

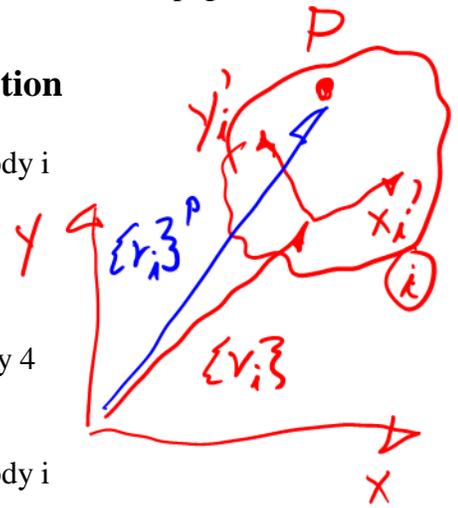
$\{r_{BODY}\}^{PNT}$

## Two-Dimensional Vector and Matrix Notation

$\{r_i\}$  global position of the origin of the reference frame attached to body i

$\{r_i\}^P$  global position of point P attached to body i

example  $\{r_4\}^B = \begin{Bmatrix} x_4^B \\ y_4^B \end{Bmatrix}$  global position of point B attached to body 4



$\{\dot{r}_i\}$  global velocity of the origin of the reference frame attached to body i

$\{\dot{r}_i\}^P$  global velocity of point P attached to body i

$\{\ddot{r}_i\}$  global acceleration of the origin of the reference frame attached to body i

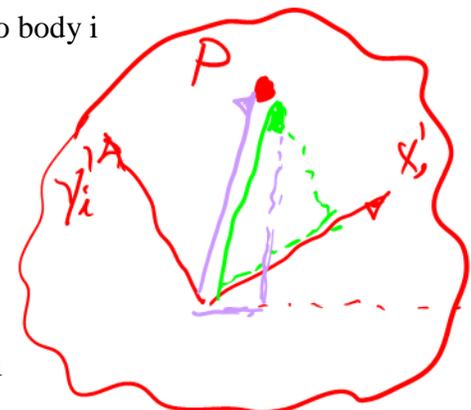
$\{\ddot{r}_i\}^P$  global acceleration of point P attached to body i

$\{\dddot{r}_i\}$  global jerk of the origin of the reference frame attached to body i

$\{\dddot{r}_i\}^P$  global jerk of point P attached to body i

$\{\overset{\cdot\cdot\cdot}{r}_i\}$  global snap of the origin of the reference frame attached to body i

$\{\overset{\cdot\cdot\cdot}{r}_i\}^P$  global snap of point P attached to body i



$\{s_i\}^P$  position of point P on body i relative to the reference frame for body i measured in local body-fixed directions

$\{ \}^P$  local directions  
blueprint CONSTANT

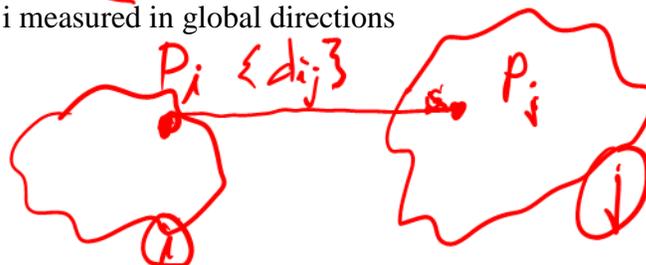
example  $\{s_4\}^{B} = \begin{Bmatrix} x_4^{B} \\ y_4^{B} \end{Bmatrix}$  location of point B on body 4 relative to the reference frame for body 4 measured in local body-fixed directions for body 4

$\{s_i\}^P$  position of point P on body i relative to the reference frame for body i but measured in global directions

no prime change as object rotates

$\{d_{ij}\}$  relative location between two points on bodies i and j measured in global directions

example  $\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P$  relative location of point P on body j with respect to point P on body i measured in global directions



$\phi_i$  attitude angle for reference frame attached to body i

$\phi_{ij}$  attitude angle of body j with respect to reference frame attached to body i

example  $\phi_{ij} = \phi_j - \phi_i$

$\dot{\phi}_i$  angular velocity of body i

$\omega_i$

$\ddot{\phi}_i$  angular acceleration of body i

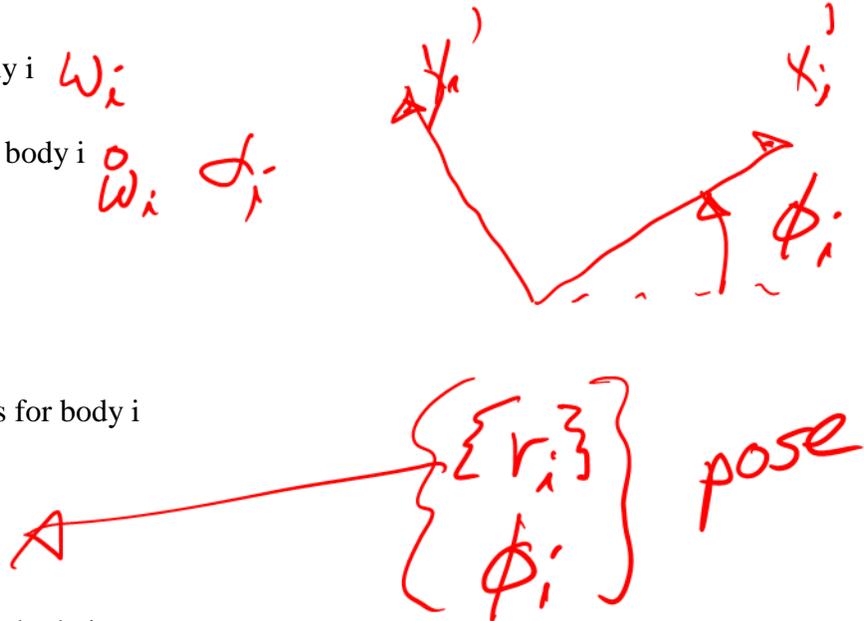
$\dot{\omega}_i \quad \alpha_i$

$\dddot{\phi}_i$  angular jerk of body i

$\ddot{\phi}_i$  angular snap of body i

$\{q_i\}$  generalized coordinates for body i

example  $\{q_i\} = \begin{Bmatrix} \{r_i\} \\ \phi_i \end{Bmatrix}$



$\{\dot{q}_i\}$  generalized velocity for body i

$\{\ddot{q}_i\}$  generalized acceleration for body i

$\{\ddot{\ddot{q}}_i\}$  generalized jerk for body i

$\{\ddot{\ddot{\ddot{q}}}_i\}$  generalized snap for body i



$[A_i]$  orthonormal rotation matrix that describes attitude of body i

$\star$  only contains  $\phi_i$

example  $[A_i] = \begin{bmatrix} \cos \phi_i & -\sin \phi_i \\ \sin \phi_i & \cos \phi_i \end{bmatrix}$

example  $\{\hat{s}_i\}^P = [A_i] \{\hat{s}_i\}'^P$  rotation matrix converts information in local body-fixed directions into global directions

$\{\hat{f}_i\}$  global direction of unit vector along local x axis attached to body i

$\{\hat{g}_i\}$  global direction of unit vector along local y axis attached to body i

example

$$[A_i] = \begin{bmatrix} \hat{f}_i \\ \hat{g}_i \end{bmatrix} \text{ unit directions of local axes for body } i$$

 $[B_i]$  second rotation matrix

example

$$[B_i] = [R][A_i] = \begin{bmatrix} -\sin \phi & -\cos \phi \\ \cos \phi & -\sin \phi \end{bmatrix}$$

 $[R]$  rotator matrix

$$[R] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$$

rotates anything  
90°

$$\phi = 90^\circ$$

 $[A_{ij}]$  rotation matrix that describes attitude of body j with respect to body i

example

$$[A_{ij}] = \begin{bmatrix} \cos \phi_{ij} & -\sin \phi_{ij} \\ \sin \phi_{ij} & \cos \phi_{ij} \end{bmatrix} = [A_i]^T [A_j]$$

 $\{F_{on i}\}^P$  force on body i acting through point P measured in global directions

no prime

 $\{F_{on i}\}^{iP}$  force on body i acting through point P measured in body-fixed directions local to body i

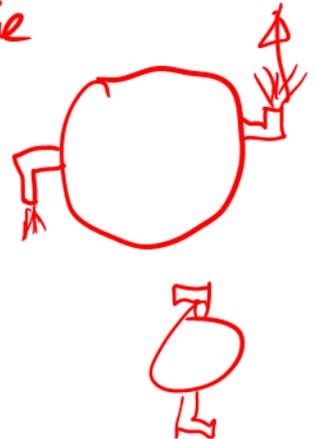
prime

 $T_{on i}$  torque on body i

 $\{Q_{on i}\}$  generalized force on body i measured in global directions

example

$$\{Q_{on i}\} = \begin{Bmatrix} \{F_{on i}\} \\ T_{on i} \end{Bmatrix}$$



**Numbering and lettering**

Bodies should be numbered consecutively beginning with 1. Body 1 is typically reserved for ground.

Points should be lettered.

Point G is typically reserved for the mass center of a body.

Point T is seldom used in that it causes confusion with the vector/matrix transpose operator.

**Subscripts and superscripts outside vector/matrix brackets**

Post-superscript prime outside vector brackets denotes information measured in local body-fixed directions.

Post-superscript letters outside vector brackets denote information related to a specific point.

Post-subscripts outside vector/matrix brackets are occasionally used for iteration or time indices.

Pre-superscripts and pre-subscripts are typically not used outside brackets.

**Subscripts and superscripts inside vector/matrix brackets**

Post-superscripts inside vector/matrix brackets are occasionally used for iteration or time indices.

Post-subscript numerals inside vector/matrix brackets are typically used for body numbers.

Post-subscript variables inside vector/matrix brackets denote partial derivative operators.

Pre-superscripts and pre-subscripts are typically not used inside brackets.

**General vector/matrix operations**

$\{ \}^T, [ ]^T$  vector/matrix transpose

$[ ]^{-1}$  matrix inverse

$\det[ ]$  determinant of matrix

$\text{tr}[ ]$  trace of matrix (sum of diagonal elements)

$\{\text{diag}[ ]\}$  diagonal elements of matrix rearranged into column vector

$[\text{diag}\{ \}]$  elements of vector placed into a diagonal matrix

$[ ]^n$  matrix to power n

$\text{norm}\{ \}$  scalar norm of vector (magnitude)

$\{\hat{a}\}$  unit vector

$[I_n]$  identity matrix of order n

$\{0\}, [0]$  vector/matrix of zeros

$[R]$  2x2 skew-symmetric rotation operator  $[R] = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$  (used for cross-product)

$$[I_3] = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$[I_2] = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$