

Two-Dimensional Generalized Forces

Moment about origin

$$T = (\{r_i\}^P - \{r_i\}) \times \{F_{on i}\}^P$$

to prime
prime

$$(\{r_i\}^P - \{r_i\}) = \{s_i\}^P = [A_i] \{s_i\}'^P$$

$$\{s_i\}^P \times \{F_{on i}\}^P = ([A_i] \{s_i\}'^P) \times \{F_{on i}\}^P = \{s_i\}'^P \times \{F_{on i}\}^P \quad \text{OK}$$

$$\{\alpha\}^T \{\alpha\} = \{\alpha\}^0 \{\alpha\}$$

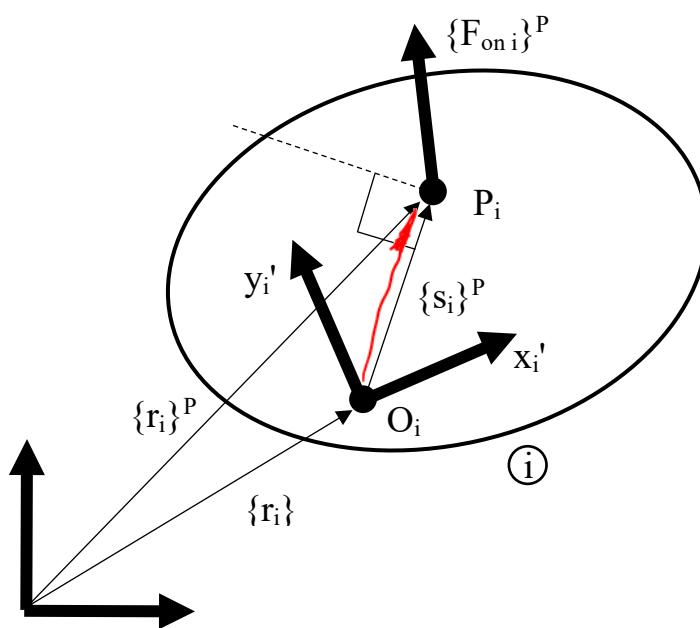
$$\cancel{\{s_i\}'^P \times \{F_{on i}\}^P} = \cancel{\{s_i\}^P \times \{F_{on i}\}'^P} \quad \text{NO!}$$

$$[R][A_i]\{s_i\}'^P = [B_i]\{s_i\}'^P \perp \{s_i\}^P$$

$([B_i]\{s_i\}'^P) \circ \{F_{on i}\}^P$ component of $\{F_{on i}\}^P$ perpendicular to $\{s_i\}^P$ multiplied by (norm $\{s_i\}^P$)

$$T = (\{r_i\}^P - \{r_i\}) \times \{F_{on i}\}^P$$

$$T = ([B_i]\{s_i\}'^P)^T \{F_{on i}\}^P = (\{F_{on i}\}^P)^T [B_i]\{s_i\}'^P$$

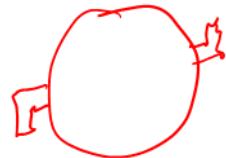


T due to
 $\{F_{on i}\}^P$
about O_i

Generalized force on body i about origin

$$\{q_i\} = \begin{Bmatrix} \{r_i\} \\ \phi_i \end{Bmatrix}$$

$$\{Q_{on\ i}\} = \begin{Bmatrix} \{F_{on\ i}\} \\ T_{on\ i} \end{Bmatrix}$$



Pure force

$$\{Q_{on\ i}\} = \left\{ \left([B_i]_{S_i}^P, P \right)^T \{F_{on\ i}\}^P \right\}$$

Pure moment

$$\{Q_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ T_{on\ i} \end{Bmatrix}$$

Translational spring-damper-actuator

$$\{d_{ij}\} = \{r_j\}^P - \{r_i\}^P \quad \text{Note similarity to double revolute constraint}$$

$$\{\dot{d}_{ij}\} = \{\dot{r}_j\}^P - \{\dot{r}_i\}^P$$

$$\{\ddot{d}_{ij}\} = \{\ddot{r}_j\}^P - \{\ddot{r}_i\}^P$$

$$\ell^2 = \{d_{ij}\}^T \{d_{ij}\}$$

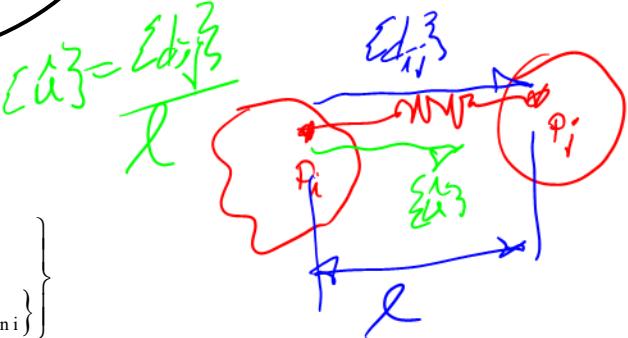
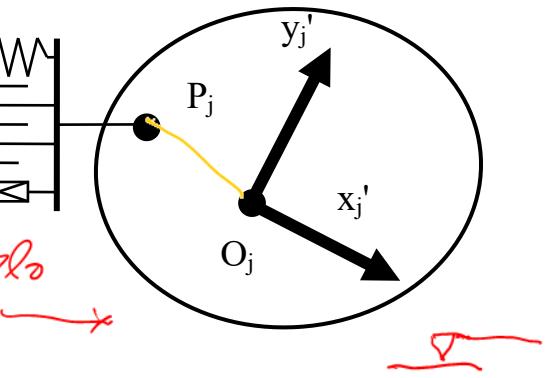
$$2\ell\dot{\ell} = 2\{d_{ij}\}^T \{\dot{d}_{ij}\}$$

$$\dot{\ell} = \{d_{ij}\}^T \{\dot{d}_{ij}\} / \ell$$

$$f = k(\ell - \ell_o) + c\dot{\ell} + f_F \text{sign}(\dot{\ell}) + f_{ACT}(\ell, \dot{\ell}, t)$$

$$\{F_{on i}\} = \frac{f}{\ell} \{d_{ij}\} \quad \{Q_{on i}\} = \left\{ \begin{array}{c} \{F_{on i}\} \\ (\underbrace{[B_i] \{s_i\}^P}_{})^T \{F_{on i}\} \end{array} \right\}$$

$$\{F_{on j}\} = -\frac{f}{\ell} \{d_{ij}\} \quad \{Q_{on j}\} = \left\{ \begin{array}{c} \{F_{on j}\} \\ (\underbrace{[B_j] \{s_j\}^P}_{})^T \{F_{on j}\} \end{array} \right\}$$



$$\ddot{\ell} = (\{d_{ij}\}^T \{\ddot{d}_{ij}\} + \{\dot{d}_{ij}\}^T \{\dot{d}_{ij}\} - \dot{\ell}^2) / \ell$$

$$\{Q_{on i}\} \neq \{Q_{on j}\}$$

$$\dot{f} = k\dot{\ell} + c\ddot{\ell} + \dot{f}_{ACT}(\ell, \dot{\ell}, \ddot{\ell}, t)$$

$$\{\dot{F}_{on i}\} = (\dot{f}\ell \{d_{ij}\} + f\ell \{\dot{d}_{ij}\} - f\dot{\ell} \{d_{ij}\}) / \ell^2$$

$$\{\dot{Q}_{on i}\} = \left\{ \begin{array}{c} \{\dot{F}_{on i}\} \\ (\underbrace{[B_i] \{s_i\}^P}_{})^T \{\dot{F}_{on i}\} - \dot{\phi}_i ([A_i] \{s_i\}^P)^T \{F_{on i}\} \end{array} \right\}$$

$$\{\dot{F}_{on j}\} = -(\dot{f}\ell \{d_{ij}\} + f\ell \{\dot{d}_{ij}\} - f\dot{\ell} \{d_{ij}\}) / \ell^2$$

third order
DAE

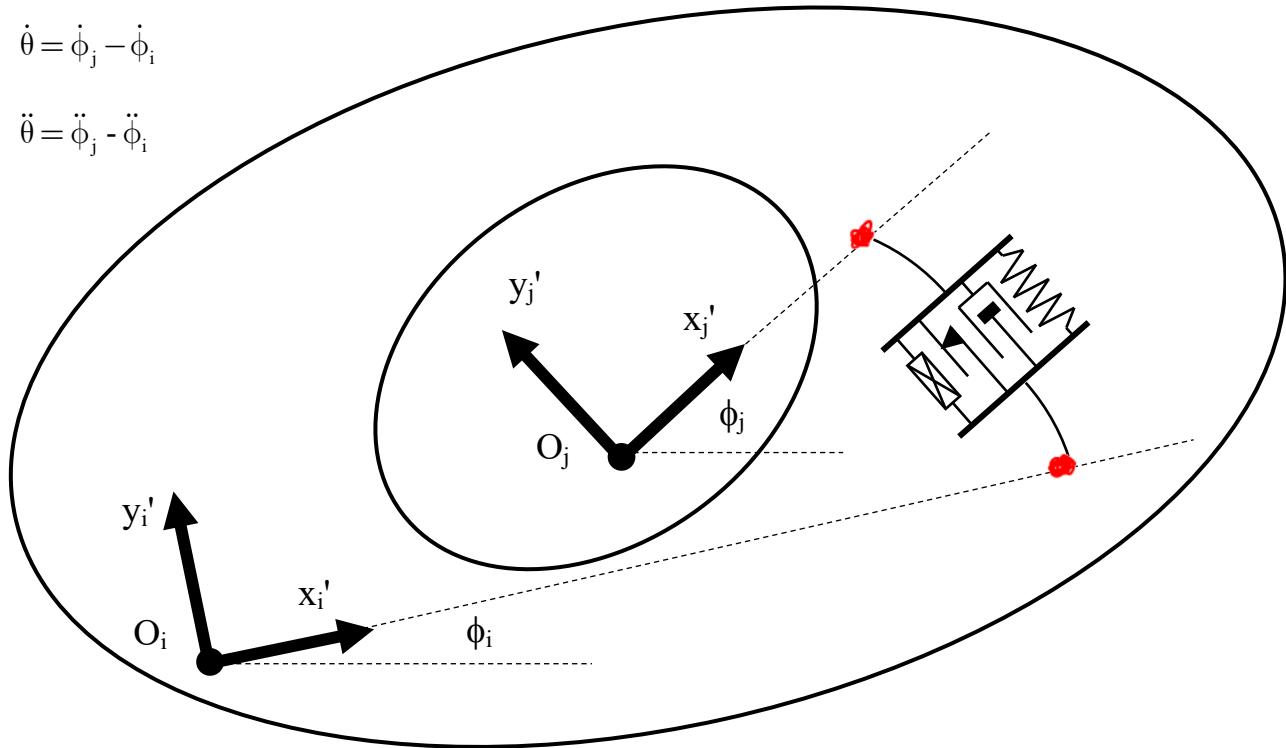
$$\{\dot{Q}_{on\ j}\} = \left\{ \begin{matrix} \{\dot{F}_{on\ j}\} \\ \left([B_j] \{s_j\}^P \right)^T \{\dot{F}_{on\ j}\} - \dot{\phi}_j \left([A_j] \{s_j\}^P \right)^T \{F_{on\ j}\} \end{matrix} \right\}$$

Rotational spring-damper-actuator

$$\theta = \phi_j - \phi_i \quad \text{Note similarity to relative angle constraint}$$

$$\dot{\theta} = \dot{\phi}_j - \dot{\phi}_i$$

$$\ddot{\theta} = \ddot{\phi}_j - \ddot{\phi}_i$$



$$T = k_\theta (\theta - \theta_o) + c_\theta \dot{\theta} + T_F \text{sign}(\dot{\theta}) + T_{ACT}(\theta, \dot{\theta}, t)$$

$$\{Q_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ T \end{Bmatrix}$$

$$\{Q_{on\ j}\} = - \begin{Bmatrix} 0 \\ 0 \\ T \end{Bmatrix}$$

$$\dot{T} = k_\theta \dot{\theta} + c_\theta \ddot{\theta} + T_{ACT}(\theta, \dot{\theta}, \ddot{\theta}, t)$$

$$\{\dot{Q}_{on\ i}\} = \begin{Bmatrix} 0 \\ 0 \\ \dot{T} \end{Bmatrix}$$

$$\{\dot{Q}_{onj}\} = - \begin{Bmatrix} 0 \\ 0 \\ \dot{T} \end{Bmatrix}$$