint frame for P on body i

$\{\hat{h}_i\}^P$ global direction of unit vector along z axis at joint frame for P on body i

example $\lfloor C_i \rfloor^P = \left[\{\hat{f}_i\}^P \quad \{\hat{g}_i\}^P \quad \{\hat{h}_i\}^P \right]$ global unit directions for local axes at joint frame for P on body i

$\{\hat{f}_i\}'^P$ local direction of unit vector along x axis at joint frame for P on body i

$\{\hat{g}_i\}'^P$ local direction of unit vector along y axis at joint frame for P on body i

$\{\hat{h}_i\}'^P$ local direction of unit vector along y axis at joint frame for P on body i

example $\lfloor C_i \rfloor'^P = \left[\{\hat{f}_i\}'^P \quad \{\hat{g}_i\}'^P \quad \{\hat{h}_i\}'^P \right]$ global unit directions for local axes at joint frame for P on body i

$\{\hat{f}_i\}''^P$ joint frame direction of unit vector along x axis at joint frame for P on body i

$$\{\hat{f}_i\}''^P = [1 \quad 0 \quad 0]^T$$

$\{\hat{g}_i\}''^P$ joint frame direction of unit vector along y axis at joint frame for P on body i

$$\{\hat{g}_i\}''^P = [0 \quad 1 \quad 0]^T$$

$\{\hat{h}_i\}''^P$ joint frame direction of unit vector along z axis at joint frame for P on body i

$$\{\hat{h}_i\}''^P = [0 \quad 0 \quad 1]^T$$

$\{F_{on\ i}\}^P$ force on body i acting through point P measured in global directions

$\{F_{on\ i}\}'^P$ force on body i acting through point P measured in local body-fixed directions

$\{T_{on\ i}\}$ torque on body i measured in global directions

$\{T_{on\ i}\}'$ torque on body i measured in local body-fixed directions

Numbering and lettering

Bodies should be numbered consecutively beginning with 1. Body 1 is typically reserved for ground.

Points should be lettered.

Point G is typically reserved for the mass center of a body.

Point T is seldom used in that it causes confusion with the vector/matrix transpose operator.

Subscripts and superscripts outside vector/matrix brackets

Post-superscript prime outside vector brackets denotes information measured in local body-fixed directions.

Post-superscript letters outside vector brackets denote information related to a specific point.

Post-subscripts outside vector/matrix brackets are occasionally used for iteration or time indices.

Pre-superscripts and pre-subscripts are typically not used outside brackets.

Subscripts and superscripts inside vector/matrix brackets

Post-superscripts inside vector/matrix brackets are occasionally used for iteration or time indices.

Post-subscript numerals inside vector/matrix brackets are typically used for body numbers.

Post-subscript variables inside vector/matrix brackets denote partial derivative operators.

Pre-superscripts and pre-subscripts are typically not used inside brackets.

General vector/matrix operations

$\{\ \}^T, [\ ]^T$	vector/matrix transpose
$[\ ]^{-1}$	matrix inverse
$\det[\ ]$	determinant of matrix
$\mathrm{tr}[\ ]$	trace of matrix (sum of diagonal elements)
$\{\mathrm{diag}[\ ]\}$	diagonal elements of matrix rearranged into column vector
$[\mathrm{diag}\{\ \}]$	elements of vector placed into a diagonal matrix
$[\ ]^n$	matrix to power n
$\mathrm{norm}\{\ \}$	scalar norm of vector (magnitude)
$\{\hat{a}\}$	unit vector
$[I_n]$	identity matrix of order n
$\{0\},[0]$	vector/matrix of zeros
$[\tilde{a}]$	skew- symmetric operator for cross products

$$\{a\} = \begin{Bmatrix} a_x \\ a_y \\ a_z \end{Bmatrix} \qquad [\tilde{a}] = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix}$$