

~~$\phi_2 - \phi_2 \text{ start} - \sqrt{2}t = 0$~~

Notes_08_12

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insufficient

Two-Dimensional Forward Dynamics

Dynamically driven

$$\boxed{nc < nq}$$

$[\Phi_q]$ does not have full row rank

$nc \times nq$

cannot solve kinematics only

~~$\{\ddot{q}\} = [\Phi_q]^{-1} \{\gamma\}$~~

given $\{\dot{q}\}$

~~$\{\dot{q}\}$~~

~~$\{Q\}_{\text{APPLIED}}$~~

~~$[M]$~~

compute $[\Phi_q]$

~~$[\Phi_q]$~~

~~$\{\gamma\}$~~

$$\{\ddot{q}\}$$

 $nc \times 1$
 8×1

$$\{\dot{q}\}$$

 $nc \times nq$
 8×9

$$\{\gamma\}$$

 $nc \times 1$
 8×1

use forward time integration



$$\begin{bmatrix} [M]_{nq \times nq} & [\Phi_q]_{nq \times nc}^T \\ [\Phi_q]_{nc \times nc} & [0]_{nc \times nc} \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\}_{nq \times 1} \\ \{\lambda\}_{nc \times 1} \end{Bmatrix} = \begin{Bmatrix} \{Q\}_{\text{APPLIED}}_{nq \times 1} \\ \{\gamma\}_{nc \times 1} \end{Bmatrix}$$

$$[\text{EOM}] = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}_{(nc+nq) \times (nc+nq)}$$

still square

$$\{\{\ddot{q}\}\} = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{Q\}_{\text{APPLIED}} \\ \{\gamma\} \end{Bmatrix}$$

must use forward time integration of $\{\ddot{q}\}$ to get $\{\dot{q}\}$ $\{q\}$ at next time step



Differential-Algebraic Equations (DAE)

differential equations for dynamics $[\mathbf{M}] \quad [\Phi_q]^T \begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \{Q\}_{APPLIED}$

algebraic equations for kinematics $[\Phi_q] \quad [0] \begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \{\gamma\}$

Kinematics

generalized coordinates $\{q\} = \begin{Bmatrix} \{q_2\} \\ \{q_3\} \\ \{q_4\} \end{Bmatrix} \quad \{q_i\} = \begin{Bmatrix} x_i \\ y_i \\ \phi_i \end{Bmatrix} \quad \text{size } nq \times 1, nq = 3(nL-1)$

constraints $\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{DRIVER} \end{Bmatrix} \quad \text{size } nc \times 1 \quad nc = nk + nd$

Jacobian $[\Phi_q] \quad \text{element } i,j = \frac{\partial \Phi_i}{\partial q_j} \quad \text{size } nc \times nq$

Inverse dynamics

$nc = nq$ for kinematically driven problem

know driver motion at any time t , find $\{q\} \quad \{\dot{q}\} \quad \{\ddot{q}\}$

position solution $\{q\}_{NEW} = \{q\}_{OLD} + [\Phi_q]^{-1} \{\Phi\}$

velocity solution $\{\dot{q}\} = [\Phi_q]^{-1} \{v\} \quad \{v\} = \begin{Bmatrix} \{v\}_{KINEMATIC} \\ \{v\}_{DRIVER} \end{Bmatrix} \quad \text{size } nc \times 1$

$\{v\}_{KINEMATIC} = \{0_{nk \times 1}\} \quad \{v\}_{DRIVER} = -\{\Phi_t\}_{DRIVER}$

acceleration solution $\{\ddot{q}\} = [\Phi_q]^{-1} \{\gamma\} \quad \{\gamma\} = \begin{Bmatrix} \{\gamma\}_{KINEMATIC} \\ \{\gamma\}_{DRIVER} \end{Bmatrix} \quad \text{size } nc \times 1$

$\{\gamma\}_{KINEMATIC}$ based on $\{\Phi\}_{KINEMATIC} \quad \{\gamma\}_D = -\{\Phi_{tt}\}_{DRIVER}$

constraint forces $\{\lambda\} = ([\Phi_q]^T)^{-1} (\{Q\}_{APPLIED} - [\mathbf{M}] \{\ddot{q}\})$

for kinematic consistency

1) integration error
2) stiff differential eqn's

LARGEST \rightarrow
SMALLEST \rightarrow

Forward dynamics

use initial conditions for $\{q\}$ $\{\dot{q}\}$ and integrate forward in time using small increments

solve kinematics and dynamics simultaneously

$$\begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix} \begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \begin{Bmatrix} \{Q\}_{APPLIED} \\ \{\gamma\} \end{Bmatrix}$$

insufficient constraints to fully control mobility

$$\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{DRIVER} \end{Bmatrix}$$

Jacobian not full row rank

$$\begin{bmatrix} \Phi_q \\ nc \times nq \end{bmatrix}$$

EOM matrix has full row rank

$$[EOM] = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}_{(nc+nq) \times (nc+nq)}$$

example four bar mechanism with $M=1$ and no kinematic driver defined

$nq = 9$ but $nc = 8$

same size $\{q\}_{9 \times 1}$ $\{\dot{q}\}_{9 \times 1}$ $\{\ddot{q}\}_{9 \times 1}$ $[M]_{9 \times 9}$ $\{Q\}_{APPLIED \ 9 \times 1}$

smaller $\{\Phi\}_{KINEMATIC \ 8 \times 1}$ $\{\gamma\}_{KINEMATIC \ 8 \times 1} = \{0_{8 \times 1}\}$ $\{\lambda\}_{KINEMATIC \ 8 \times 1}$ $[\Phi_q]_{KINEMATIC \ 8 \times 9}$

$$[EOM] = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}_{17 \times 17}$$

must know $\{q\}$ $\{\dot{q}\}$ $[M]$ $\{Q\}_{APPLIED}$ at current time step

can compute $\{\gamma\}$ $[\Phi_q]$ at current time step

use DAE to find $\{\ddot{q}\}$ and $\{\lambda\}$

$$\begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \left(\begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix} \right)^{-1} \begin{Bmatrix} \{Q\}_{APPLIED} \\ \{\gamma\} \end{Bmatrix}$$

no drivers $\{\Phi\}_{DRIVER}$ no driver forces $\{\lambda\}_{DRIVER}$

integrate $\{\ddot{q}\}$ to predict $\{q\}$ $\{\dot{q}\}$ at next time step

example five bar mechanism with $M=2$ and no kinematic drivers defined

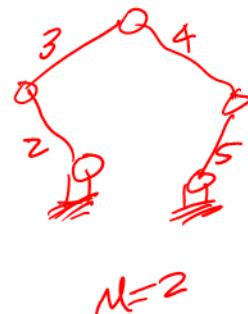
$$nq = 12 \quad \text{but} \quad nc = 10$$

$$\begin{array}{ccccc} \{q\}_{12 \times 1} & \{\dot{q}\}_{12 \times 1} & \{\ddot{q}\}_{12 \times 1} & [M]_{12 \times 12} & \{Q\}_{12 \times 1}^{\text{APPLIED}} \end{array}$$

$$\begin{array}{ccccc} \{\Phi\}_{10 \times 1}^{\text{KINEMATIC}} & \{v\}_{10 \times 1}^{\text{KINEMATIC}} = \{0_{10 \times 1}\} & \{\gamma\}_{10 \times 1}^{\text{KINEMATIC}} & \{\lambda\}_{10 \times 1}^{\text{KINEMATIC}} & [\Phi_q]_{10 \times 12}^{\text{KINEMATIC}} \end{array}$$

$$\underbrace{[EOM]}_{22 \times 22} = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}$$

$$\text{no drivers } \{\Phi\}_{\text{DRIVER}} \quad \text{no driver forces } \{\lambda\}_{\text{DRIVER}}$$



example five bar mechanism with $M=2$ and one kinematic driver defined

$$\underline{nq = 12} \quad \text{but} \quad \underline{nc = 11}$$

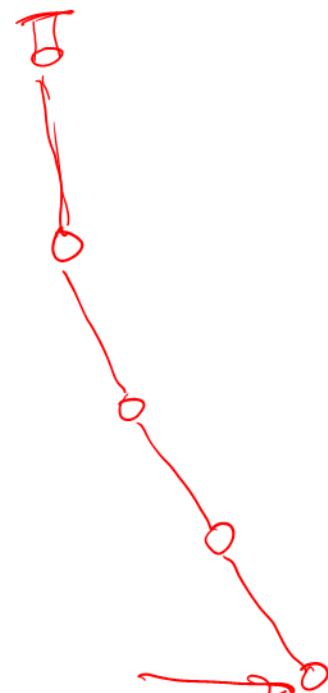
$$\text{same size} \quad \begin{array}{ccccc} \{q\}_{12 \times 1} & \{\dot{q}\}_{12 \times 1} & \{\ddot{q}\}_{12 \times 1} & [M]_{12 \times 12} & \{Q\}_{12 \times 1}^{\text{APPLIED}} \end{array}$$

$$\begin{array}{ccc} \{\Phi\}_{11 \times 1} = \begin{cases} \{\Phi\}_{\text{KINEMATIC}} \\ \Phi_{\text{DRIVER}} \end{cases} & \{v\}_{11 \times 1} = \begin{cases} \{0_{10 \times 1}\} \\ v_{\text{DRIVER}} \end{cases} & \{\gamma\}_{11 \times 1} = \begin{cases} \{\gamma\}_{\text{KINEMATIC}} \\ \gamma_{\text{DRIVER}} \end{cases} \end{array}$$

$$\begin{array}{ccc} \{\lambda\}_{11 \times 1} = \begin{cases} \{\lambda\}_{\text{KINEMATIC}} \\ \lambda_{\text{DRIVER}} \end{cases} & [\Phi_q]_{11 \times 12} = \begin{bmatrix} [\Phi_q]_{\text{KINEMATIC}} \\ [\Phi_q]_{\text{DRIVER}} \end{bmatrix} \end{array}$$

$$\underbrace{[EOM]}_{23 \times 23} = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}$$

$$\text{one driver } \Phi_{\text{DRIVER}} \quad \text{one driver force } \lambda_{\text{DRIVER}}$$



error in $\{\dot{q}\}$ and $\{\ddot{q}\}$ accumulate over time

check residuals at each time step

1) position residuals $\{\dot{q}\} \rightarrow \{\ddot{q}\}$

2) velocity residuals $[\dot{\Phi}_q] \{\dot{q}\} = \{\dot{\gamma}\} ?$