Three-Dimensional Position, Velocity, Acceleration Solutions

Position

given $\{r_i\}$ and $\{p_i\}$

$$[E_i] = \begin{bmatrix} -e_1 & e_0 & -e_3 & e_2 \\ -e_2 & e_3 & e_0 & -e_1 \\ -e_3 & -e_2 & e_1 & e_0 \end{bmatrix} \qquad [G_i] = \begin{bmatrix} -e_1 & e_0 & e_3 & -e_2 \\ -e_2 & -e_3 & e_0 & e_1 \\ -e_3 & e_2 & -e_1 & e_0 \end{bmatrix} \qquad [A_i] = [E_i][G_i]^T$$

$$\{r_i\}^P = \{r_i\} + [A_i]\{s_i\}'^P$$

Velocity

given $\{\dot{r}_i\}$ and $\{\dot{p}_i\}$

$$\{\omega_i\}' = 2[G_i]\{\dot{p}_i\} \qquad \{\dot{p}_i\} = \tfrac{1}{2}[G_i]^T\{\omega_i\}'$$

$$\{\dot{r}_i\}^P = \{\dot{r}_i\} + [A_i][\tilde{\omega}_i]'\{s_i\}'^P$$

Partial derivatives

$$\lfloor *_{pi} \rfloor = 2\lfloor *_{\pi' i} \rfloor [G_i] \qquad \lfloor *_{\pi' i} \rfloor = \tfrac{1}{2}\lfloor *_{pi} \rfloor [G_i]^T$$

Acceleration

given $\{\ddot{r}_i\}$ and $\{\ddot{p}_i\}$

$$\{\dot{\omega}_i\}' = 2[G_i]\{\ddot{p}_i\} \qquad \{\ddot{p}_i\} = \tfrac{1}{2}[G_i]^T\{\dot{\omega}_i\}' - \tfrac{1}{4}\{p_i\}\{\omega_i\}'^T\{\omega_i\}'$$

$$\{\ddot{r}_i\}^P = \{\ddot{r}_i\} + [A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{s_i\}'^P + [A_i][\dot{\tilde{\omega}}_i]'\{s_i\}'^P$$

Kinematically driven motion

1.0 Initialize

1.1 constants

1.2 global locations of fixed points $\{r_1\}^P$

1.3 local blueprint locations $\{s_i\}'^P$

2.0 Initial estimates

2.1 global locations of origins $\{r_i\}$

2.2 Euler parameters $\{p_i\}$

2.3 assemble into $\{q\} = \begin{Bmatrix} \{r_2\} \\ \{p_2\} \\ \{r_3\} \\ \{p_3\} \\ \vdots \end{Bmatrix}$

3.0 Explicit time loop

4.0 Position solution

4.1 rip new values for $\{r_i\}$ and $\{p_i\}$ from $\{q\}_k$

4.2 form attitude information $[A_i]$ $[G_i]$ $\{\hat{f}_i\}$ $\{\hat{g}_i\}$ $\{\hat{h}_i\}$

4.3 compute global locations for all points $\{r_i\}^P$

4.4 extract θ for fixed revolute rotation drvier from $[A_i]$ $[A_j]$

4.5 evaluate constraints $\{\Phi\}_{KINEMATIC}$ $\{\Phi\}_{EULER}$ $\{\Phi\}_{DRIVER}$

4.6 assemble all position constraints $\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{EULER} \\ \{\Phi\}_{DRIVER} \end{Bmatrix}$

4.7 evaluate partial derivatives $\begin{matrix} [\Phi_{ri}]_{KINEMATIC} & [\Phi_{ri}]_{EULER} & [\Phi_{ri}]_{DRIVER} \\ [\Phi_{pi}]_{KINEMATIC} & [\Phi_{pi}]_{EULER} & [\Phi_{pi}]_{DRIVER} \end{matrix}$

4.8 assemble Jacobian

$$[\Phi_q] = \begin{bmatrix} [\Phi_{r2}]_{KINEMATIC} & [\Phi_{p2}]_{KINEMATIC} & [\Phi_{r3}]_{KINEMATIC} & [\Phi_{p3}]_{KINEMATIC} & \cdots \\ [\Phi_{r2}]_{EULER} & [\Phi_{p2}]_{EULER} & [\Phi_{r3}]_{EULER} & [\Phi_{p3}]_{EULER} & \cdots \\ [\Phi_{r2}]_{DRIVER} & [\Phi_{p2}]_{DRIVER} & [\Phi_{r3}]_{DRIVER} & [\Phi_{p3}]_{DRIVER} & \cdots \end{bmatrix}$$

4.9 Newton-Raphson update $\{q\}_{k+1} = \{q\}_k - [\Phi_q]^{-1}\{\Phi\}$

4.10 check convergence - repeat 4.1 through 4.9 if needed

5.0 Euler parameter velocity solution

5.1 evaluate $\{\nu\}_{KINEMATIC} = \{0\} \qquad \{\nu\}_{EULER} = \{0\} \qquad \{\nu\}_{DRIVER} = -\{\Phi_t\}_{DRIVER}$

5.2 assemble $\{\nu\} = \begin{Bmatrix} \{\nu\}_{KINEMATIC} \\ \{\nu\}_{EULER} \\ \{\nu\}_{DRIVER} \end{Bmatrix}$

5.3 compute $\{\dot{q}\} = [\Phi_q]^{-1}\{\nu\}$ for $\{\dot{q}\} = \begin{Bmatrix} \{\dot{r}_2\} \\ \{\dot{p}_2\} \\ \{\dot{r}_3\} \\ \{\dot{p}_3\} \\ \vdots \end{Bmatrix}$

5.4 determine $\{\omega_i\}' = 2[G_i]\{\dot{p}_i\}$ and form $[\tilde{\omega}_i]'$

6.0 Euler parameter acceleration solution

6.1 evaluate $\{\gamma\}_{KINEMATIC} \qquad \{\gamma\}_{EULER} \qquad \{\gamma\}_{DRIVER}$

6.2 assemble $\{\gamma\} = \begin{Bmatrix} \{\gamma\}_{KINEMATIC} \\ \{\gamma\}_{EULER} \\ \{\gamma\}_{DRIVER} \end{Bmatrix}$

6.3 compute $\{\ddot{q}\} = \left[\Phi_q\right]^{-1}\{\gamma\}$ for $\{\ddot{q}\} = \begin{Bmatrix} \{\ddot{r}_2\} \\ \{\ddot{p}_2\} \\ \{\ddot{r}_3\} \\ \{\ddot{p}_3\} \\ \vdots \end{Bmatrix}$

6.4 determine $\{\dot{\omega}_i\}' = 2[G_i]\{\ddot{p}_i\}$ and form $[\tilde{\dot{\omega}}_i]'$

7.0 ALTERNATE - Local angular velocity solution

7.1 use same $\{\nu\}_{KINEMATIC} = \{0\} \qquad \{\nu\}_{DRIVER}$

7.2 assemble $\{\nu\} = \begin{Bmatrix} \{\nu\}_{KINEMATIC} \\ \{\nu\}_{DRIVER} \end{Bmatrix}$ (do not need $\{\nu\}_{EULER}$)

7.3 use same $\begin{matrix} [\Phi_{ri}]_{KINEMATIC} & [\Phi_{ri}]_{DRIVER} \\ [\Phi_{pi}]_{KINEMATIC} & [\Phi_{pi}]_{DRIVER} \end{matrix}$

7.4 assemble $[\Phi_r] = \begin{bmatrix} [\Phi_{r2}]_{KINEMATIC} & [\Phi_{r3}]_{KINEMATIC} & \cdots \\ [\Phi_{r2}]_{DRIVER} & [\Phi_{r3}]_{DRIVER} & \cdots \end{bmatrix}$ for $\{\dot{r}\} = \begin{Bmatrix} \{\dot{r}_2\} \\ \{\dot{r}_3\} \\ \vdots \end{Bmatrix}$

7.5 compute π' derivatives $\begin{matrix} [\Phi_{\pi' i}]_{KINEMATIC} = \frac{1}{2}[\Phi_{pi}]_{KINEMATIC}[G_i]^T \\ [\Phi_{\pi' i}]_{DRIVER} = \frac{1}{2}[\Phi_{pi}]_{DRIVER}[G_i]^T \end{matrix}$

7.6 assemble $[\Phi_{\pi'}] = \begin{bmatrix} [\Phi_{\pi' 2}]_{KINEMATIC} & [\Phi_{\pi' 3}]_{KINEMATIC} & \cdots \\ [\Phi_{\pi' 2}]_{DRIVER} & [\Phi_{\pi' 3}]_{DRIVER} & \cdots \end{bmatrix}$ for $\{\omega\}' = \begin{Bmatrix} \{\omega_2\}' \\ \{\omega_3\}' \\ \vdots \end{Bmatrix}$

7.7 use $\begin{bmatrix} [\Phi_r] & [\Phi_{\pi'}] \end{bmatrix} \begin{Bmatrix} \{\dot{r}\} \\ \{\omega\}' \end{Bmatrix} = \{\nu\}$

7.8 compute $\begin{Bmatrix} \{\dot{r}\} \\ \{\omega\}' \end{Bmatrix} = \begin{bmatrix} [\Phi_r] & [\Phi_{\pi'}] \end{bmatrix}^{-1} \{\nu\}$

8.0 ALTERNATE - Local angular acceleration solution

8.1 use same $\{\gamma\}_{KINEMATIC} \qquad \{\gamma\}_{DRIVER}$

8.2 assemble $\{\gamma\} = \begin{Bmatrix} \{\gamma\}_{KINEMATIC} \\ \{\gamma\}_{DRIVER} \end{Bmatrix}$ (do not need $\{\gamma\}_{EULER}$)

8.3 use $\{\gamma\} = \begin{bmatrix} [\Phi_r] & [\Phi_{\pi'}] \end{bmatrix} \begin{Bmatrix} \{\ddot{r}\} \\ \{\dot{\omega}\}' \end{Bmatrix}$ for $\{\ddot{r}\} = \begin{Bmatrix} \{\ddot{r}_2\} \\ \{\ddot{r}_3\} \\ \vdots \end{Bmatrix} \qquad \{\dot{\omega}\}' = \begin{Bmatrix} \{\dot{\omega}_2\}' \\ \{\dot{\omega}_3\} \\ \vdots \end{Bmatrix}$

8.4 compute $\begin{Bmatrix} \{\ddot{r}\} \\ \{\dot{\omega}\}' \end{Bmatrix} = \begin{bmatrix} [\Phi_r] & [\Phi_{\pi'}] \end{bmatrix}^{-1} \{\gamma\}$