Three-Dimensional Generalized Forces

Moment about local origin

$$\{T\} = \{s_i\}^P \times \{F_{on\ i}\}^P = [\tilde{s}_i]^P \{F_{on\ i}\}^P$$ global directions

$$\{T\}' = \{s_i\}'^P \times \{F_{on\ i}\}'^P = [\tilde{s}_i]'^P \{F_{on\ i}\}'^P = [\tilde{s}_i]'^P [A_i]^T \{F_{on\ i}\}^P$$ local directions

Generalized force on body i about local origin

$$\{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\} \\ \{T_{on\ i}\}' \end{Bmatrix}$$ note that force is in global directions and torque is in local directions

Pure force

$$\{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\}^P \\ [\tilde{s}_i]'^P [A_i]^T \{F_{on\ i}\}^P \end{Bmatrix}$$

Pure moment

$$\{Q_{on\ i}\} = \begin{Bmatrix} \{0_{3x1}\} \\ \{T_{on\ i}\}' \end{Bmatrix}$$

Translational spring-damper-actuator often with cylindrical or prismatic

$$\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$$

$$\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$$

$$\{\ddot{d}_{ij}\} = \{\ddot{r}_j\}^P - \{\ddot{r}_i\}^P$$

$$\ell^2 = \{d_{ij}\}^T \{d_{ij}\}$$

$$2\ell\dot{\ell} = 2\{d_{ij}\}^T \{\dot{d}_{ij}\}$$

$$\dot{\ell} = \{d_{ij}\}^T \{\dot{d}_{ij}\} / \ell$$

$$f = k(\ell - \ell_o) + c\,\dot{\ell} + f_F \mathrm{sign}(\dot{\ell}) + f_{ACT}(\ell, \dot{\ell}, t)$$

$$\{F_{on\ i}\} = \frac{f}{\ell}\{d_{ij}\} \qquad \{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\} \\ [\tilde{s}_i]'^P [A_i]^T \{F_{on\ i}\} \end{Bmatrix}$$

$$\{F_{on\ j}\} = -\frac{f}{\ell}\{d_{ij}\} \qquad \{Q_{on\ j}\} = \begin{Bmatrix} \{F_{on\ j}\} \\ [\tilde{s}_j]'^P [A_j]^T \{F_{on\ j}\} \end{Bmatrix}$$

$$\ddot{\ell} = \left(\{d_{ij}\}^T \{\ddot{d}_{ij}\} + \{\dot{d}_{ij}\}^T \{\dot{d}_{ij}\} - \dot{\ell}^2\right) / \ell$$

$$\dot{f} = k\dot{\ell} + c\,\ddot{\ell} + \dot{f}_{ACT}(\ell, \dot{\ell}, \ddot{\ell}, t)$$

$$\{\dot{F}_{on\ i}\} = \left(\dot{f}\ \ell\{d_{ij}\} + f\ \ell\{\dot{d}_{ij}\} - f\ \dot{\ell}\{d_{ij}\}\right) / \ell^2$$

$$\{\dot{Q}_{on\ i}\} = \begin{Bmatrix} \{\dot{F}_{on\ i}\} \\ [\tilde{s}_i]'^P [A_i]^T \{\dot{F}_{on\ i}\} - [\tilde{s}_i]'^P [\tilde{\omega}_i]' [A_i]^T \{F_{on\ i}\} \end{Bmatrix}$$

$$\{\dot{F}_{on\ j}\} = -\left(\dot{f}\ \ell\{d_{ij}\} + f\ \ell\{\dot{d}_{ij}\} - f\ \dot{\ell}\{d_{ij}\}\right) / \ell^2$$

$$\left\{\dot{Q}_{on\,j}\right\} = \left\{ \begin{matrix} \left\{\dot{F}_{on\,j}\right\} \\ \left[\tilde{s}_j\right]^{\prime P}\left[A_j\right]^T\left\{\dot{F}_{on\,j}\right\} - \left[\tilde{s}_j\right]^{\prime P}\left[\tilde{\omega}_j\right]^{\prime}\left[A_j\right]^T\left\{F_{on\,j}\right\} \end{matrix} \right\}$$

Rotational spring-damper-actuator about revolute, cylindrical or screw joint

θ measured +CCW about $\left\{\hat{h}_i\right\}^P$ along axis of rotation

$$\dot{\theta} = \left(\left\{\hat{h}_i\right\}^P\right)^T\left(\left\{\omega_j\right\} - \left\{\omega_i\right\}\right) = \left(\left\{\hat{h}_i\right\}^P\right)^T\left(\left[A_j\right]\left\{\omega_j\right\}^{\prime} - \left[A_i\right]\left\{\omega_i\right\}^{\prime}\right)$$

$$T = k_\theta\left(\theta - \theta_o\right) + c_\theta\,\dot{\theta} + T_F \operatorname{sign}\left(\dot{\theta}\right) + T_{ACT}\left(\theta, \dot{\theta}, t\right)$$

$$\left\{Q_{on\,i}\right\} = \left\{ \begin{matrix} \left\{0_{3x1}\right\} \\ T\left[A_i\right]^T\left\{\hat{h}_i\right\}^P \end{matrix} \right\} = \left\{ \begin{matrix} \left\{0_{3x1}\right\} \\ T\left\{\hat{h}_i\right\}^{\prime P} \end{matrix} \right\}$$

$$\left\{Q_{on\,j}\right\} = -\left\{ \begin{matrix} \left\{0_{3x1}\right\} \\ T\left[A_j\right]^T\left\{\hat{h}_i\right\}^P \end{matrix} \right\}$$

use $\left[A_j\right]$ but not $\left\{\hat{h}_j\right\}^P$ just in case $\left\{\hat{h}_j\right\}^P = -\left\{\hat{h}_i\right\}^P$

$$\ddot{\theta} = \frac{d}{dt}\left(\left(\left\{\hat{h}_i\right\}^P\right)^T\left(\left\{\omega_j\right\} - \left\{\omega_i\right\}\right)\right) = \frac{d}{dt}\left(\left(\left\{\hat{h}_i\right\}^P\right)^T\left(\left[A_j\right]\left\{\omega_j\right\}^{\prime} - \left[A_i\right]\left\{\omega_i\right\}^{\prime}\right)\right)$$

$$\dot{T} = k_\theta\dot{\theta} + c_\theta\,\ddot{\theta} + T_{ACT}\left(\theta, \dot{\theta}, \ddot{\theta}, t\right)$$

$$\left\{\dot{Q}_{on\,i}\right\} =$$

$$\left\{\dot{Q}_{on\,j}\right\} =$$

Axial Coulomb friction in cylindrical or prismatic joints

1) Start with coefficient of friction $\mu = 0$

2) Use $\left\{\hat{h}_i\right\}^{'P}$ along axis of translation

3) Compute constraint force $\{\lambda\}$ and find all components of $\left\{F_{on\,i}\right\}^{'P}$ in local directions

4) Determine normal force $F_{NORMAL} = \sqrt{\left(F_X{}^{'P}\right)^2 + \left(F_Y{}^{'P}\right)^2}$

5) Compute $f_F = \mu\, F_{NORMAL}$

6) repeat steps 3) through 5) until convergence by relaxation

Torsional Coulomb friction in revolute or cylindrical joints

1) Start with coefficients of friction $\mu = 0$ and $\mu_{END} = 0$

2) Use $\left\{\hat{h}_i\right\}^{'P}$ along axis of rotation

3) Compute constraint force $\{\lambda\}$ and find components all of $\left\{F_{on\,i}\right\}^{'P}$ in local directions

4) Determine radial force $F_{RADIAL} = \sqrt{\left(F_X{}^{'P}\right)^2 + \left(F_Y{}^{'P}\right)^2}$

5) Compute $T_F = \mu\, R\, F_{RADIAL}$ for cylindrical joint

6) Compute $T_F = \mu\, R\, F_{RADIAL} + \mu_{END}\, R_{END}\, \mathrm{abs}\left(F_Z{}^{'P}\right)$ for revolute joint

7) repeat steps 3) through 6) until convergence by relaxation

Torsional Coulomb friction in spherical joint

1) Start with coefficient of friction $\mu = 0$

2) Use $\{\hat{u}\} = \text{unit}(\{\omega_j\} - \{\omega_i\})$ for axis of rotation (note global directions)

3) Compute constraint force $\{\lambda\}$ and find $\{F_{on\_i}\}_{TOTAL}$ (note global directions)

4) Determine axial force $F_{AXIAL} = \{\hat{u}\}^T \{F_{on\_i}\}_{TOTAL}$

5) Determine radial force $F_{RADIAL} = \sqrt{(\text{norm}\{F_{on\_i}\}_{TOTAL})^2 - F_{AXIAL}{}^2}$

6) Compute $\{T_{F\,on\_i}\} = \mu\, R\, (F_{RADIAL} + \alpha F_{AXIAL})\{\hat{u}\}$ for $\alpha \le 1$ (note global directions)

7) $\{T_{F\,on\_j}\} = -\{T_{F\,on\_i}\}$

8) repeat steps 2) through 5) until convergence by relaxation