

inverse dynamics

Matrix Dynamic Analysis for Four Bar

The four bar linkage shown below operates in a vertical plane. Each link is a uniform bar with 2 cm by 2 cm square cross-section stainless steel. Assume that the masses of the bearings and the effects of friction are negligible. Do not neglect the effects of gravity.

$$\theta_2 = 45 \text{ deg}$$

$$\omega_2 = 20 \text{ rad/s CW}$$

$$\alpha_2 = 100 \text{ rad/s/s CCW}$$

$$\theta_3 = 20 \text{ deg}$$

$$\omega_3 = 12.82 \text{ rad/s CCW}$$

$$\alpha_3 = 39.6 \text{ rad/s/s CW}$$

$$\theta_4 = 117.4 \text{ deg}$$

$$\omega_4 = 6.20 \text{ rad/s CW}$$

$$\alpha_4 = 482.5 \text{ rad/s/s CCW}$$

$$m_2 = 0.248 \text{ kg}$$

$$J_{G2}' = 1.405 \text{ kg.cm}^2$$

$$\rho = 7.75 \text{ g/cm}^3$$

$$m_3 = 0.372 \text{ kg}$$

$$J_{G3}' = 4.588 \text{ kg.cm}^2$$

$$m_4 = 0.341 \text{ kg}$$

$$J_{G4}' = 3.552 \text{ kg.cm}^2$$

$$V_{G2} = 56.57 - j 56.57 \text{ cps}$$

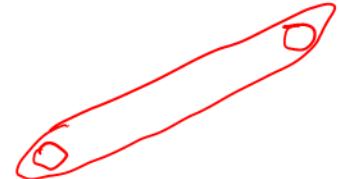
$$A_{G2} = -1414.2 - j 848.5 \text{ cps}^2$$

$$V_{G3} = 86.83 - j 40.86 \text{ cps}$$

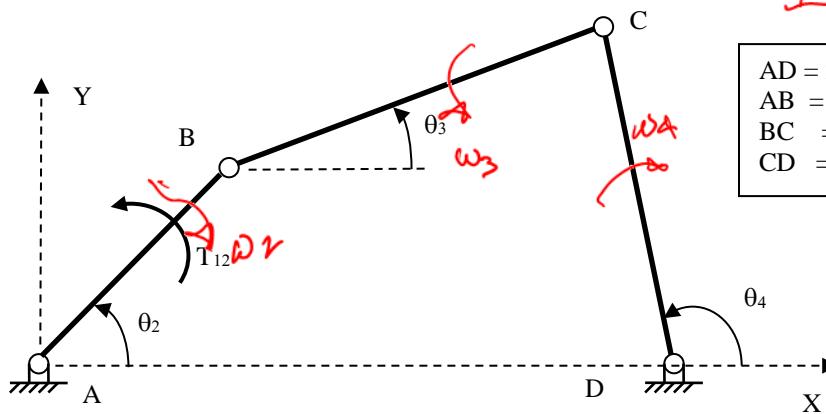
$$A_{G3} = -3672.5 - j 2260.9 \text{ cps}^2$$

$$V_{G4} = 30.27 - j 15.71 \text{ cps}$$

$$A_{G4} = -2262.4 - j 1412.4 \text{ cps}^2$$



AD = 22 cm
 AB = 8 cm
 BC = 12 cm
 CD = 11 cm

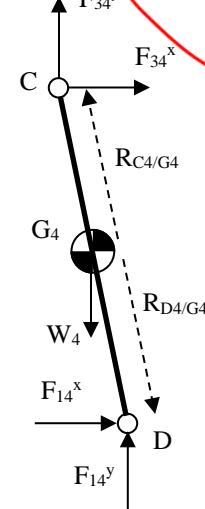
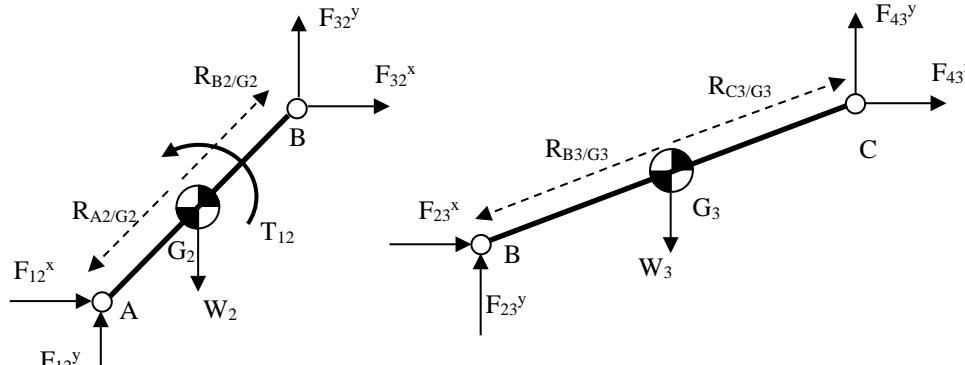


bearing force

F_A F_B F_C F_D

motor torque

T_{12}



ΣF on 2 right +

$$F_{12}^x + F_{32}^x = m_2 A_{G2}^x$$

 ΣF on 2 up +

$$F_{12}^y + F_{32}^y + W_2 = m_2 A_{G2}^y$$

 ΣM on 2 about G₂ CCW +

$$- F_{12}^x r_{A2/G2}^y + F_{12}^y r_{A2/G2}^x - F_{32}^x r_{B2/G2}^y + F_{32}^y r_{B2/G2}^x + T_{12} = J_{G2}' \alpha_2$$

 ΣF on 3 right +

$$F_{23}^x + F_{43}^x = m_3 A_{G3}^x$$

 ΣF on 3 up +

$$F_{23}^y + F_{43}^y + W_3 = m_3 A_{G3}^y$$

 ΣM on 3 about G₃ CCW +

$$- F_{23}^x r_{B3/G3}^y + F_{23}^y r_{B3/G3}^x - F_{43}^x r_{C3/G3}^y + F_{43}^y r_{C3/G3}^x = J_{G3}' \alpha_3$$

unknown bearing forces and motor torque

 ΣF on 4 right +

$$F_{34}^x + F_{14}^x = m_4 A_{G4}^x$$

 ΣF on 4 up +

$$F_{34}^y + F_{14}^y + W_4 = m_4 A_{G4}^y$$

 ΣM on 4 about G₄ CCW +

$$- F_{34}^x r_{C4/G4}^y + F_{34}^y r_{C4/G4}^x - F_{14}^x r_{D4/G4}^y + F_{14}^y r_{D4/G4}^x = J_{G4}' \alpha_4$$

distributed matrix

EFC

$$\left[\begin{array}{cccccccccc} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & F_{12}^x \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & F_{12}^y \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & F_{23}^x \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & F_{23}^y \\ 0 & 0 & -r_{B3/G3}^y & r_{B3/G3}^x & r_{C3/G3}^y & -r_{C3/G3}^x & 0 & 0 & 0 & F_{34}^x \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & F_{34}^y \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & F_{14}^x \\ 0 & 0 & 0 & 0 & -r_{C4/G4}^y & r_{C4/G4}^x & -r_{D4/G4}^y & r_{D4/G4}^x & 0 & F_{14}^y \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & T_{12} \end{array} \right] = \left[\begin{array}{c} m_2 A_{G2}^x \\ m_2 A_{G2}^y - W_2 \\ J_{G2}' \alpha_2 \\ m_3 A_{G3}^x \\ m_3 A_{G3}^y - W_3 \\ J_{G3}' \alpha_3 \\ m_4 A_{G4}^x \\ m_4 A_{G4}^y - W_4 \\ J_{G4}' \alpha_4 \end{array} \right]$$

mass accel minus external forces

$$W_2 = -j 2.433 \text{ N}$$

$$m_2 A_{G2} = -3.507 - j 2.104 \text{ N}$$

$$J_{G2}' \alpha_2 = 1.405 \text{ N.cm}$$

$$W_3 = -j 3.649 \text{ N}$$

$$m_3 A_{G3} = -13.662 - j 8.411 \text{ N}$$

$$J_{G3}' \alpha_3 = -1.817 \text{ N.cm}$$

$$W_4 = -j 3.345 \text{ N}$$

$$m_4 A_{G4} = -7.715 - j 4.816 \text{ N}$$

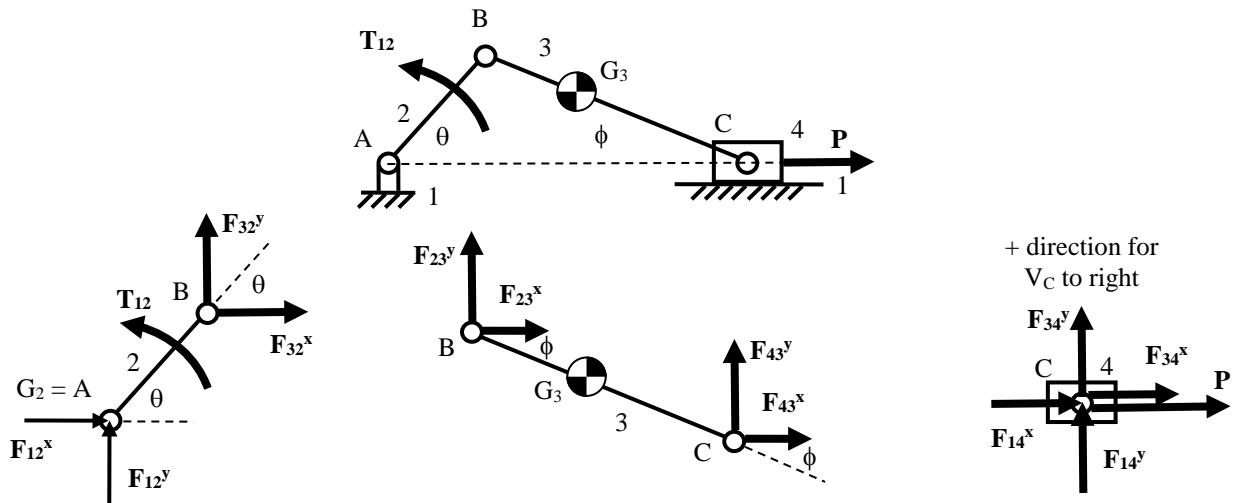
$$J_{G4}' \alpha_4 = 17.138 \text{ N.cm}$$

driver force

$$\left[\begin{array}{cccccccccc} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 & F_{12}^x \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & F_{12}^y \\ 2.828 & -2.828 & 2.828 & -2.828 & 0 & 0 & 0 & 0 & 1 & F_{23}^x \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 & F_{23}^y \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 & F_{34}^x \\ 0 & 0 & 2.052 & -5.638 & 2.052 & -5.638 & 0 & 0 & 0 & F_{34}^y \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & F_{14}^x \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & F_{14}^y \\ 0 & 0 & 0 & 0 & -4.883 & -2.531 & 4.883 & 2.531 & 0 & T_{12} \end{array} \right] = \left[\begin{array}{c} -3.507 \\ 0.329 \\ 1.405 \\ -13.662 \\ -4.762 \\ -1.817 \\ -7.715 \\ -1.471 \\ 17.138 \end{array} \right]$$

$$\begin{Bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ F_{14}^x \\ F_{14}^y \\ T_{12} \end{Bmatrix} = \begin{Bmatrix} -22.236 \text{ N} \\ -6.221 \text{ N} \\ -18.729 \text{ N} \\ -6.550 \text{ N} \\ -5.067 \text{ N} \\ -1.788 \text{ N} \\ -2.648 \text{ N} \\ +0.317 \text{ N} \\ +81.135 \text{ N.cm} \end{Bmatrix}$$

Matrix Dynamic Analysis for Slider Crank



$$\Sigma F \text{ on 2 right } + \quad F_{12}^x + F_{32}^x = m_2 A_{G2}^x = 0$$

$$\Sigma F \text{ on 2 up } + \quad F_{12}^y + F_{32}^y = m_2 A_{G2}^y = 0$$

$$\Sigma M \text{ on 2 about A CCW } + \quad - (F_{32}^x \sin\theta) AB + (F_{32}^y \cos\theta) AB + T_{12} = J_{G2} \alpha_2$$

$$\Sigma F \text{ on 3 right } + \quad F_{23}^x + F_{43}^x = m_3 A_{G3}^x$$

$$\Sigma F \text{ on 3 up } + \quad F_{23}^y + F_{43}^y = m_3 A_{G3}^y$$

$$\Sigma M \text{ on 3 about G}_3 \text{ CCW } + \quad - (F_{23}^x \sin\phi) BG_3 - (F_{23}^y \cos\phi) BG_3 \\ + (F_{43}^x \sin\phi) CG_3 + (F_{43}^y \cos\phi) CG_3 = J_{G3} \alpha_3$$

$$\Sigma F \text{ on 4 right } + \quad F_{14}^x + F_{34}^x + P = m_4 A_{G4}^x$$

$$\Sigma F \text{ on 4 up } + \quad F_{14}^y + F_{34}^y = m_4 A_{G4}^y = 0$$

$$\text{friction} \quad F_{14}^x = -\mu \text{ abs}(F_{14}^y) \text{ sign}(V_C)$$



$$\begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & +AB\sin\theta & -AB\cos\theta & 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & -BG_3 \sin\phi & -BG_3 \cos\phi & +CG_3 \sin\phi & +CG_3 \cos\phi & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} F_{12}^x \\ F_{12}^y \\ F_{23}^x \\ F_{23}^y \\ F_{34}^x \\ F_{34}^y \\ T_{12} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ J_{G2}\alpha_2 \\ m_3 A_{G3}^x \\ m_3 A_{G3}^y \\ J_{G3}\alpha_3 \\ m_4 A_{G4}^x - P \\ 0 \\ 0 \end{bmatrix}$$