

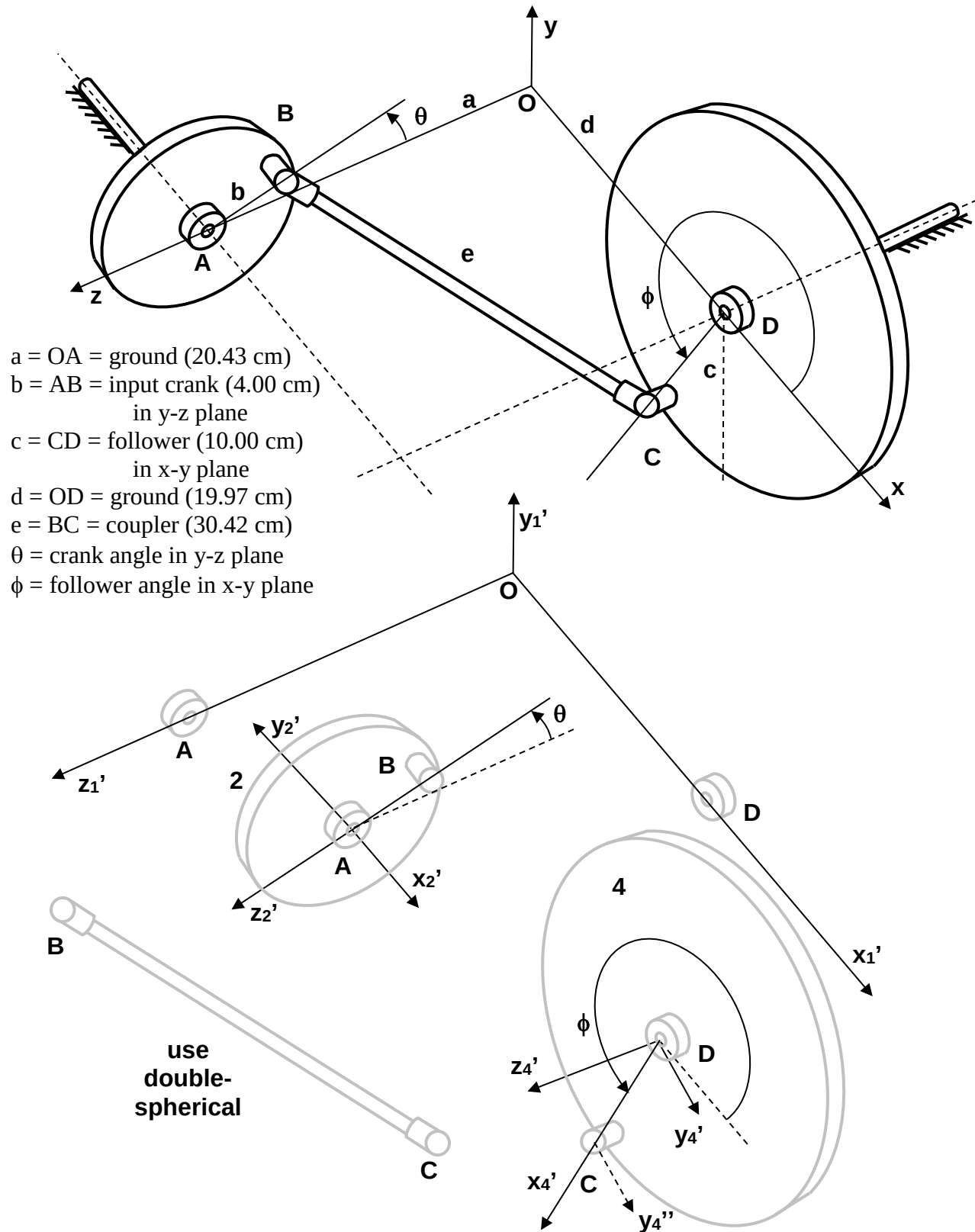
RSSR Generalized Coordinates

A = revolute R

B = spherical S

C = spherical S

D = revolute R



CONSTRAINTS

$$\{\mathbf{q}\}_{14 \times 1} = \begin{Bmatrix} \{\mathbf{r}_2\} \\ \{\mathbf{p}_2\} \\ \{\mathbf{r}_4\} \\ \{\mathbf{p}_4\} \end{Bmatrix} \quad \{\Phi\}_{14 \times 1} = \left\{ \begin{array}{lll} \begin{array}{l} \{\mathbf{r}_1\}^A - \{\mathbf{r}_2\}^A \\ \{\hat{\mathbf{g}}_2\}^T \{\hat{\mathbf{f}}_1\} \\ \{\hat{\mathbf{h}}_2\}^T \{\hat{\mathbf{f}}_1\} \end{array} & \begin{array}{l} \text{revA} \\ y_2' \perp x_1' \\ z_2' \perp x_1' \end{array} & \begin{array}{l} A, r_1 - r_2, i = 2, j = 1 \\ g_2 f_1, i = 2, j = 1 \\ h_2 f_1, i = 2, j = 1 \end{array} \\ \\ \begin{array}{l} \{\mathbf{d}_{ij}\} = \{\mathbf{r}_4\}^C - \{\mathbf{r}_2\}^B \quad \{\mathbf{d}_{ij}\}^T \{\mathbf{d}_{ij}\} - L^2 \\ \{\mathbf{r}_1\}^D - \{\mathbf{r}_4\}^D \\ \{\hat{\mathbf{f}}_4\}^T \{\hat{\mathbf{h}}_1\} \\ \{\hat{\mathbf{g}}_4\}^T \{\hat{\mathbf{h}}_1\} \end{array} & \begin{array}{l} \text{double - sph BC} \\ \text{revD} \\ x_4' \perp z_1' \\ y_4' \perp z_1' \end{array} & \begin{array}{l} i = B2, j = C4 \\ D, r_1 - r_4, i = 4, j = 1 \\ f_4 h_1, i = 4, j = 1 \\ g_4 h_1, i = 4, j = 1 \end{array} \\ \\ \begin{array}{l} \{\mathbf{p}_2\}^T \{\mathbf{p}_2\} - 1 \\ \{\mathbf{p}_4\}^T \{\mathbf{p}_4\} - 1 \end{array} & \begin{array}{l} \text{Euler parameters} \\ \text{Euler parameters} \end{array} & \begin{array}{l} p_2 \\ p_4 \end{array} \\ \\ \theta - C - f(t) & \text{driver} & \text{driver} \end{array} \right\}$$

CONSTANTS

$$\{\mathbf{r}_1\}^A = \begin{Bmatrix} 0 \\ 0 \\ a \end{Bmatrix} \quad \{\mathbf{r}_1\}^D = \begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix}$$

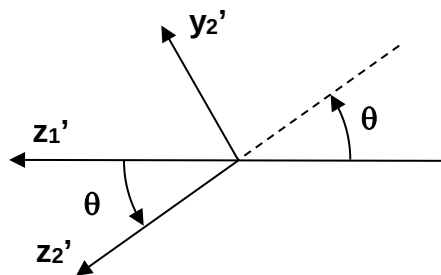
$$\{\mathbf{s}_2\}'^A = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{\mathbf{s}_2\}'^B = \begin{Bmatrix} 0 \\ 0 \\ -b \end{Bmatrix} \quad BC = e \quad \{\mathbf{s}_4\}'^C = \begin{Bmatrix} c \\ 0 \\ 0 \end{Bmatrix} \quad \{\mathbf{s}_4\}'^D = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}$$

INITIAL ESTIMATES

$$\{\mathbf{r}_2\} = \begin{Bmatrix} 0 \\ 0 \\ a \end{Bmatrix} \quad \chi_2 = \theta = 0 \text{ deg} \quad \{\hat{\mathbf{u}}_2\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \quad \{\mathbf{p}_2\} = \begin{Bmatrix} \mathbf{e}_0 \\ \mathbf{e}_1 \\ \mathbf{e}_2 \\ \mathbf{e}_3 \end{Bmatrix} = \begin{Bmatrix} C \frac{\chi}{2} \\ u S \frac{\chi}{2} \\ v S \frac{\chi}{2} \\ w S \frac{\chi}{2} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{r_4\} = \begin{Bmatrix} d \\ 0 \\ 0 \end{Bmatrix} \quad \chi_4 = \phi = 270 \text{ deg} \quad \{\hat{u}_4\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} \quad \{p_4\} = \begin{Bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \begin{Bmatrix} C \frac{\chi}{2} \\ uS \frac{\chi}{2} \\ vS \frac{\chi}{2} \\ wS \frac{\chi}{2} \end{Bmatrix} = \begin{Bmatrix} -0.7071 \\ 0 \\ 0 \\ 0.7071 \end{Bmatrix}$$

FIXED REVOLUTE DRIVER

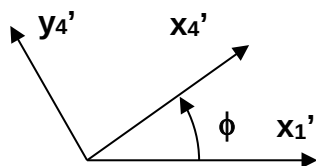


$$\{\hat{h}_1\}^T \{\hat{h}_2\} = C\theta$$

$$\{\hat{h}_1\}^T \{\hat{g}_2\} = S\theta$$

$$\theta = \arctan 2 \left(\frac{\{\hat{h}_1\}^T \{\hat{g}_2\}}{\{\hat{h}_1\}^T \{\hat{h}_2\}} \right)$$

OUTPUT LINK



$$\{\hat{f}_1\}^T \{\hat{f}_4\} = C\phi$$

$$\{\hat{f}_1\}^T \{\hat{g}_4\} = C(\phi + 90^\circ) = -S\phi$$

$$\phi = \arctan 2 \left(\frac{-\{\hat{f}_1\}^T \{\hat{g}_4\}}{\{\hat{f}_1\}^T \{\hat{f}_4\}} \right)$$

JACOBIAN

$-[I_3]$	$2[A_2][\tilde{s}_2]'^A[G_2]$	$0_{3 \times 3}$	$0_{3 \times 4}$	$A, r_1 - r_2, i = 2, j = 1$
$0_{1 \times 3}$	$-2\{f_1\}'^T[A_1]^T[A_2][\tilde{g}_2]'[G_2]$	$0_{1 \times 3}$	$0_{1 \times 4}$	$g_2 f_1, i = 2, j = 1$
$0_{1 \times 3}$	$-2\{f_1\}'^T[A_1]^T[A_2][\tilde{h}_2]'[G_2]$	$0_{1 \times 3}$	$0_{1 \times 4}$	$h_2 f_1, i = 2, j = 1$
$-2\{d_{ij}\}^T$	$4\{d_{ij}\}^T[A_2][\tilde{s}_2]'^B[G_2]$	$2\{d_{ij}\}^T$	$-4\{d_{ij}\}^T[A_4][\tilde{s}_4]'^C[G_4]$	$i = B2, j = C4$
$0_{3 \times 3}$	$0_{3 \times 4}$	$-[I_3]$	$2[A_4][\tilde{s}_4]'^D[G_4]$	$D, r_1 - r_4, i = 4, j = 1$
$0_{1 \times 3}$	$0_{1 \times 4}$	$0_{1 \times 3}$	$-2\{h_1\}'^T[A_1]^T[A_4][\tilde{f}_4]'[G_4]$	$f_4 h_1, i = 4, j = 1$
$0_{1 \times 3}$	$0_{1 \times 4}$	$0_{1 \times 3}$	$-2\{h_1\}'^T[A_1]^T[A_4][\tilde{g}_4]'[G_4]$	$g_4 h_1, i = 4, j = 1$
$0_{1 \times 3}$	$2\{p_2\}^T$	$0_{1 \times 3}$	$0_{1 \times 4}$	p_2
$0_{1 \times 3}$	$0_{1 \times 4}$	$0_{1 \times 3}$	$2\{p_4\}^T$	p_4
$0_{1 \times 3}$	$-2\sin \frac{\theta}{2} \quad 2e_0 u \quad 2e_0 v \quad 2e_0 w$	$0_{1 \times 3}$	$0_{1 \times 4}$	driver

$$\frac{\partial}{\partial \{p_i\}} = 2 \left(\frac{\partial}{\partial \{\pi_i\}'} \right) [G_i]$$

VELOCITY

$$\{v\} = \left\{ \begin{array}{l} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ +f_t \end{array} \right\} \left\{ \begin{array}{l} A, r_1 - r_2, i = 2, j = 1 \\ g_2 f_1, i = 2, j = 1 \\ h_2 f_1, i = 2, j = 1 \\ i = B2, j = C4 \\ D, r_1 - r_4, i = 4, j = 1 \\ f_4 h_1, i = 4, j = 1 \\ g_4 h_1, i = 4, j = 1 \\ p_2 \\ p_4 \\ \text{driver} \end{array} \right.$$

ACCELERATION

$$\{\gamma\} = \left\{ \begin{array}{l} [A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]'\{s_2\}'^A \\ -\{f_1\}'^T ([A_1]^T [A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{g_2\}' \\ -\{f_1\}'^T ([A_1]^T [A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{h_2\}' \\ \\ -2\{\dot{d}_{ij}\}^T \{\dot{d}_{ij}\} + 2\{d_{ij}\}^T \{\gamma^s\} \\ \\ [A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{s_4\}'^D \\ -\{h_1\}'^T ([A_1]^T [A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]')\{f_4\}' \\ -\{h_1\}'^T ([A_1]^T [A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]')\{g_4\}' \\ \\ -2\{\dot{p}_2\}^T \{\dot{p}_2\} \\ -2\{\dot{p}_4\}^T \{\dot{p}_4\} \\ \\ + f_u \end{array} \right\} \begin{array}{l} A, r_1 - r_2, i = 2, j = 1 \\ g_2 f_1, i = 2, j = 1 \\ h_2 f_1, i = 2, j = 1 \\ \\ i = B2, j = C4 \\ \\ D, r_1 - r_4, i = 4, j = 1 \\ f_4 h_1, i = 4, j = 1 \\ g_4 h_1, i = 4, j = 1 \\ \\ p_2 \\ p_4 \\ \\ \text{driver} \end{array}$$

$$\text{dot-1} \quad \{\Phi\} = \{\hat{a}_i\}^T \{\hat{a}_j\} \quad \gamma = -\{a_j\}'^T ([A_j]^T [A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]' + 2[\tilde{\omega}_j]'^T [A_j]^T [A_i][\tilde{\omega}_i]' + [\tilde{\omega}_j]'[\tilde{\omega}_j]'[A_j]^T [A_i]) \{a_i\}'$$

$$\text{sphere} \quad \{\Phi\} = \{r_j\}^p - \{r_i\}^p \quad \gamma = [A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{s_i\}'^p - [A_j][\tilde{\omega}_j]'[\tilde{\omega}_j]'\{s_j\}'^p$$

$$\text{double-spherical} \quad \{d_{ij}\} = \{r_j\}^p - \{r_i\}^p \quad \{\Phi\} = \{d_{ij}\}^T \{d_{ij}\} - L^2 \quad \gamma = -2\{\dot{d}_{ij}\}^T \{\dot{d}_{ij}\} + 2\{d_{ij}\}^T \{\gamma^s\}$$

JERK

$$\{\eta\} = \left\{ \begin{array}{l} [A_2][H_2]\{s_2\}'^A \\ -\{f_1\}'^T [A_1]^T [A_2][H_2]\{g_2\}' \\ -\{f_1\}'^T [A_1]^T [A_2][H_2]\{h_2\}' \\ -6\{\dot{d}_{ij}\}^T \{\ddot{d}_{ij}\} + 2\{d_{ij}\}^T \{\eta^s\} \\ [A_4][H_4]\{s_4\}'^D \\ -\{h_1\}'^T [A_1]^T [A_4][H_4]\{f_4\}' \\ -\{h_1\}'^T [A_1]^T [A_4][H_4]\{g_4\}' \\ -6\{\ddot{p}_2\}^T \{\dot{p}_2\} \\ -6\{\ddot{p}_4\}^T \{\dot{p}_4\} \\ -\dot{\theta}^3/4 + f_{ttt} \quad \text{for link 2} \end{array} \right\} \begin{array}{l} A, r_1 - r_2, i = 2, j = 1 \\ g_2 f_1, i = 2, j = 1 \\ h_2 f_1, i = 2, j = 1 \\ i = B2, j = C4 \\ D, r_1 - r_4, i = 4, j = 1 \\ f_4 h_1, i = 4, j = 1 \\ g_4 h_1, i = 4, j = 1 \\ p_2 \\ p_4 \\ \text{driver} \end{array}$$

$$\text{jerk product} \quad [H_i] = 2[\tilde{\omega}_i]'[\tilde{\dot{\omega}}_i]' + [\tilde{\dot{\omega}}_i]'[\tilde{\omega}_i]' + [\tilde{\omega}_i]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'$$

$$\text{dot} - 1 \quad \left(\begin{array}{l} \eta = -\{a_j\}'^T [A_j]^T [A_i][H_i]\{a_i\}' - \{a_i\}'^T [A_i]^T [A_j][H_j]\{a_j\}' \\ -3\{a_j\}'^T ([\tilde{\omega}_j]'[\tilde{\omega}_j]' - \tilde{\dot{\omega}}_j') [A_j]^T [A_i][\tilde{\omega}_i]'\{a_i\}' - 3\{a_i\}'^T ([\tilde{\omega}_i]'[\tilde{\omega}_i]' - \tilde{\dot{\omega}}_i') [A_i]^T [A_j][\tilde{\omega}_j]'\{a_j\}' \end{array} \right)$$

$$\text{sphere} \quad \eta = [A_i][H_i]\{s_i\}'^p - [A_j][H_j]\{s_j\}'^p$$

$$\text{double-spherical} \quad \{d_{ij}\} = \{r_j\}^p - \{r_i\}^p \quad \{\Phi\} = \{d_{ij}\}^T \{d_{ij}\} - L^2 \quad \eta = -6\{\dot{d}_{ij}\}^T \{\ddot{d}_{ij}\} + 2\{d_{ij}\}^T \{\eta^s\}$$

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