Three-Dimensional Constraints

General

$$\{\Phi\}=\{0\}$$

$$\{\dot{\Phi}\}=\{0\}$$

$$\{\dot{\Phi}\}=[\Phi_q]\{\dot{q}\}+\{\Phi_t\}=\{0\}$$

$$[\Phi_q]\{\dot{q}\}=\{\nu\} \qquad \{\nu\}\equiv -\{\Phi_t\}$$

$$\{\ddot{\Phi}\}=\{0\}$$

$$\{\ddot{\Phi}\}=[\Phi_q]\{\ddot{q}\}+\left([\Phi_q]\{\dot{q}\}\right)_q\{\dot{q}\}+2[\Phi_{qt}]\{\dot{q}\}+\{\Phi_{tt}\}=\{0\}$$

$$[\Phi_q]\{\ddot{q}\}=\{\gamma\} \qquad \{\gamma\}\equiv -\left([\Phi_q]\{\dot{q}\}\right)_q\{\dot{q}\}-2[\Phi_{qt}]\{\dot{q}\}-\{\Phi_{tt}\}$$

$$\{\dddot{\Phi}\}=\{0\}$$

$$[\Phi_q]\{\dddot{q}\}=\{\eta\}$$

$$\{\eta\}\equiv -3\left([\Phi_q]\{\dot{q}\}\right)_q\{\ddot{q}\}-\left(\left([\Phi_q]\{\dot{q}\}\right)_q\{\dot{q}\}\right)_q\{\dot{q}\}-3[\Phi_{qt}]\{\ddot{q}\}-3\left([\Phi_{qt}]\{\dot{q}\}\right)_q\{\dot{q}\}-3[\Phi_{qtt}]\{\dot{q}\}-\{\Phi_{ttt}\}$$

$$\{\ddddot{\Phi}\}=\{0\}$$

$$[\Phi_q]\{\ddddot{q}\}=\{\sigma\}$$

$$\begin{aligned}\{\sigma\}\equiv\ & -4\left([\Phi_q]\{\dddot{q}\}\right)_q\{\dot{q}\}-3\left([\Phi_q]\{\ddot{q}\}\right)_q\{\ddot{q}\}-6\left(\left([\Phi_q]\{\dot{q}\}\right)_q\{\dot{q}\}\right)_q\{\ddot{q}\}-\left(\left(\left([\Phi_q]\{\dot{q}\}\right)_q\{\dot{q}\}\right)_q\{\dot{q}\}\right)_q\{\dot{q}\}\\ & -4[\Phi_{qt}]\{\dddot{q}\}-12\left([\Phi_{qt}]\{\dot{q}\}\right)_q\{\ddot{q}\}-4\left(\left([\Phi_{qt}]\{\dot{q}\}\right)_q\{\dot{q}\}\right)_q\{\dot{q}\}-6[\Phi_{qtt}]\{\ddot{q}\}-6\left([\Phi_{qtt}]\{\dot{q}\}\right)_q\{\dot{q}\}\\ & -4[\Phi_{qttt}]\{\dot{q}\}-\{\Phi_{tttt}\}\end{aligned}$$

Scleronomic constraints

independent of time such as mechanical joints

$$\{\gamma\} \equiv -\left(\left[\Phi_q\right]\{\dot{q}\}\right)_q \{\dot{q}\}$$

$$\{\eta\} \equiv \{\dot{\gamma}\} - \left(\left[\Phi_q\right]\{\ddot{q}\}\right)_q \{\dot{q}\}$$

$$\{\sigma\} \equiv \{\dot{\eta}\} - \left(\left[\Phi_q\right]\{\dddot{q}\}\right)_q \{\dot{q}\}$$

use same $\{\gamma\}$ for both $\{\dot{\omega}'\}$ and $\{\ddot{p}\}$ solutions

must use $\{\eta_{\pi'}\}$ for $\{\ddot{\omega}'\}$ and different $\{\eta_p\}$ for $\{\dddot{p}\}$ solutions

must use $\{\sigma_{\pi'}\}$ for $\{\dddot{\omega}'\}$ and different $\{\sigma_p\}$ for $\{\ddddot{p}\}$ solutions

Spherical

$$\{\Phi\}_{SPH} = \{r_j\}^P - \{r_i\}^P = \{0_{3x1}\}$$

$$\left[\Phi_{ri}\right]_{SPH} = -\left[I_3\right]$$
$$\left[\Phi_{\pi'i}\right]_{SPH} = \left[A_i\right]\left[\tilde{s}_i\right]'^P$$
$$\left[\Phi_{pi}\right]_{SPH} = 2\left[A_i\right]\left[\tilde{s}_i\right]'^P\left[G_i\right]$$

$$\left[\Phi_{rj}\right]_{SPH} = \left[I_3\right]$$
$$\left[\Phi_{\pi'j}\right]_{SPH} = -\left[A_j\right]\left[\tilde{s}_j\right]'^P$$
$$\left[\Phi_{pj}\right]_{SPH} = -2\left[A_j\right]\left[\tilde{s}_j\right]'^P\left[G_j\right]$$

$$\{\nu\}_{SPH} = \{0_{3x1}\}$$

$$\{\gamma\}_{SPH} = \left[A_i\right]\left[\tilde{\omega}_i\right]'\left[\tilde{\omega}_i\right]'\{s_i\}'^P - \left[A_j\right]\left[\tilde{\omega}_j\right]'\left[\tilde{\omega}_j\right]'\{s_j\}'^P$$

$$[H_i]' = 2[\tilde{\omega}_i]'[\dot{\tilde{\omega}}_i]' + [\dot{\tilde{\omega}}_i]'[\tilde{\omega}_i]' + [\tilde{\omega}_i]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'$$

$$\{\eta\}_{SPH}=[A_i][H_i]'\{s_i\}'^P-[A_j][H_j]'\{s_j\}'^P \qquad \{\ddot{\omega}\}' \text{ version}$$

$$\{\eta\}_{SPH\_p}=\{\eta\}_{SPH}-\tfrac{1}{4}[A_i]\Big([H_i]'-3\big[\dot{\tilde{\omega}}_i\big]'\big[\tilde{\omega}_i\big]'\Big)\{s_i\}'^P+\tfrac{1}{4}[A_j]\Big([H_j]'-3\big[\dot{\tilde{\omega}}_j\big]'\big[\tilde{\omega}_j\big]'\Big)\{s_j\}'^P \qquad \{\ddot{p}\} \text{ version}$$

$$\begin{aligned}[W_i]'=&\big[\ddot{\tilde{\omega}}_i\big]'[\tilde{\omega}_i]'+3\big[\dot{\tilde{\omega}}_i\big]'\big[\dot{\tilde{\omega}}_i\big]'+3[\tilde{\omega}_i]'\big[\ddot{\tilde{\omega}}_i\big]'+\big[\dot{\tilde{\omega}}_i\big]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'+2[\tilde{\omega}_i]'\big[\dot{\tilde{\omega}}_i\big]'[\tilde{\omega}_i]'+3[\tilde{\omega}_i]'[\tilde{\omega}_i]'\big[\dot{\tilde{\omega}}_i\big]'\\&+[\tilde{\omega}_i]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'\end{aligned}$$

$$\{\sigma\}_{SPH}=[A_i][W_i]'\{s_i\}'^P-[A_j][W_j]'\{s_j\}'^P \qquad \{\dddot{\omega}\}' \text{ version}$$

$$\{\kappa F_i\}_{SPH}=\{0_{3x1}\}$$

$$\{\kappa T_i\}_{SPH}=[\tilde{s}_i]'^P[\tilde{\omega}_i]'[A_i]^T\{\lambda\}_{SPH}$$

$$\{\kappa F_j\}_{SPH}=\{0_{3x1}\}$$

$$\{\kappa T_j\}_{SPH}=-[\tilde{s}_j]'^P[\tilde{\omega}_j]'[A_j]^T\{\lambda\}_{SPH}$$

Double spherical

$$\Phi_{SPH\_SPH}=\{d_{ij}\}^T\{d_{ij}\}-L^2=0 \qquad L=\text{constant length}$$

$$\text{for} \quad \{d_{ij}\}=\{r_j\}^P-\{r_i\}^P \quad \text{and} \quad \{\dot{d}_{ij}\}=\{\dot{r}_j\}^P-\{\dot{r}_i\}^P$$

$$[\Phi_{ri}]_{SPH\_SPH}=-2\{d_{ij}\}^T$$

$$[\Phi_{\pi' i}]_{SPH\_SPH}=2\{d_{ij}\}^T[A_i][\tilde{s}_i]'^P$$

$$[\Phi_{pi}]_{SPH\_SPH}=4\{d_{ij}\}^T[A_i][\tilde{s}_i]'^P[G_i]$$

$$[\Phi_{rj}]_{SPH\_SPH}=2\{d_{ij}\}^T$$

$$[\Phi_{\pi' j}]_{SPH\_SPH}=-2\{d_{ij}\}^T[A_j][\tilde{s}_j]'^P$$

$$[\Phi_{pj}]_{SPH\_SPH}=-4\{d_{ij}\}^T[A_j][\tilde{s}_j]'^P[G_j]$$

$$\nu_{SPH\_SPH}=0$$

$$\gamma_{SPH\_SPH} = -2\left\{\dot{d}_{ij}\right\}^T\left\{\dot{d}_{ij}\right\} + 2\left\{d_{ij}\right\}^T\{\gamma\}_{SPH}$$

$$\eta_{SPH\_SPH} = -6\left\{\dot{d}_{ij}\right\}^T\left\{\ddot{d}_{ij}\right\} + 2\left\{d_{ij}\right\}^T\{\eta\}_{SPH} \qquad \left\{\ddot{\omega}\right\}' \text{ version}$$

$$\begin{aligned}\eta_{SPH\_SPH\_p} = \eta_{SPH\_SPH} &- \tfrac{1}{2}\left\{d_{ij}\right\}^T[A_i]\left([H_i]'-3\left[\dot{\tilde{\omega}}_i\right]'\left[\tilde{\omega}_i\right]'\right)\{s_i\}'^P \\ &+ \tfrac{1}{2}\left\{d_{ij}\right\}^T[A_j]\left([H_j]'-3\left[\dot{\tilde{\omega}}_j\right]'\left[\tilde{\omega}_j\right]'\right)\{s_j\}'^P\end{aligned} \qquad \left\{\ddot{p}\right\} \text{ version}$$

$$\sigma_{SPH\_SPH} = -6\left\{\ddot{d}_{ij}\right\}^T\left\{\ddot{d}_{ij}\right\} - 8\left\{\dot{d}_{ij}\right\}^T\left\{\dddot{d}_{ij}\right\} + 2\left\{d_{ij}\right\}^T\{\sigma\}_{SPH} \qquad \left\{\ddot{\omega}\right\}' \text{ version}$$

$$\left\{\boldsymbol{\kappa}F_i\right\}_{SPH\_SPH} = -2\boldsymbol{\lambda}_{SPH\_SPH}\left\{\dot{d}_{ij}\right\}$$

$$\left\{\kappa T_i\right\}_{SPH\_SPH} = +2\lambda_{SPH\_SPH}\begin{pmatrix}\left([A_i]\left[\tilde{s}_i\right]'^P\right)^T\left\{\dot{d}_{ij}\right\} + \left([A_i]\left[\tilde{\omega}_i\right]'\left[\tilde{s}_i\right]'^P\right)^T\left\{d_{ij}\right\} \\ +\left[\tilde{s}_i\right]'^P\left[\tilde{\omega}_i\right]'\left\{s_i\right\}'^P + \left([A_i]\left[\tilde{s}_i\right]'^P\right)^T\left[A_j\right]\left[\tilde{\omega}_j\right]'\left\{s_j\right\}'^P\end{pmatrix}$$

$$\left\{\boldsymbol{\kappa}F_j\right\}_{SPH\_SPH} = 2\boldsymbol{\lambda}_{SPH\_SPH}\left\{\dot{d}_{ij}\right\}$$

$$\left\{\kappa T_j\right\}_{SPH\_SPH} = -2\lambda_{SPH\_SPH}\begin{pmatrix}\left([A_j]\left[\tilde{s}_j\right]'^P\right)^T\left\{\dot{d}_{ij}\right\} + \left([A_j]\left[\tilde{\omega}_j\right]'\left[\tilde{s}_j\right]'^P\right)^T\left\{d_{ij}\right\} \\ -\left[\tilde{s}_j\right]'^P\left[\tilde{\omega}_j\right]'\left\{s_j\right\}'^P - \left([A_j]\left[\tilde{s}_j\right]'^P\right)^T\left[A_i\right]\left[\tilde{\omega}_i\right]'\left\{s_i\right\}'^P\end{pmatrix}$$

Dot-1

$$\Phi_{DOT\_1} = \left\{a_i\right\}^T\left\{a_j\right\} = 0$$

for $\left\{a_i\right\} = \left\{r_i\right\}^Q - \left\{r_i\right\}^P$ and $\left\{a_j\right\} = \left\{r_j\right\}^Q - \left\{r_j\right\}^P$

$$\left[\Phi_{ri}\right]_{DOT\_1} = \left[0_{1x3}\right]$$

$$\left[\Phi_{\pi' i}\right]_{DOT\_1} = -\left\{a_j\right\}'^T\left[A_j\right]^T\left[A_i\right]\left[\tilde{a}_i\right]' = -\left\{a_j\right\}^T\left[A_i\right]\left[\tilde{a}_i\right]'$$

$$\left[\Phi_{pi}\right]_{DOT\_1} = -2\left\{a_j\right\}'^T\left[A_j\right]^T\left[A_i\right]\left[\tilde{a}_i\right]'\left[G_i\right] = -2\left\{a_j\right\}^T\left[A_i\right]\left[\tilde{a}_i\right]'\left[G_i\right]$$

$$\left[\Phi_{rj}\right]_{DOT\_1} = \left[0_{1x3}\right]$$

$$\left[\Phi_{\pi' j}\right]_{DOT\_1} = -\left\{a_i\right\}'^T\left[A_i\right]^T\left[A_j\right]\left[\tilde{a}_j\right]' = -\left\{a_i\right\}^T\left[A_j\right]\left[\tilde{a}_j\right]'$$

$$\left[\Phi_{pj}\right]_{DOT\_1} = -2\left\{a_i\right\}'^T\left[A_i\right]^T\left[A_j\right]\left[\tilde{a}_j\right]'\left[G_j\right] = -2\left\{a_i\right\}^T\left[A_j\right]\left[\tilde{a}_j\right]'\left[G_j\right]$$

$$\nu_{DOT\_1} = 0$$

$$\gamma_{DOT\_1} = -\left\{a_j\right\}'^T\left(\left[A_j\right]^T\left[A_i\right]\left[\tilde{\omega}_i\right]'\left[\tilde{\omega}_i\right]' + 2\left[\tilde{\omega}_j\right]'^T\left[A_j\right]^T\left[A_i\right]\left[\tilde{\omega}_i\right]' + \left[\tilde{\omega}_j\right]'\left[\tilde{\omega}_j\right]'\left[A_j\right]^T\left[A_i\right]\right)\left\{a_i\right\}'$$

$$\begin{aligned}\eta_{DOT\_1} = &-\{a_j\}'^T[A_j]^T[A_i][H_i]'\{a_i\}' - \{a_i\}'^T[A_i]^T[A_j][H_j]'\{a_j\}' \\ &-3\{a_j\}'^T(\,[\tilde{\omega}_j]'[\tilde{\omega}_j]' - [\dot{\tilde{\omega}}_j]'\,)[A_j]^T[A_i][\tilde{\omega}_i]'\{a_i\}' \qquad \left\{\ddot{\omega}\right\}' \text{ version} \\ &-3\{a_i\}'^T(\,[\tilde{\omega}_i]'[\tilde{\omega}_i]' - [\dot{\tilde{\omega}}_i]'\,)[A_i]^T[A_j][\tilde{\omega}_j]'\{a_j\}'\end{aligned}$$

$$\begin{aligned}\eta_{DOT\_1\_p} = \eta_{DOT\_1} &+ \tfrac{1}{4}\left\{a_j\right\}'^T[A_j]^T[A_i]\left([H_i]' - 3\left[\dot{\tilde{\omega}}_i\right]'\left[\tilde{\omega}_i\right]'\right)\{a_i\}' \qquad \left\{\ddot{p}\right\} \text{ version} \\ &+ \tfrac{1}{4}\{a_i\}'^T[A_i]^T[A_j]\left([H_j]' - 3\left[\dot{\tilde{\omega}}_j\right]'\left[\tilde{\omega}_j\right]'\right)\{a_j\}'\end{aligned}$$

$$\begin{aligned}\sigma_{DOT\_1} = &-\{a_j\}'^T[A_j]^T[A_i][W_i]'\{a_i\}' - \{a_i\}'^T[A_i]^T[A_j][W_j]'\{a_j\}' \\ &+4\{a_j\}'^T[\tilde{\omega}_j]'[A_j]^T[A_i]\left([H_i]' + [\ddot{\tilde{\omega}}_i]'\right)\{a_i\}' \\ &+4\{a_i\}'^T[\tilde{\omega}_i]'[A_i]^T[A_j]\left([H_j]' + [\ddot{\tilde{\omega}}_j]'\right)\{a_j\}' \qquad \left\{\ddot{\omega}\right\}' \text{ version} \\ &-6\{a_j\}'^T(\,[\tilde{\omega}_j]'[\tilde{\omega}_j]' - [\dot{\tilde{\omega}}_j]'\,)[A_j]^T[A_i](\,[\tilde{\omega}_i]'[\tilde{\omega}_i]' + [\dot{\tilde{\omega}}_i]'\,)\{a_i\}'\end{aligned}$$

$$\left\{\boldsymbol{\kappa}F_i\right\}_{DOT\_1} = \left\{0_{3x1}\right\}$$

$$\left\{\kappa T_i\right\}_{DOT\_1} = -\lambda_{DOT\_1}\left(\left(\left[A_i\right]\left[\tilde{a}_i\right]'\right)^T\left[A_j\right]\left[\tilde{\omega}_j\right]'\left\{a_j\right\}' + \left(\left[A_i\right]\left[\tilde{\omega}_i\right]'\left[\tilde{a}_i\right]'\right)^T\left[A_j\right]\left\{a_j\right\}'\right)$$

$$\left\{\boldsymbol{\kappa}F_j\right\}_{DOT\_1} = \left\{0_{3x1}\right\}$$

$$\left\{\kappa T_j\right\}_{DOT\_1} = -\lambda_{DOT\_1}\left(\left(\left[A_j\right]\left[\tilde{a}_j\right]'\right)^T\left[A_i\right]\left[\tilde{\omega}_i\right]'\left\{a_i\right\}' + \left(\left[A_j\right]\left[\tilde{\omega}_j\right]'\left[\tilde{a}_j\right]'\right)^T\left[A_i\right]\left\{a_i\right\}'\right)$$

Dot-2

$$\Phi_{DOT\_2}=\{a_i\}^T\{d_{ij}\}=0$$

for $\{d_{ij}\}=\{r_j\}^P-\{r_i\}^P$ and $\{\dot{d}_{ij}\}=\{\dot{r}_j\}^P-\{\dot{r}_i\}^P$ and $\{a_i\}=\{r_i\}^Q-\{r_i\}^P$

$$[\Phi_{ri}]_{DOT\_2}=-\{a_i\}'^T[A_i]^T=-\{a_i\}^T$$

$$[\Phi_{\pi'i}]_{DOT\_2}=\{a_i\}'^T[\tilde{s}_i]'^P-\{d_{ij}\}^T[A_i][\tilde{a}_i]'$$

$$[\Phi_{pi}]_{DOT\_2}=2\left(\{a_i\}'^T[\tilde{s}_i]'^P-\{d_{ij}\}^T[A_i][\tilde{a}_i]'\right)[G_i]$$

$$[\Phi_{rj}]_{DOT\_2}=\{a_i\}'^T[A_i]^T=\{a_i\}^T$$

$$[\Phi_{\pi'j}]_{DOT\_2}=-\{a_i\}'^T[A_i]^T[A_j][\tilde{s}_j]'^P=-\{a_i\}^T[A_j][\tilde{s}_j]'^P$$

$$[\Phi_{pj}]_{DOT\_2}=-2\{a_i\}'^T[A_i]^T[A_j][\tilde{s}_j]'^P[G_j]=-2\{a_i\}^T[A_j][\tilde{s}_j]'^P[G_j]$$

$$\nu_{DOT\_2}=0$$

$$\gamma_{DOT\_2}=-2\{\dot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'\{a_i\}'-\{d_{ij}\}^T[A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{a_i\}'+\{a_i\}'^T[A_i]^T\{\gamma\}_{SPH}$$

$$\eta_{DOT\_2}=-3\{\ddot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'\{a_i\}'-3\{\dot{d}_{ij}\}^T[A_i]\left([\tilde{\omega}_i]'[\tilde{\omega}_i]'+[\dot{\tilde{\omega}}_i]'\right)\{a_i\}' -\{d_{ij}\}^T[A_i][H_i]'\{a_i\}'+\{a_i\}'^T[A_i]^T\{\eta\}_{SPH}$$

$\{\ddot{\omega}\}'$ version

$$\eta_{DOT\_2\_p}=\eta_{DOT\_2}-\tfrac{1}{4}\{a_i\}'^T\left([H_i]'-3[\dot{\tilde{\omega}}_i]'[\tilde{\omega}_i]'\right)\{s_i\}'^P +\tfrac{1}{4}\{d_{ij}\}^T[A_i]\left([H_i]'-3[\dot{\tilde{\omega}}_i]'[\tilde{\omega}_i]'\right)\{a_i\}' +\tfrac{1}{4}\{a_i\}'^T[A_i]^T[A_j]\left([H_j]'-3[\dot{\tilde{\omega}}_j]'[\tilde{\omega}_j]'\right)\{s_j\}'^P$$

$\{\ddot{p}\}$ version

$$\sigma_{DOT\_2}=-4\{\dddot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'\{a_i\}'-6\{\ddot{d}_{ij}\}^T[A_i]\left([\tilde{\omega}_i]'[\tilde{\omega}_i]'+[\dot{\tilde{\omega}}_i]'\right)\{a_i\}' -4\{\dot{d}_{ij}\}^T[A_i]\left([H_i]'+[\ddot{\tilde{\omega}}_i]'\right)\{a_i\}'-\{d_{ij}\}'^T[A_i][W_i]'\{a_i\}' +\{a_i\}'^T[A_i]^T\{\sigma\}_{SPH}$$

$\{\dddot{\omega}\}'$ version

$$\{\kappa F_i\}_{DOT\_2}=-\lambda_{DOT\_2}[A_i][\tilde{\omega}_i]'\{a_i\}'$$

$$\{\kappa T_i\}_{DOT\_2}=-\lambda_{DOT\_2}\begin{pmatrix}\left([A_i][\tilde{a}_i]'\right)^T\{\dot{d}_{ij}\}+\left([A_i][\tilde{\omega}_i]'[\tilde{a}_i]'\right)^T\{d_{ij}\}\\ +[\tilde{a}_i]'[\tilde{\omega}_i]'\{s_i\}'^P+\left([A_i][\tilde{a}_i]'\right)^T[A_j][\tilde{\omega}_j]'\{s_j\}'^P\end{pmatrix}$$

$$\{\kappa F_j\}_{DOT\_2} = \lambda_{DOT\_2}[A_i][\tilde{\omega}_i]'\{a_i\}'$$

$$\{\kappa T_j\}_{DOT\_2} = -\lambda_{DOT\_2}\left(\left([A_j][\tilde{s}_j]'^P\right)^T[A_i][\tilde{\omega}_i]'\{a_i\}' + \left([A_j][\tilde{\omega}_j]'[\tilde{s}_j]'^P\right)^T[A_i]\{a_i\}'\right)$$

Euler parameters

$$\Phi_{EUL} = \{p_i\}^T\{p_i\} - 1 = 0$$

$$[\Phi_{ri}]_{EUL} = [0_{1x3}]$$
$$[\Phi_{pi}]_{EUL} = 2\{p_i\}^T$$

$$\nu_{EUL} = 0$$

$$\gamma_{EUL} = -2\{\dot{p}_i\}^T\{\dot{p}_i\}$$

$$\eta_{EUL} = -6\{\ddot{p}_i\}^T\{\dot{p}_i\}$$ $\{\dddot{p}\}$ version

Fixed revolute rotation driver (angle θ about fixed axis [u v w]^T)

angle θ about fixed axis $\{\hat{u}\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix}$ $\qquad \{p\} = \begin{Bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \begin{Bmatrix} C\frac{\theta}{2} \\ uS\frac{\theta}{2} \\ vS\frac{\theta}{2} \\ wS\frac{\theta}{2} \end{Bmatrix}$

$$\Phi_{FRRD} = \theta - f(t) = 0$$

recommend dot products to get θ four-quadrant

or $\theta = 2\cos^{-1}e_0$ or $\theta = 2\sin^{-1}\left(\sqrt{e_1^2+e_2^2+e_3^2}\right)$ or $\theta = 2\tan^{-1}\left(\frac{\sqrt{e_1^2+e_2^2+e_3^2}}{e_0}\right)$

$$[\Phi_{ri}]_{FRRD} = [0_{1x3}]$$
$$[\Phi_{\pi' i}]_{FRRD} = \{\hat{u}\}^T$$
$$[\Phi_{pi}]_{FRRD} = 2\{\hat{u}\}^T[G_i]$$

$\nu_{FRRD} = f_t$

$\gamma_{FRRD} = f_{tt}$

$\eta_{FRRD} = f_{ttt}$ $\{\ddot{\omega}\}'$ version

$\eta_{FRRD} = -\dfrac{\dot{\theta}^3}{4} + f_{ttt}$ $\{\ddot{p}\}$ version

$\{\kappa F_i\}_{FRRD} = \{0_{3x1}\}$

$\{\kappa T_i\}_{FRRD} = \{0_{3x1}\}$

3D relative distance driver (see double spherical)

$\Phi_{3DRDD} = \{d_{ij}\}^T \{d_{ij}\} - (f(t))^2 = 0$ $f(t) > 0$

for $\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$ and $\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$

$[\Phi_{qi}]_{3DRDD} = [\Phi_{qi}]_{SPH\_SPH}$

$[\Phi_{qj}]_{3DRDD} = [\Phi_{qj}]_{SPH\_SPH}$

$\nu_{3DRDD} = 2\ f\ f_t$

$\gamma_{3DRDD} = \gamma_{SPH\_SPH} + 2\ f_t^{\ 2} + 2\ f\ f_{tt}$

$\eta_{3DRDD} = \eta_{SPH\_SPH} + 6\ f_t\ f_{tt} + 2\ f\ f_{ttt}$ $\{\ddot{\omega}\}'$ version

Dot-2 distance driver (see dot-2)

$\Phi_{D2DD} = \{a_i\}^T \{d_{ij}\} / L\ -\ f(t) = 0$ $L = |a_i| =$ constant length

for $\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$ and $\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$ and $\{a_i\} = \{r_i\}^Q - \{r_i\}^P$

$$\left[\Phi_{qi}\right]_{D2DD} = \frac{\left[\Phi_{qi}\right]_{DOT\_2}}{L}$$

$$\left[\Phi_{qj}\right]_{D2DD} = \frac{\left[\Phi_{qj}\right]_{DOT\_2}}{L}$$

$$\nu_{D2DD} = f_t$$

$$\gamma_{D2DD} = \frac{\gamma_{DOT\_2}}{L} + f_{tt}$$

$$\eta_{D2DD} = \frac{\eta_{DOT\_2}}{L} + f_{ttt}$$ $\{\ddot{\omega}\}'$ version

General revolute rotation driver

$$\Phi_{GRRD} = \theta - f(t) = 0$$ $\{\ddot{\omega}\}'$ version

Screw distance-angle (see dot-2 and revolute rotation driver)

$\Phi_{SDA} = \{a_i\}^T \{d_{ij}\} / L - \lambda\ \theta = 0$ $L = |a_i|$ = constant length, λ = screw pitch, θ = screw angle

for $\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$ and $\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$ and $\{a_i\} = \{r_i\}^Q - \{r_i\}^P$

$$\left[\Phi_{ri}\right]_{SDA} = \frac{\left[\Phi_{ri}\right]_{DOT\_2}}{L}$$

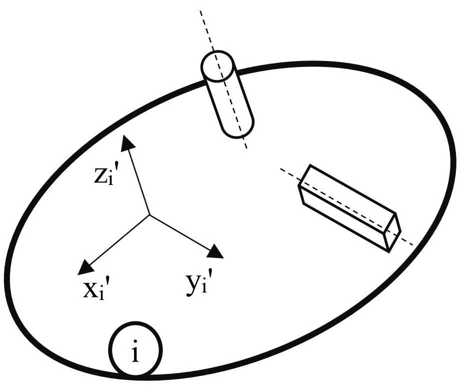
$$\left[\Phi_{\pi' i}\right]_{SDA} = \frac{\left[\Phi_{\pi' i}\right]_{DOT\_2}}{L} - \lambda \begin{bmatrix} u & v & w \end{bmatrix}$$

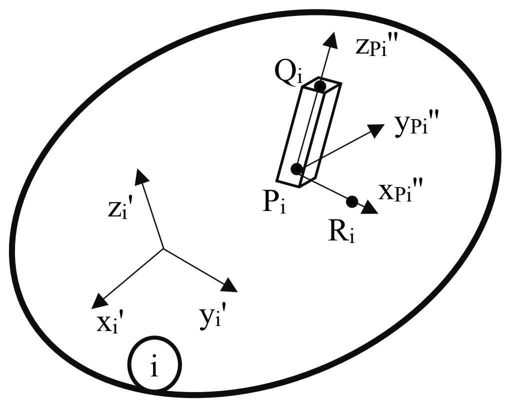
$$\left[\Phi_{pi}\right]_{SDA} = \frac{\left[\Phi_{pi}\right]_{DOT\_2}}{L} - \lambda \begin{bmatrix} -2\sqrt{1-e_0^2} & 2e_0 u & 2e_0 v & 2e_0 w \end{bmatrix}$$

Local Joint Definition Frames

do not need secondary joint frames for spherical (S) and double spherical (S-S)

need secondary joint frames if axes of constraints for universal U, revolute R, cylindrical C, prismatic P, or helical (screw) H **are not parallel** to local body-fixed direction





$$[C_i]'^P = \left[\left\{\hat{f}_i\right\}'^P \quad \{\hat{g}_i\}'^P \quad \left\{\hat{h}_i\right\}'^P \right]$$

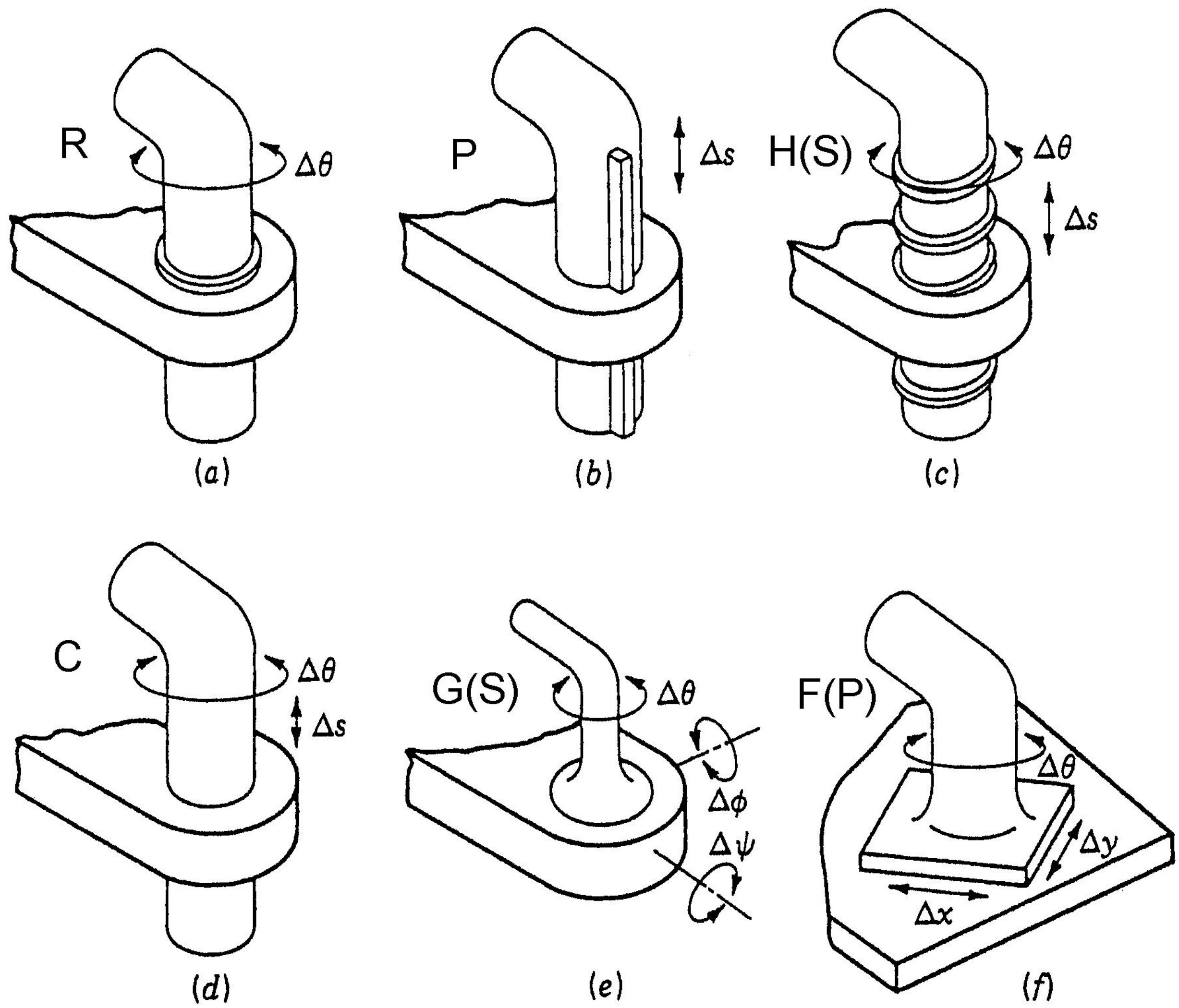
$$[C_i]'^P = \left[\text{unit}\left(\{s_i\}'^R - \{s_i\}'^P\right) \qquad \left\{\hat{h}_i\right\}'^P \times \left\{\hat{f}_i\right\}'^P \qquad \text{unit}\left(\{s_i\}'^Q - \{s_i\}'^P\right) \right]$$

$$[C_i]^P = \left[\left\{\hat{f}_i\right\}^P \quad \{\hat{g}_i\}^P \quad \left\{\hat{h}_i\right\}^P \right] = [A_i][C_i]'^P$$

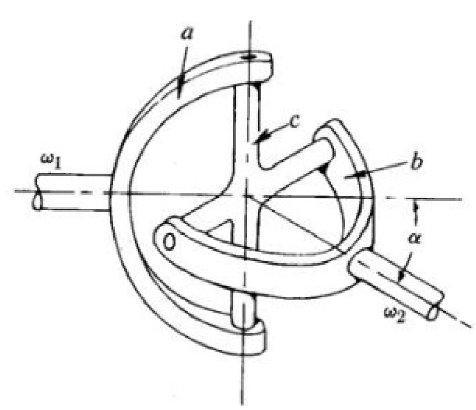
$$\left\{F_{\text{on i}}\right\}^P{}'' = \left([C_i]'^P\right)^T [A_i]^T \left\{F_{\text{on i}}\right\}^P$$

$$\left\{T_{\text{on i}}\right\}'' = \left([C_i]'^P\right)^T \left\{T_{\text{on i}}\right\}' \qquad \left\{T_{\text{on i}}\right\} = [A_i]\left\{T_{\text{on i}}\right\}'$$

Three-Dimensional Joint Constraints

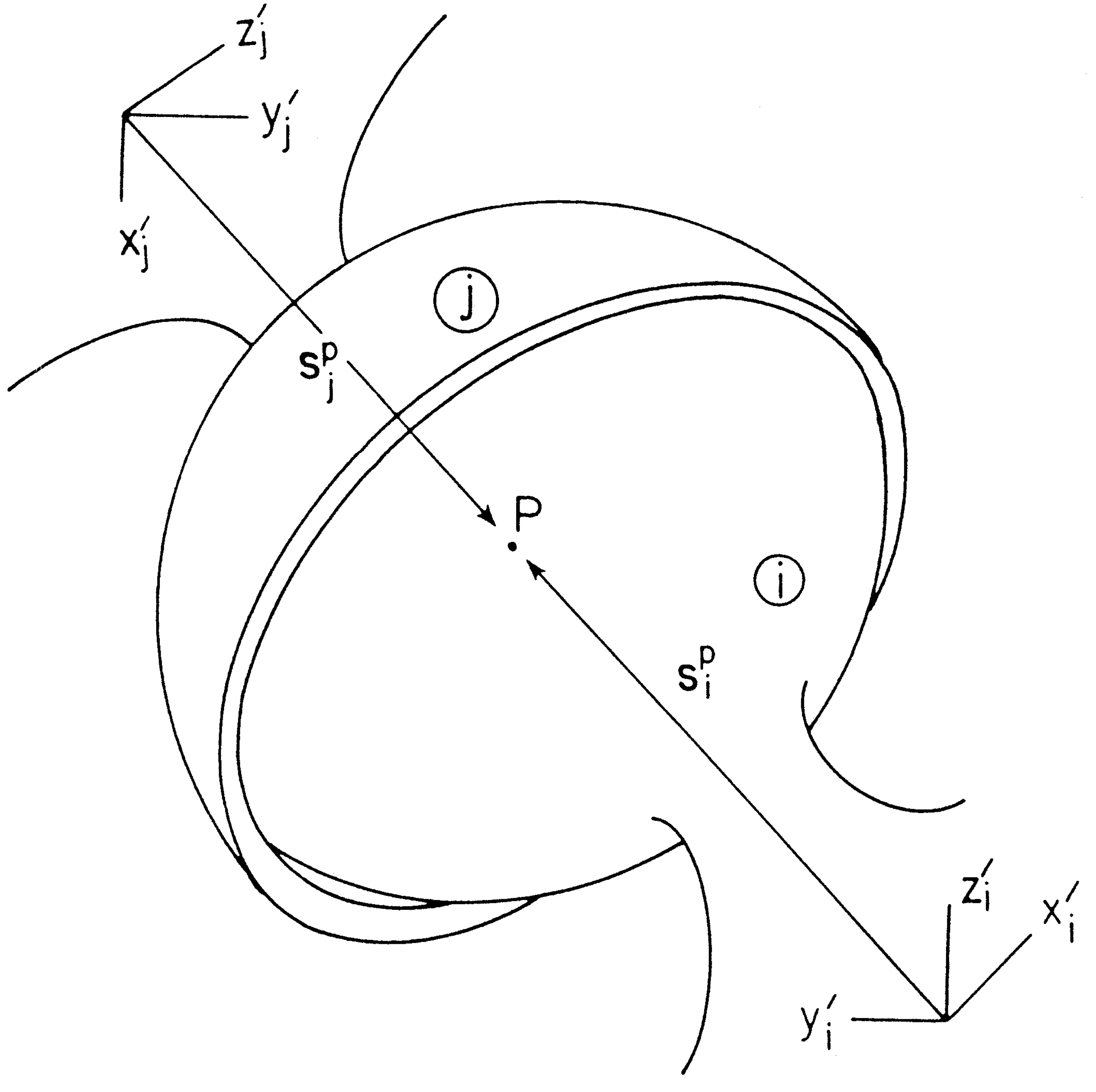


		Restricts	Allows
Revolute	R	5	1
Prismatic	P	5	1
Helical (Screw)	H(S)	5	1
Cylindrical	C	4	2
Universal (Hooke)	U(H)	4	2
Globe (Spherical)	G(S)	3	3
Flat (Planar)	F(P)	3	3

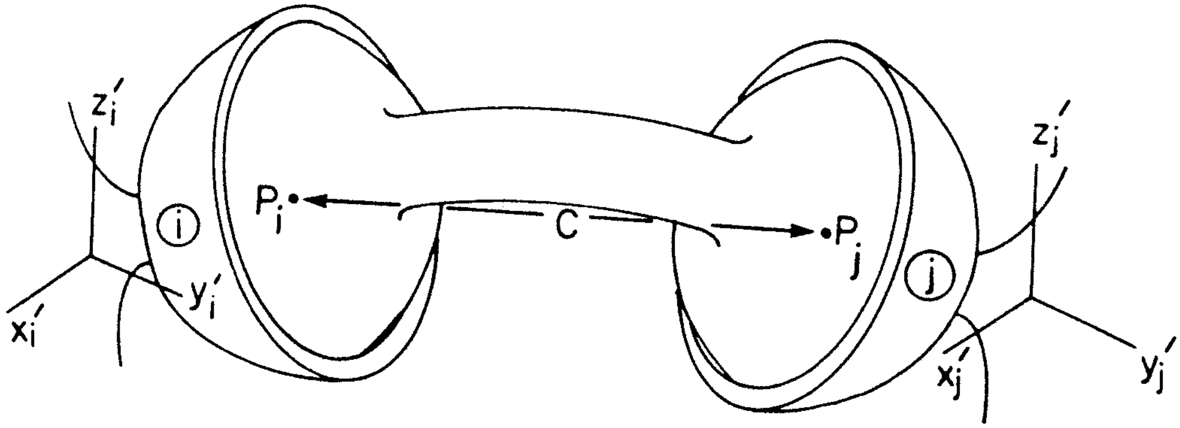


3D mobility

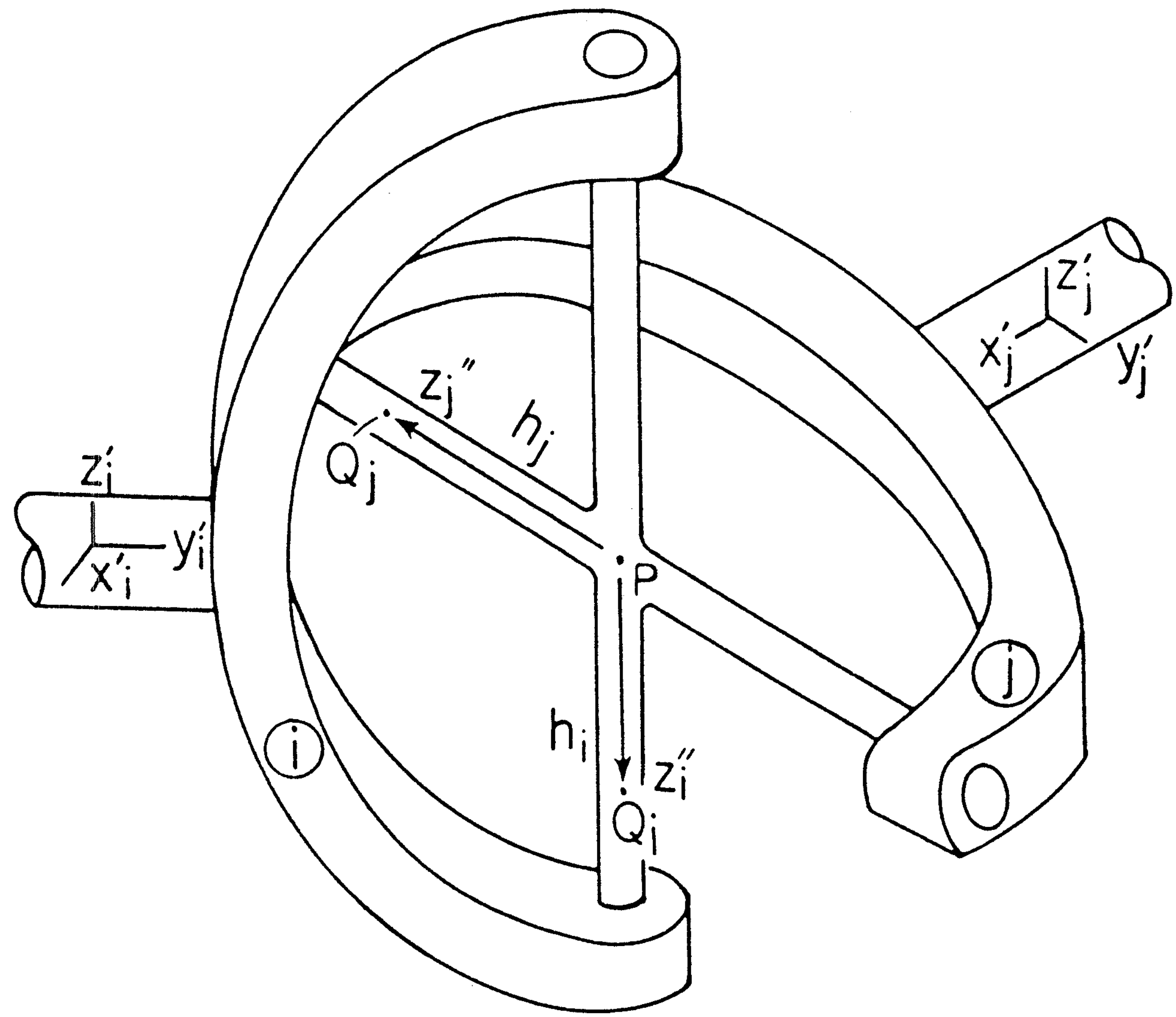
M = 6 nML - 5 nJ1 - 4 nJ2 - 3 nJ3



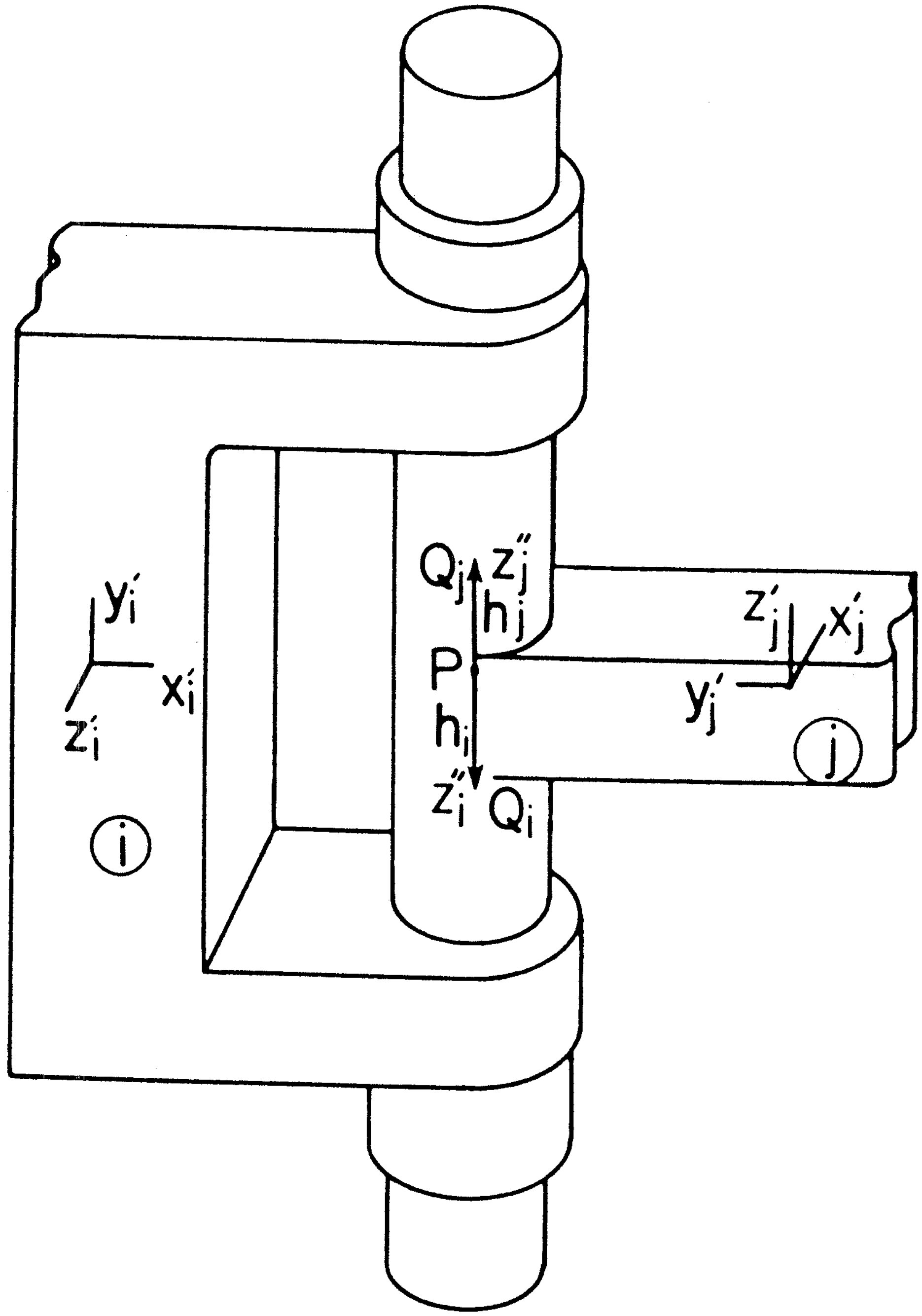
Spherical S (Globe G)



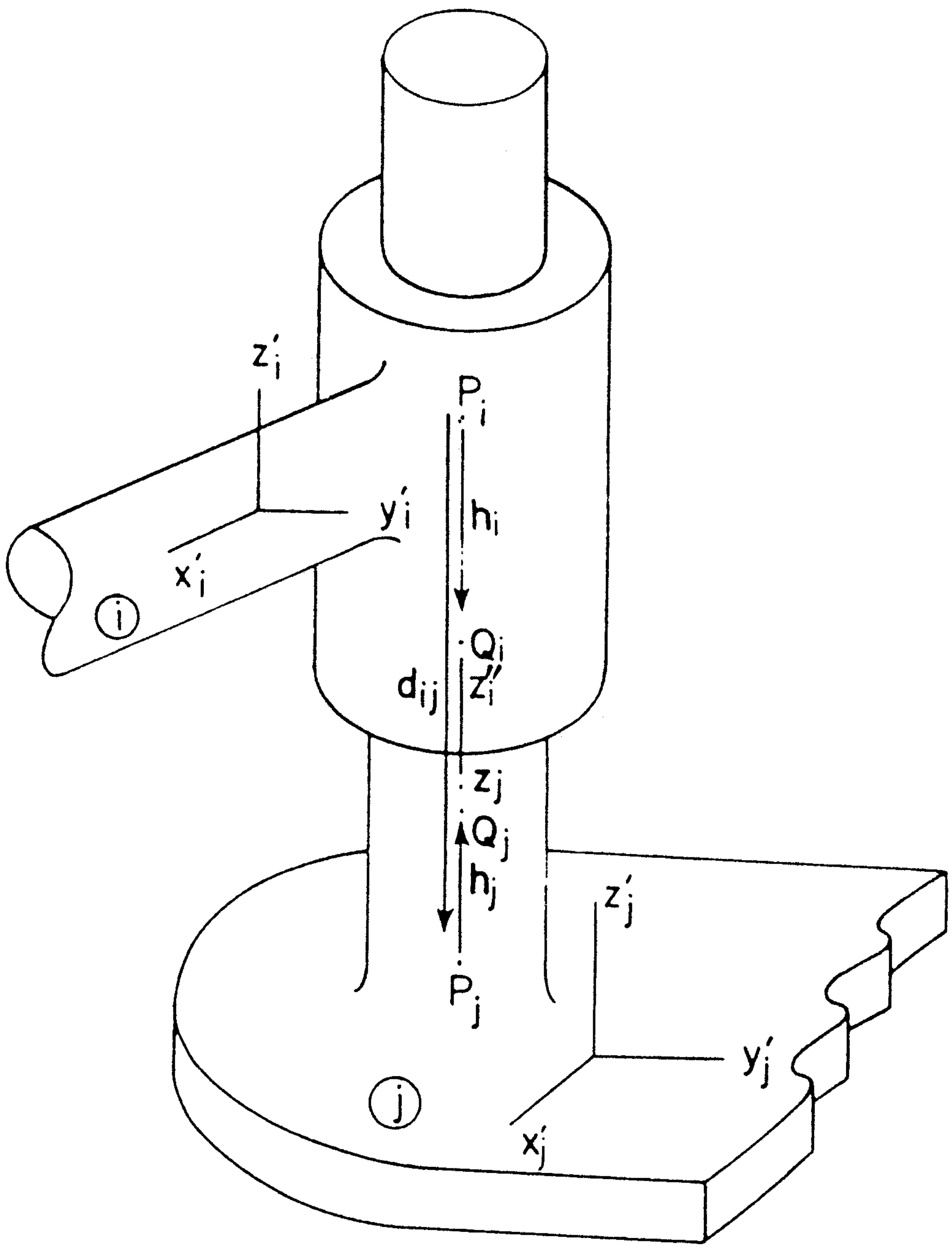
Double Spherical SS



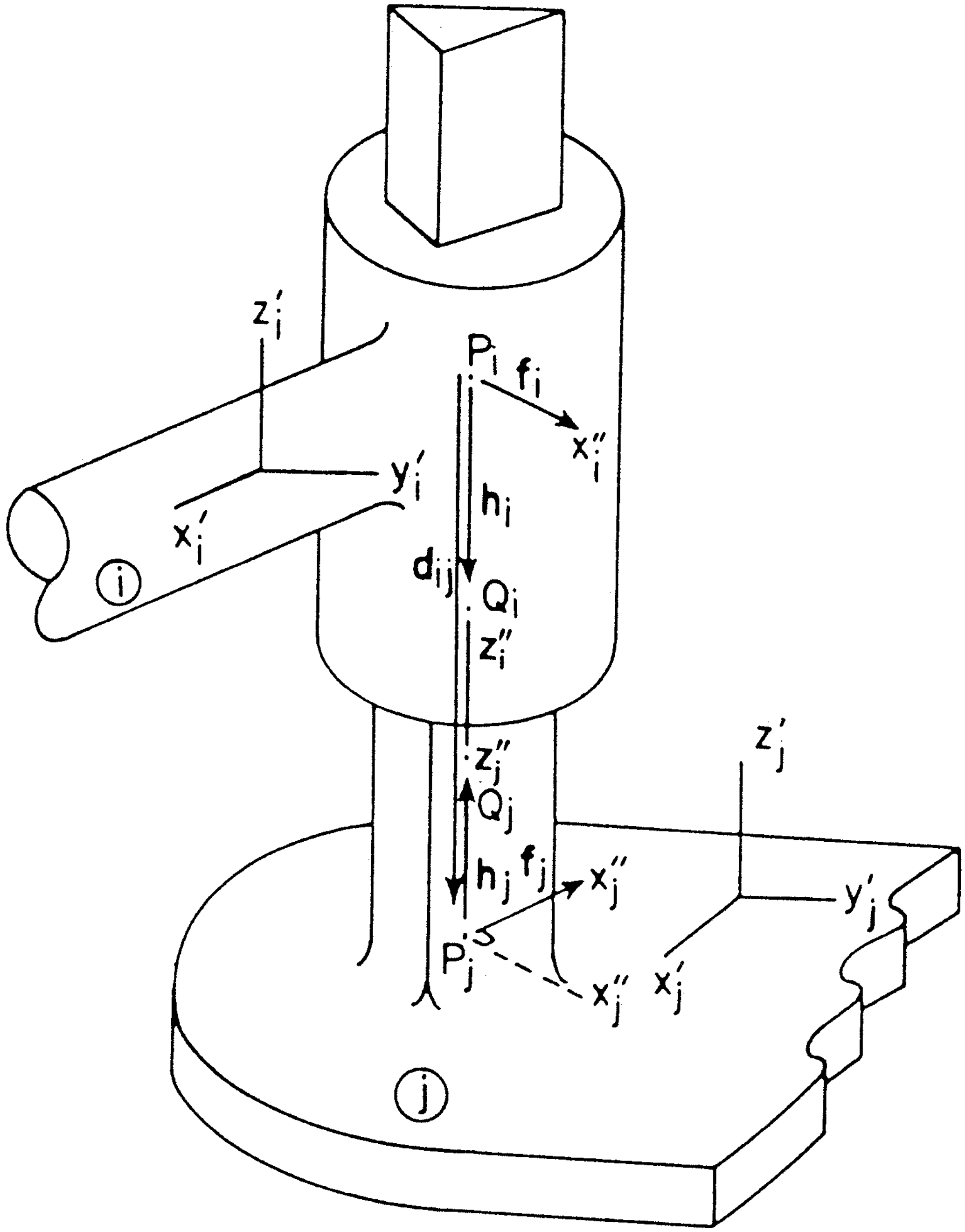
Universal U (Hooke H)



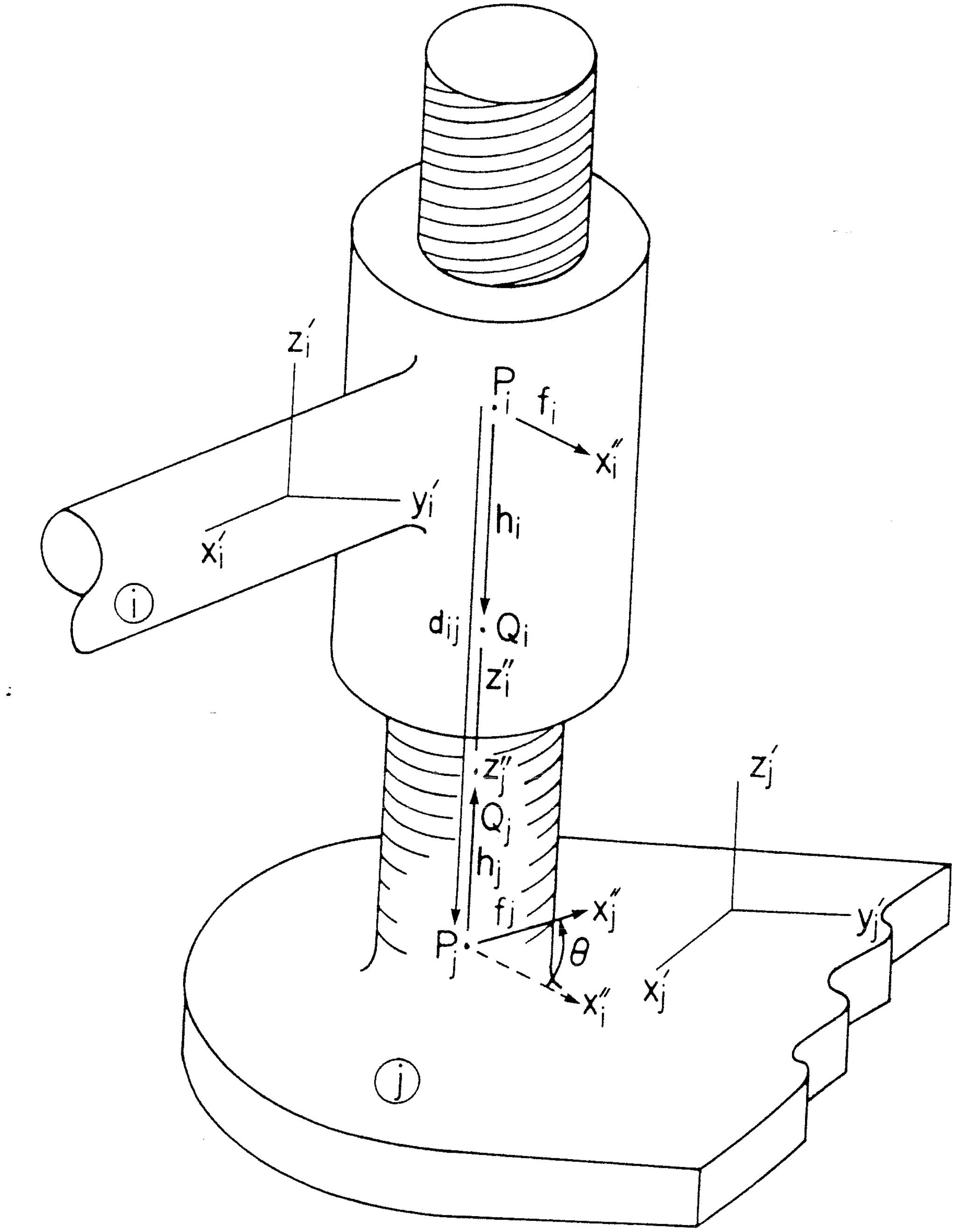
Revolute R



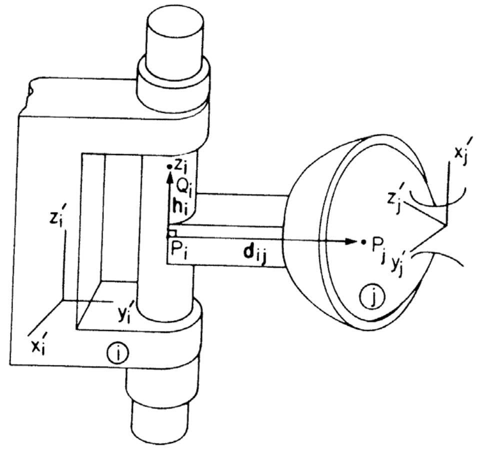
Cylindrical C



Prismatic P (Translational T)



Screw S (Helical H)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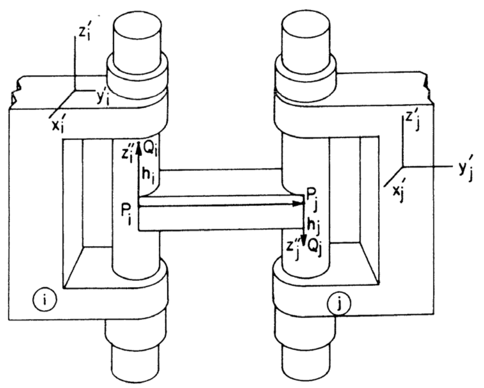
Q_i
h_i
z_i'
P_i
d_{ij}
P_j
x_j'
z_j'
y_j'
y_i'
x_i'
i
j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y_i'
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z_i''
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P_j
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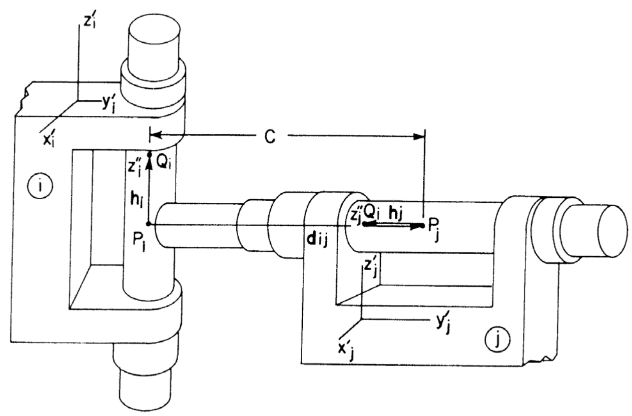
z_j'
y_j'
x_j'
i
j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x_i'
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Q_i
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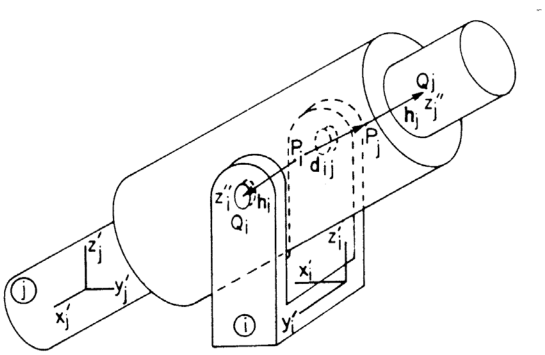
x_j'
i
j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h_j
P_j
P_i
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Q_i
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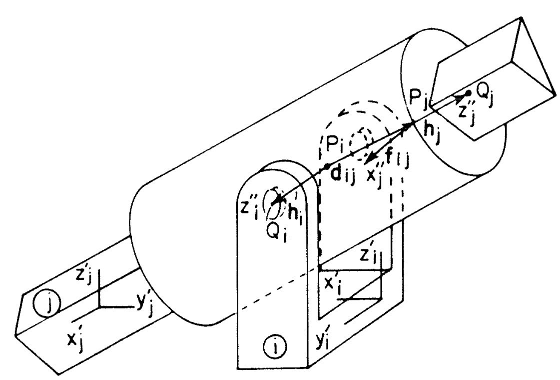
z_j'
y_j'
x_j'
j
i


P_j
Q_j
z_j''
h_j
P_i
f_{ij}
d_{ij}
x_j'
z_i''
h_i
Q_i
z_i'
x_i'
y_i'
z_j'
y_j'
x_j'
j
i