**Three-Dimensional Coordinate Transformations**

 



[A] matrices are orthonormal [A] -1 = [A] T

● all columns are unit vectors

● all columns are mutually orthogonal

● all rows are unit vectors

● all rows are mutually orthogonal

● det [A] = +1

**Single rotations**







# Coordinate Frames at the Corners of a Wedge

Adapted from Introduction to Robotics, J.J. Craig, Addison-Wesley, 1989

x2’

y2’

z2’

x1’

y1’

z1’

x3’

y3’

z3’

a

b

c



 

x4’

y4’

z4’

x5’

y5’

z5

x6’

y6’

z6’

a

b

c



 

**Euler Angles**

**Euler sequences** – ZXZ (original), ZYZ, YXY, YZY, XYX, XZX

**ZXZ sequence (1 about global z - 2 about intermediate x - 3 about local z)**







**ZYZ sequence**

**YXY sequence**

**YZY sequence**

**XYX sequence**

**XZX sequence**

**Cardan-Bryant-Tait sequences** – XYZ, XZY, YXZ, YZX, ZXY, ZYX

**XYZ sequence (x about global x - y about intermediate y - z about local z)**





**ZYX sequence (z about global z - y about intermediate y - x about local x)**





**XZY sequence (x about global x - z about intermediate z - y about local y)**





**YZX sequence**

**YXZ sequence**

**ZXY sequence**

**Derivatives of Cardan-Bryant-Tait angles** – ZYX

**ZYX sequence (z about global z - y about intermediate y - x about local x)**











 













 

**Chasles’ Angle and Euler Parameters**

Rotation χ about unit direction  ****



**Euler parameters (unit quarternion)**

 















**Velocity**



















**Acceleration**





















**Jerk**



















**Snap**









**Numerical partial derivatives of rotation matrices with respect to Euler parameters can produce different results**





**Rodriguez Parameters**

