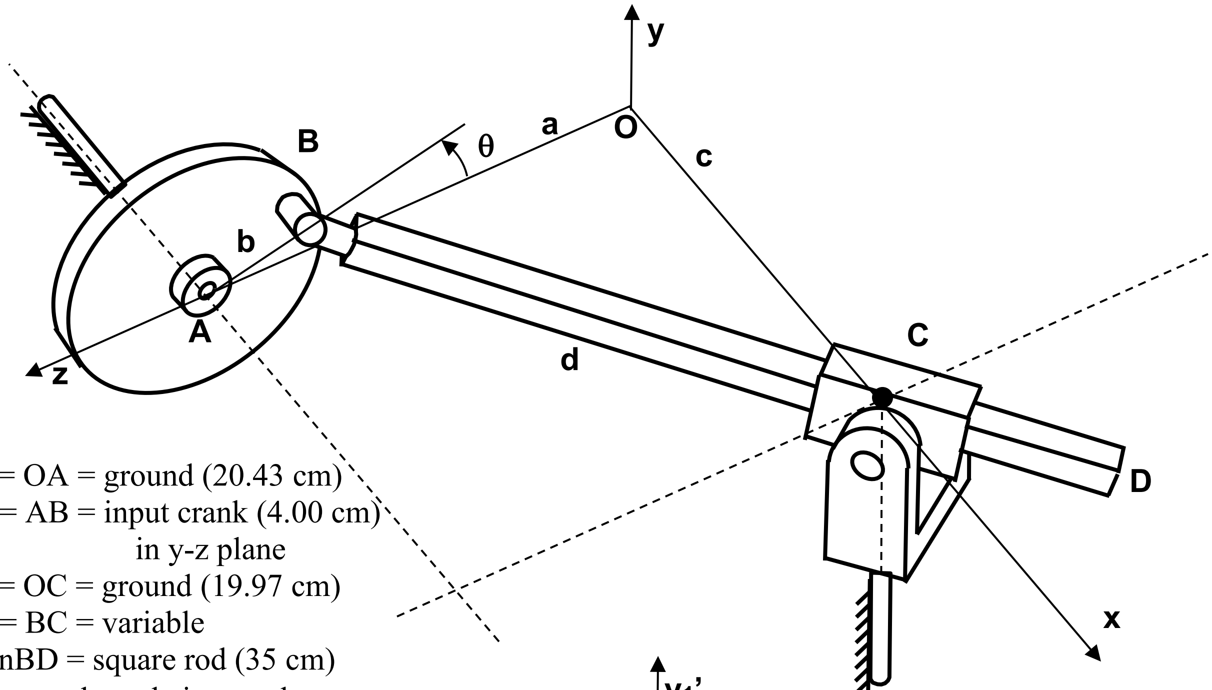
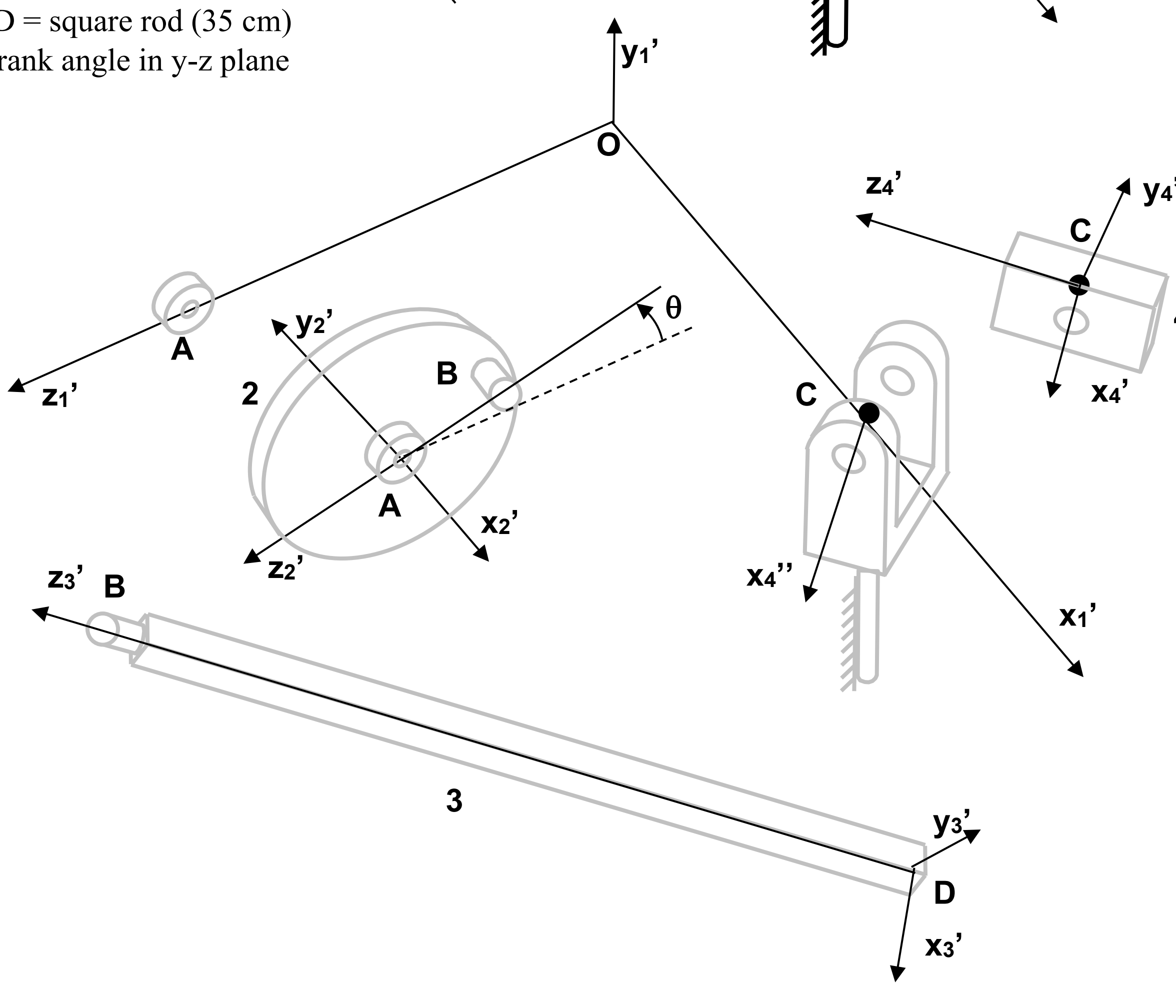
RSPU Generalized Coordinates

A = revolute R B = spherical S C = prismatic P, universal U



a = OA = ground (20.43 cm)
b = AB = input crank (4.00 cm)
in y-z plane
c = OC = ground (19.97 cm)
d = BC = variable
lenBD = square rod (35 cm)
θ = crank angle in y-z plane



CONSTRAINTS

$$
\underset{21x1}{\{q\}} = \begin{Bmatrix} \{r_2\} \\ \{p_2\} \\ \{r_3\} \\ \{p_3\} \\ \{r_4\} \\ \{p_4\} \end{Bmatrix}
\qquad
\underset{21x1}{\{\Phi\}} = \left\{ \begin{array}{c} \{r_1\}^A - \{r_2\}^A \\ \{\hat{g}_2\}^T\{\hat{f}_1\} \\ \{\hat{h}_2\}^T\{\hat{f}_1\} \\ \\ \{r_2\}^B - \{r_3\}^B \\ \\ \{\hat{f}_4\}^T\{\hat{h}_3\} \\ \{\hat{g}_4\}^T\{\hat{h}_3\} \\ \{d_{ij}\} = \{r_3\}^B - \{r_4\}^C \quad \{\hat{f}_4\}^T\{d_{ij}\} \\ \{d_{ij}\} = \{r_3\}^B - \{r_4\}^C \quad \{\hat{g}_4\}^T\{d_{ij}\} \\ \{\hat{f}_4\}^T\{\hat{g}_3\} \\ \\ \{r_1\}^C - \{r_4\}^C \\ \{\hat{f}_4\}^T\{\hat{g}_1\} \\ \\ \{p_2\}^T\{p_2\} - 1 \\ \{p_3\}^T\{p_3\} - 1 \\ \{p\ \ \}^T\{p_4\} - 1 \\ \\ \theta - C - f(t) \end{array} \right\}
\begin{array}{cc} \text{revA} & A, r_1 - r_2, i = A2, j = A1 \\ y_2' \perp x_1' & g_2 f_1, i = 2, j = 1 \\ z_2' \perp x_1' & h_2 f_1, i = 2, j = 1 \\ & \\ \text{sphB} & B, r_2 - r_3, i = B3, j = B2 \\ & \\ x_4' \perp z_3' & f_4 h_3, i = 4, j = 3 \\ y_4' \perp z_3' & g_4 h_3, i = 4, j = 3 \\ x_4' \perp d_{ij} & f_4 d_{ij}, i = C4, j = B3 \\ y_4' \perp d_{ij} & g_4 d_{ij}, i = C4, j = B3 \\ x_4' \perp y_3' & f_4 g_3, i = 4, j = 3 \\ & \\ \text{univC} & C, r_1 - r_4, i = C4, j = C1 \\ x_4' \perp y_1' & f_4 g_1, i = 4, j = 1 \\ & \\ \text{Euler parameters} & p_2 \\ \text{Euler parameters} & p_3 \\ \text{Euler parameters} & p_4 \\ & \\ \text{driver} & \text{driver} \end{array}
$$

CONSTANTS

$$
\{r_1\}^A = \begin{Bmatrix} 0 \\ 0 \\ a \end{Bmatrix} \quad \{r_1\}^C = \begin{Bmatrix} c \\ 0 \\ 0 \end{Bmatrix}
$$

$$
\{s_2\}'^A = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{s_2\}'^B = \begin{Bmatrix} 0 \\ 0 \\ -b \end{Bmatrix} \quad \{s_3\}'^B = \begin{Bmatrix} 0 \\ 0 \\ \text{lenBD} \end{Bmatrix} \quad \{s_3\}'^D = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{s_4\}'^C = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix}
$$

INITIAL ESTIMATES

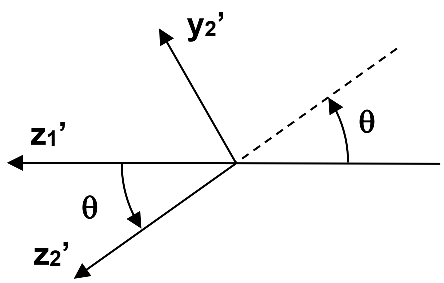
$$\{r_2\} = \begin{Bmatrix} 0 \\ 0 \\ a \end{Bmatrix} \qquad \chi_2 = \theta = 5 \text{ deg} \qquad \{\hat{u}_2\} = \begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \qquad \{p_2\} = \begin{Bmatrix} e_0 \\ e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \begin{Bmatrix} C\frac{\chi}{2} \\ uS\frac{\chi}{2} \\ vS\frac{\chi}{2} \\ wS\frac{\chi}{2} \end{Bmatrix} = \begin{Bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{\hat{f}_3\} = \text{unit}\begin{Bmatrix} a-b \\ 0 \\ c \end{Bmatrix} \quad \{\hat{g}_3\} = \text{unit}\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix} \quad \{\hat{h}_3\} = \text{unit}\begin{Bmatrix} -c \\ 0 \\ a-b \end{Bmatrix} \quad \{r_3\} = \{r_1\}^C + \left(\sqrt{(a-b)^2 + c^2} - \text{lenBD}\right)\{\hat{h}_3\}$$

$$e_0^2 = \left(\text{tr}[A] + 1\right)/4 \qquad \begin{Bmatrix} e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \frac{1}{4e_0}\begin{Bmatrix} a_{32} - a_{23} \\ a_{13} - a_{31} \\ a_{21} - a_{12} \end{Bmatrix}$$

$$\{r_4\} = \begin{Bmatrix} c \\ 0 \\ 0 \end{Bmatrix} \qquad \{p_4\} = \{p_3\}$$

FIXED REVOLUTE DRIVER



$$\{\hat{h}_1\}^T\{\hat{h}_2\} = C\theta$$

$$\{\hat{h}_1\}^T\{\hat{g}_2\} = S\theta$$

$$\theta = a\tan 2\left(\frac{\{\hat{h}_1\}^T\{\hat{g}_2\}}{\{\hat{h}_1\}^T\{\hat{h}_2\}}\right)$$

OUTPUT

$$d^2 = \{d_{ij}\}^T\{d_{ij}\}$$

$$2d\dot{d} = \{\dot{d}_{ij}\}^T\{d_{ij}\} + \{d_{ij}\}^T\{\dot{d}_{ij}\} = 2\{d_{ij}\}^T\{\dot{d}_{ij}\}$$

$$\dot{d} = \{d_{ij}\}^T\{\dot{d}_{ij}\}/d$$

$$d\ddot{d} + \dot{d}^2 = \{\dot{d}_{ij}\}^T\{\dot{d}_{ij}\} + \{d_{ij}\}^T\{\ddot{d}_{ij}\}$$

$$\ddot{d} = \left(\{d_{ij}\}^T\{\ddot{d}_{ij}\} + \{\dot{d}_{ij}\}^T\{\dot{d}_{ij}\} - \dot{d}^2\right)/d$$

$$d\dddot{d}+3\dot{d}\ddot{d}=\left\{\ddot{d}_{ij}\right\}^{T}\left\{\dot{d}_{ij}\right\}+\left\{\dot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}+\left\{\dot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}+\left\{d_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}=\left\{d_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}+3\left\{\dot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}$$

$$\dddot{d}=\left(\left\{d_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}+3\left\{\dot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}-3\dot{d}\ddot{d}\right)/\,d$$

$$d\ddddot{d}+\dot{d}\dddot{d}+3\dot{d}\dddot{d}+3\ddot{d}\ddot{d}=\left\{d_{ij}\right\}^{T}\left\{\ddddot{d}_{ij}\right\}+\left\{\dot{d}_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}+3\left\{\ddot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}+3\left\{\dot{d}_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}$$

$$d\ddddot{d}+4\dot{d}\dddot{d}+3\ddot{d}\ddot{d}=\left\{d_{ij}\right\}^{T}\left\{\ddddot{d}_{ij}\right\}+4\left\{\dot{d}_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}+3\left\{\ddot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}$$

$$\ddddot{d}=\left(\left\{d_{ij}\right\}^{T}\left\{\ddddot{d}_{ij}\right\}+4\left\{\dot{d}_{ij}\right\}^{T}\left\{\dddot{d}_{ij}\right\}+3\left\{\ddot{d}_{ij}\right\}^{T}\left\{\ddot{d}_{ij}\right\}-4\dot{d}\dddot{d}-3\ddot{d}\ddot{d}\right)/\,d$$

JACOBIAN

$$
[\Phi_q] = \begin{bmatrix}
-[I_3] & 2[A_2][\tilde{s}_2]'^{A}[G_2] & 0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x4} \\
0_{1x3} & -2\{f_1\}'^{T}[A_1]^{T}[A_2][\tilde{g}_2]'[G_2] & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & -2\{f_1\}'^{T}[A_1]^{T}[A_2][\tilde{h}_2]'[G_2] & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
 & & & & & \\
[I_3] & -2[A_2][\tilde{s}_2]'^{B}[G_2] & -[I_3] & 2[A_3][\tilde{s}_3]'^{B}[G_3] & 0_{1x3} & 0_{1x4} \\
 & & & & & \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{f_4\}'^{T}[A_4]^{T}[A_3][\tilde{h}_3]'[G_3] & 0_{1x3} & -2\{h_3\}'^{T}[A_3]^{T}[A_4][\tilde{f}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{g_4\}'^{T}[A_4]^{T}[A_3][\tilde{h}_3]'[G_3] & 0_{1x3} & -2\{h_3\}'^{T}[A_3]^{T}[A_4][\tilde{g}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & \{f_4\}'^{T}[A_4]^{\mathbf{T}} & -2\{f_4\}'^{T}[A_4]^{\mathbf{T}}[A_3][\tilde{s}_3]'^{B}[G_3] & -\{f_4\}'^{T}[A_4]^{\mathbf{T}} & 2\{f_4\}'^{T}[\tilde{s}_4]'^{C}[G_4]-2\{d_{ij}\}^{\mathbf{T}}[A_4][\tilde{f}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & \{g_4\}'^{T}[A_4]^{\mathbf{T}} & -2\{g_4\}'^{T}[A_4]^{\mathbf{T}}[A_3][\tilde{s}_3]'^{B}[G_3] & -\{g_4\}'^{T}[A_4]^{\mathbf{T}} & 2\{g_4\}'^{T}[\tilde{s}_4]'^{C}[G_4]-2\{d_{ij}\}^{\mathbf{T}}[A_4][\tilde{g}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{f_4\}'^{T}[A_4]^{T}[A_3][\tilde{g}_3]'[G_3] & 0_{1x3} & -2\{g_3\}'^{T}[A_3]^{T}[A_4][\tilde{f}_4]'[G_4] \\
 & & & & & \\
0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x4} & -[I_3] & 2[A_4][\tilde{s}_4]'^{C}[G_4] \\
0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{g_1\}'^{T}[A_1]^{T}[A_4][\tilde{f}_4]'[G_4] \\
 & & & & & \\
0_{1x3} & 2\{p_2\}^{T} & 0_{3x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & 0_{1x4} & 0_{1x3} & 2\{p_3\}^{T} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} & 0_{1x3} & 2\{p_4\}^{T} \\
 & & & & & \\
0_{1x3} & -2\sin\tfrac{\theta}{2} \quad 2e_0u \quad 2e_0v \quad 2e_0w & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4}
\end{bmatrix}
$$

$$A, r_1 - r_2, i = A2, j = A1$$
$$g_2 f_1, i = 2, j = 1$$
$$h_2 f_1, i = 2, j = 1$$

$$B, r_2 - r_3, i = B3, j = B2$$

$$f_4 h_3, i = 4, j = 3$$
$$g_4 h_3, i = 4, j = 3$$
$$f_4 d_{ij}, i = C4, j = B3$$
$$g_4 d_{ij}, i = C4, j = B3$$
$$f_4 g_3, i = 4, j = 3$$

$$C, r_1 - r_4, i = C4, j = C1$$
$$f_4 g_1, i = 4, j = 1$$

$$p_2$$
$$p_3$$
$$p_4$$

driver

$$\frac{\partial}{\partial\{p_i\}} = 2\left(\frac{\partial}{\partial\{\pi_i\}'}\right)[G_i]$$

VELOCITY

$$\{\nu\} = \left[\begin{array}{c} 0 \\ 0 \\ 0 \\ \\ 0 \\ \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ \\ 0 \\ 0 \\ \\ 0 \\ 0 \\ 0 \\ 0 \\ +f_t \end{array}\right] \begin{array}{c} A, r_1 - r_2, i = A2, j = A1 \\ g_2 f_1, i = 2, j = 1 \\ h_2 f_1, i = 2, j = 1 \\ \\ B, r_2 - r_3, i = B3, j = B2 \\ \\ f_4 h_3, i = 4, j = 3 \\ g_4 h_3, i = 4, j = 3 \\ f_4 d_{ij}, i = C4, j = B3 \\ g_4 d_{ij}, i = C4, j = B3 \\ f_4 g_3, i = 4, j = 3 \\ \\ C, r_1 - r_4, i = C4, j = C1 \\ f_4 g_1, i = 4, j = 1 \\ \\ p_2 \\ p_3 \\ p_4 \\ \\ \text{driver} \end{array}$$

ACCELERATION

$$
\{\gamma\} = \left\{
\begin{array}{c}
[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]'\{s_2\}'^{A} \\
-\{f_1\}'^{T}([A_1]^{T}[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{g_2\}' \\
-\{f_1\}'^{T}([A_1]^{T}[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{h_2\}' \\
\\
[A_3][\tilde{\omega}_3]'[\tilde{\omega}_3]'\{s_3\}'^{B} - [A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]'\{s_2\}'^{B} \\
\\
-\{h_3\}'^{T}\left([A_3]^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]' + 2[\tilde{\omega}_3]'^{T}[A_3]^{T}[A_4][\tilde{\omega}_4]' + [\tilde{\omega}_3]'[\tilde{\omega}_3]'[A_3]^{T}[A_4]\right)\{f_4\}' \\
-\{h_3\}'^{T}\left([A_3]^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]' + 2[\tilde{\omega}_3]'^{T}[A_3]^{T}[A_4][\tilde{\omega}_4]' + [\tilde{\omega}_3]'[\tilde{\omega}_3]'[A_3]^{T}[A_4]\right)\{g_4\}' \\
\{\gamma^{s}\} = [A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{s_4\}'^{C} - [A_3][\tilde{\omega}_3]'[\tilde{\omega}_3]'\{s_3\}'^{B} \\
-2\{\dot{d}_{ij}\}^{T}[A_4][\tilde{\omega}_4]'\{f_4\}' - \{d_{ij}\}^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{f_4\}' + \{f_4\}'^{T}[A_4]^{T}\{\gamma^{s}\} \\
-2\{\dot{d}_{ij}\}^{T}[A_4][\tilde{\omega}_4]'\{g_4\}' - \{d_{ij}\}^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{g_4\}' + \{g_4\}'^{T}[A_4]^{T}\{\gamma^{s}\} \\
-\{g_3\}'^{T}\left([A_3]^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]' + 2[\tilde{\omega}_3]'^{T}[A_3]^{T}[A_4][\tilde{\omega}_4]' + [\tilde{\omega}_3]'[\tilde{\omega}_3]'[A_3]^{T}[A_4]\right)\{f_4\}' \\
\\
[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{s_4\}'^{C} \\
-\{g_1\}'^{T}([A_1]^{T}[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]')\{f_4\}' \\
\\
-2\{\dot{p}_2\}^{T}\{\dot{p}_2\} \\
-2\{\dot{p}_3\}^{T}\{\dot{p}_3\} \\
-2\{\dot{p}_4\}^{T}\{\dot{p}_4\} \\
\\
+f_{tt}
\end{array}
\right\}
\begin{array}{c}
A, r_1 - r_2, i = A2, j = A1 \\
g_2 f_1, i = 2, j = 1 \\
h_2 f_1, i = 2, j = 1 \\
\\
B, r_2 - r_3, i = B3, j = B2 \\
\\
f_4 h_3, i = 4, j = 3 \\
g_4 h_3, i = 4, j = 3 \\
\\
f_4 d_{ij}, i = C4, j = B3 \\
g_4 d_{ij}, i = C4, j = B3 \\
f_4 g_3, i = 4, j = 3 \\
\\
C, r_1 - r_4, i = C4, j = C1 \\
f_4 g_1, i = 4, j = 1 \\
\\
p_2 \\
p_3 \\
p_4 \\
\\
\text{driver}
\end{array}
$$

$$
\text{dot} - 1 \qquad \{\Phi\} = \{a_i\}^{T}\{a_j\} \qquad \gamma = -\{a_j\}'^{T}\left([A_j]^{T}[A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]' + 2[\tilde{\omega}_j]'^{T}[A_j]^{T}[A_i][\tilde{\omega}_i]' + [\tilde{\omega}_j]'[\tilde{\omega}_j]'[A_j]^{T}[A_i]\right)\{a_i\}'
$$

sphere $\{\Phi\} = \{r_j\}^P - \{r_j\}^P$ $\{\gamma\} = [A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{s_i\}'^P - [A_j][\tilde{\omega}_j]'[\tilde{\omega}_j]'\{s_j\}'^P$

dot – 2 $\{\boldsymbol{\gamma}\} = -2\{\dot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'\{a_i\}' - \{d_{ij}\}^T[A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{a_i\}' + \{a_i\}'^T[A_i]^T\{\gamma^s\}$

JERK

$$
\{\eta\} = \left\{ \begin{array}{c}
[A_2][H_2]\{s_2\}'^{A} \\
-\{f_1\}'^{T}[A_1]^{T}[A_2][H_2]\{g_2\}' \\
-\{f_1\}'^{T}[A_1]^{T}[A_2][H_2]\{h_2\}' \\
\\
[A_3][H_3]\{s_3\}'^{B} - [A_2][H_2]\{s_2\}'^{B} \\
\\
\left(\begin{array}{l}
-\{h_3\}'^{T}[A_3]^{T}[A_4][H_4]\{f_4\}' - \{f_4\}'^{T}[A_4]^{T}[A_3][H_3]\{h_3\}' \\
-3\{h_3\}'^{T}\left([\tilde{\omega}_3]'[\tilde{\omega}_3]' - [\dot{\tilde{\omega}}_3]'\right)[A_3]^{T}[A_4][\tilde{\omega}_4]'\{f_4\}' - 3\{f_4\}'^{T}\left([\tilde{\omega}_4]'[\tilde{\omega}_4]' - [\dot{\tilde{\omega}}_4]'\right)[A_4]^{T}[A_3][\tilde{\omega}_3]'\{h_3\}'
\end{array} \right) \\
\left(\begin{array}{l}
-\{h_3\}'^{T}[A_3]^{T}[A_4][H_4]\{g_4\}' - \{g_4\}'^{T}[A_4]^{T}[A_3][H_3]\{h_3\}' \\
-3\{h_3\}'^{T}\left([\tilde{\omega}_3]'[\tilde{\omega}_3]' - [\dot{\tilde{\omega}}_3]'\right)[A_3]^{T}[A_4][\tilde{\omega}_4]'\{g_4\}' - 3\{g_4\}'^{T}\left([\tilde{\omega}_4]'[\tilde{\omega}_4]' - [\dot{\tilde{\omega}}_4]'\right)[A_4]^{T}[A_3][\tilde{\omega}_3]'\{h_3\}'
\end{array} \right) \\
\{\gamma^{s}\} = [A_4][H_4]\{s_4\}'^{C} - [A_3][H_3]\{s_3\}'^{B} \\
-3\{\ddot{d}_{ij}\}^{T}[A_4][\tilde{\omega}_4]'\{f_4\}' - 3\{\dot{d}_{ij}\}^{T}[A_4]\left([\tilde{\omega}_4]'[\tilde{\omega}_4]' + [\dot{\tilde{\omega}}_4]'\right)\{f_4\}' - \{d_{ij}\}^{T}[A_4][H_4]\{f_4\}' + \{f_4\}'^{T}[A_4]^{T}\{\eta^{s}\} \\
-3\{\ddot{d}_{ij}\}^{T}[A_4][\tilde{\omega}_4]'\{g_4\}' - 3\{\dot{d}_{ij}\}^{T}[A_4]\left([\tilde{\omega}_4]'[\tilde{\omega}_4]' + [\dot{\tilde{\omega}}_4]'\right)\{g_4\}' - \{d_{ij}\}^{T}[A_4][H_4]\{g_4\}' + \{g_4\}'^{T}[A_4]^{T}\{\eta^{s}\} \\
\left(\begin{array}{l}
-\{g_3\}'^{T}[A_3]^{T}[A_4][H_4]\{f_4\}' - \{f_4\}'^{T}[A_4]^{T}[A_3][H_3]\{g_3\}' \\
-3\{g_3\}'^{T}\left([\tilde{\omega}_3]'[\tilde{\omega}_3]' - [\dot{\tilde{\omega}}_3]'\right)[A_3]^{T}[A_4][\tilde{\omega}_4]'\{f_4\}' - 3\{f_4\}'^{T}\left([\tilde{\omega}_4]'[\tilde{\omega}_4]' - [\dot{\tilde{\omega}}_4]'\right)[A_4]^{T}[A_3][\tilde{\omega}_3]'\{g_3\}'
\end{array} \right) \\
\\
[A_4][H_4]\{s_4\}'^{C} \\
-\{g_1\}'^{T}[A_1]^{T}[A_4][H_4]\{f_4\}' \\
\\
-6\{\ddot{p}_2\}^{T}\{\dot{p}_2\} \\
-6\{\ddot{p}_3\}^{T}\{\dot{p}_3\} \\
-6\{\ddot{p}_4\}^{T}\{\dot{p}_4\} \\
\\
-\dot{\theta}^{3}/4 + f_{ttt} \quad \text{for link 2}
\end{array} \right\}
\quad
\begin{array}{c}
A, r_1 - r_2, i = A2, j = A1 \\
g_2 f_1, i = 2, j = 1 \\
h_2 f_1, i = 2, j = 1 \\
\\
B, r_2 - r_3, i = B3, j = B2 \\
\\
\left(f_4 h_3, i = 4, j = 3 \right) \\
\left(g_4 h_3, i = 4, j = 3 \right) \\
\\
f_4 d_{ij}, i = C4, j = B3 \\
g_4 d_{ij}, i = C4, j = B3 \\
\left(f_4 g_3, i = 4, j = 3 \right) \\
\\
C, r_1 - r_4, i = C4, j = C1 \\
f_4 g_1, i = 4, j = 1 \\
\\
p_2 \\
p_3 \\
p_4 \\
\\
\text{driver}
\end{array}
$$

jerk product $[H_i] = 2[\tilde{\omega}_i]'[\dot{\tilde{\omega}}_i]' + [\dot{\tilde{\omega}}_i]'[\tilde{\omega}_i]' + [\tilde{\omega}_i]'[\tilde{\omega}_i]'[\tilde{\omega}_i]'$

dot − 1 $$\eta = \begin{pmatrix} -\{a_j\}'^T[A_j]^T[A_i][H_i]\{a_i\}' - \{a_i\}'^T[A_i]^T[A_j][H_j]\{a_j\}' \\ -3\{a_j\}'^T([\tilde{\omega}_j]'[\tilde{\omega}_j]' - [\dot{\tilde{\omega}}_j]')[A_j]^T[A_i][\tilde{\omega}_i]'\{a_i\}' - 3\{a_i\}'^T([\tilde{\omega}_i]'[\tilde{\omega}_i]' - [\dot{\tilde{\omega}}_i]')[A_i]^T[A_j][\tilde{\omega}_j]'\{a_j\}' \end{pmatrix}$$

sphere $\{\eta\} = [A_i][H_i]\{s_i\}'^P - [A_j][H_j]\{s_j\}'^P$

dot − 2 $$-3\{\ddot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'\{a_i\}' - 3\{\dot{d}_{ij}\}^T[A_i][\tilde{\omega}_i]'[\tilde{\omega}_i]'\{a_i\}' - 3\{\dot{d}_{ij}\}^T[A_i][\dot{\tilde{\omega}}_i]'\{a_i\}' - \{d_{ij}\}^T[A_i][H_i]\{a_i\}' + \{a_i\}'^T[A_i]^T\{\eta^s\}$$

RSPU - Inertial Properties

link 2

density $\rho = 1.18\ \text{g/cm}^3$ plexiglass

diameter D = 5 inch = 12.7 cm

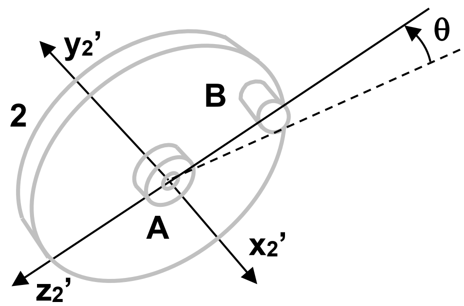
t = 0.5 inch = 1.27 cm

$m_2 = \rho t \pi D^2/4 = 189.84\ \text{g} = 0.18984\ \text{kg}$

$J_{XX\_2} = mR^2/2 = mD^2/8 = 3.8274\ \text{kg.cm}^2$

$J_{YY\_2} = mR^2/4 + mt^2/12 = 1.9177\ \text{kg.cm}^2$

$J_{ZZ\_2} = mR^2/4 + mt^2/12 = 1.9177\ \text{kg.cm}^2$



link 3

density $\rho = 1.18\ \text{g/cm}^3$ plexiglass

square rod width w = 0.5 inch = 1.27 cm

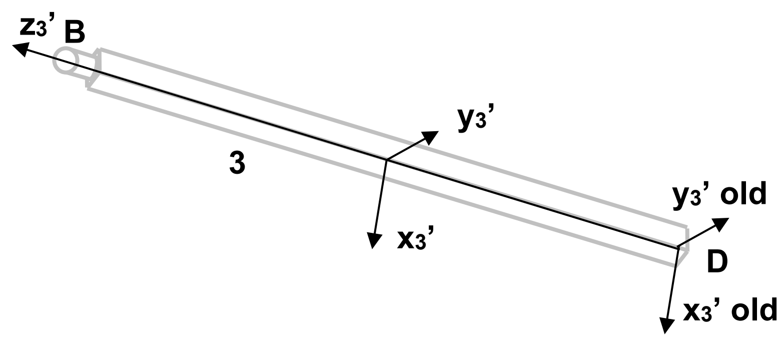
length L = lenBD = 35 cm

$m_3 = \rho L w^2/4 = 66.61\ \text{g} = 0.06661\ \text{kg}$

$J_{XX\_3} = mL^2/12 = 6.8000\ \text{kg.cm}^2$

$J_{YY\_3} = mL^2/12 = 6.8000\ \text{kg.cm}^2$

$J_{ZZ\_3} = m\left(w^2 + w^2\right)/12 = 0.017906\ \text{kg.cm}^2$



shift centroid $\{s_3\}'^{B} = \begin{Bmatrix} 0 \\ 0 \\ +\text{lenBD}/2 \end{Bmatrix}$ $\{s_3\}'^{D} = \begin{Bmatrix} 0 \\ 0 \\ -\text{lenBD}/2 \end{Bmatrix}$

initial estimate $\{r_3\} = \{r_1\}^C + \left(\sqrt{(a-b)^2 + c^2} - \text{lenBD}/2\right)\{\hat{h}_3\}$

link 4

density $\rho = 1.18\ \text{g/cm}^3$ plexiglass

cube width w = 2.5 inch = 6.35 cm

$m_4 = \rho w^3 = 302.14\ \text{g} = 0.30214\ \text{kg}$ (ignore hole)

$J_{XX\_4} = m\left(w^2 + w^2\right)/12 = 2.0305\ \text{kg.cm}^2$

$J_{YY\_4} = m\left(w^2 + w^2\right)/12 = 2.0305\ \text{kg.cm}^2$

$J_{ZZ\_4} = m\left(w^2 + w^2\right)/12 = 2.0305\ \text{kg.cm}^2$