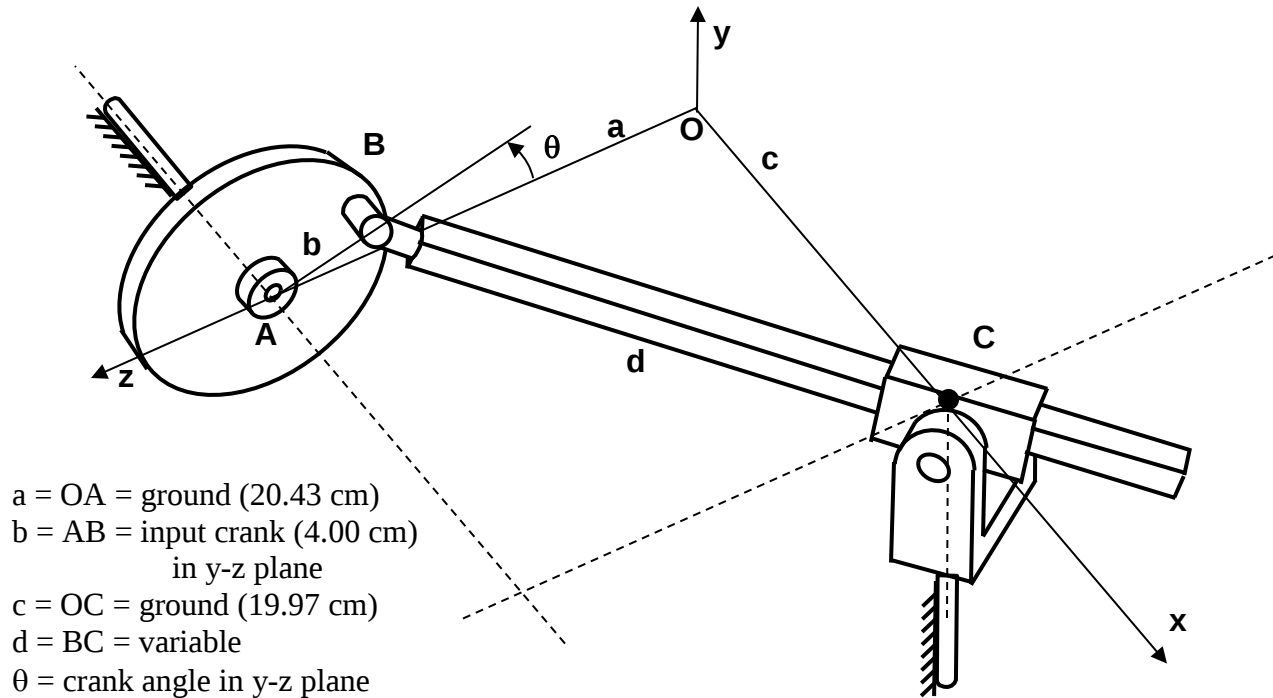


RSPU Geometric Method

A = revolute R B = spherical S C = prismatic P, universal U



Position solution - given θ , find d

$$x_B = 0 \quad y_B = bS\theta \quad z_B = a - bC\theta \quad x_C = c \quad y_C = 0 \quad z_C = 0$$

$$d^2 = (x_B - x_C)^2 + (y_B - y_C)^2 + (z_B - z_C)^2$$

$$d^2 = (-c)^2 + (bS\theta)^2 + (a - bC\theta)^2$$

$$d^2 = c^2 + b^2S^2\theta + a^2 - 2abC\theta + b^2C^2\theta = a^2 + b^2 + c^2 - 2abC\theta$$

$$d = \sqrt{a^2 + b^2 + c^2 - 2abC\theta}$$

Velocity solution - given $\dot{\theta}$, find $\dot{d}$

$$d/dt(d^2 = a^2 + b^2 + c^2 - 2abC\theta)$$

$$2d\dot{d} = 2ab\dot{\theta}S\theta$$

$$\dot{d} = ab\dot{\theta}S\theta/d$$

Acceleration solution - given $\ddot{\theta}$, find $\ddot{d}$

$$d/dt(\dot{d} = ab\dot{\theta}S\theta)$$

$$\ddot{d} + \dot{d}^2 = ab\ddot{\theta}S\theta + ab\dot{\theta}^2C\theta = ab(\ddot{\theta}S\theta + \dot{\theta}^2C\theta)$$

$$\ddot{d} = (ab(\ddot{\theta}S\theta + \dot{\theta}^2C\theta) - \dot{d}^2)/d$$

Jerk solution - given $\ddot{\theta}$, find $\ddot{d}$

$$d/dt(\ddot{d} + \dot{d}^2 = ab(\ddot{\theta}S\theta + \dot{\theta}^2C\theta))$$

$$\ddot{d} + \dot{d}\ddot{d} + 2\dot{d}\dot{d} = ab(\ddot{\theta}S\theta + \dot{\theta}\ddot{\theta}C\theta + 2\dot{\theta}\ddot{\theta}C\theta - \dot{\theta}^3S\theta)$$

$$\ddot{d} + 3\dot{d}\ddot{d} = ab((\ddot{\theta} - \dot{\theta}^3)S\theta + 3\dot{\theta}\ddot{\theta}C\theta)$$

$$\ddot{d} = (ab((\ddot{\theta} - \dot{\theta}^3)S\theta + 3\dot{\theta}\ddot{\theta}C\theta) - 3\dot{d}\ddot{d})/d$$

Snap solution - given $\ddot{\ddot{\theta}}$, find $\ddot{\ddot{d}}$

$$d/dt(\ddot{d} + 3\dot{d}\ddot{d} = ab((\ddot{\theta} - \dot{\theta}^3)S\theta + 3\dot{\theta}\ddot{\theta}C\theta))$$

$$\ddot{\ddot{d}} + \dot{\ddot{d}}\ddot{d} + 3\dot{d}\ddot{\ddot{d}} + 3\dot{d}\ddot{d} = ab((\ddot{\ddot{\theta}} - 3\dot{\theta}^2\ddot{\theta})S\theta + (\ddot{\theta}\ddot{\theta} - \dot{\theta}^4)C\theta + 3\ddot{\theta}^2C\theta + 3\dot{\theta}\ddot{\theta}C\theta - 3\dot{\theta}^2\ddot{\theta}S\theta)$$

$$\ddot{\ddot{d}} + 4\dot{d}\ddot{\ddot{d}} + 3\dot{d}^2 = ab((\ddot{\ddot{\theta}} - 6\dot{\theta}^2\ddot{\theta})S\theta + (4\ddot{\theta}\ddot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4)C\theta)$$

$$\ddot{\ddot{d}} = (ab((\ddot{\ddot{\theta}} - 6\dot{\theta}^2\ddot{\theta})S\theta + (4\ddot{\theta}\ddot{\theta} + 3\ddot{\theta}^2 - \dot{\theta}^4)C\theta) - 4\dot{d}\ddot{\ddot{d}} - 3\dot{d}^2)/d$$