ME 581 – C01 **Name** ________________________

1) Estimate generalized coordinates $\{q\} = \{x_2 \quad y_2 \quad \phi_2 \quad x_3 \quad y_3 \quad \phi_3 \quad x_4 \quad y_4 \quad \phi_4\}^T$ for local coordinate frames attached to the web cutter shown below. Use units of cm.

Link	1	2	3	4
Origin $\{r_i\}$	$\{0, 0\}^T$	$\{0, 0\}^T$	$\{3.5, 2\}^T$	$\{3.5, 16.23\}^T$
Angle ϕ_i	0 deg	30 deg	90 deg	-60 deg

2) Complete the table of constant local body-fixed locations of specific points. Use units of cm.

	$\{s_1\}'$	$\{s_2\}'$	$\{s_3\}'$	$\{s_4\}'$
A	$\{0, 0\}^T$	$\{0, 0\}^T$		
B		$\{4, 0\}^T$	$\{0, 0\}^T$	
C			$\{14.23, 0\}^T$	$\{0, 0\}^T$
D	$\{13.21, -2.03\}^T$			$\{20.32, 0\}^T$
P			$\{16.26, -12.20\}^T$	
Q				$\{7.11, 10.16\}^T$

3) Symbolically write a constraint vector $\{\Phi\}$ for this mechanism. The driver constraint should start ϕ_2 at 30 degrees and rotate at constant 60 rpm CCW for web motion from right to left.

4) Use MATLAB to compute residuals of $\{\Phi\}$ using values from parts 1) and 2) above. You may cut/paste code from Notes_04_06.

maximum abs $\{\Phi\}$ 0.6624 cm

5) Symbolically write the entire Jacobian matrix $\lfloor\Phi_q\rfloor$.

6) Use MATLAB to evaluate the determinant of the Jacobian from part 5) using values from parts 1) and 2) above.

det $\lfloor\Phi_q\rfloor$ -144.5768 cm^2

(BC) (CD) $\sin\gamma = 144.6$ cm^2 for $\gamma \sim 30$ deg

7) Numerically estimate the Jacobian matix $\lfloor\Phi_q\rfloor$ using test_jac.m and compare to part 6).

det $\lfloor\Phi_q\rfloor$ using test_jac -144.5756 cm^2

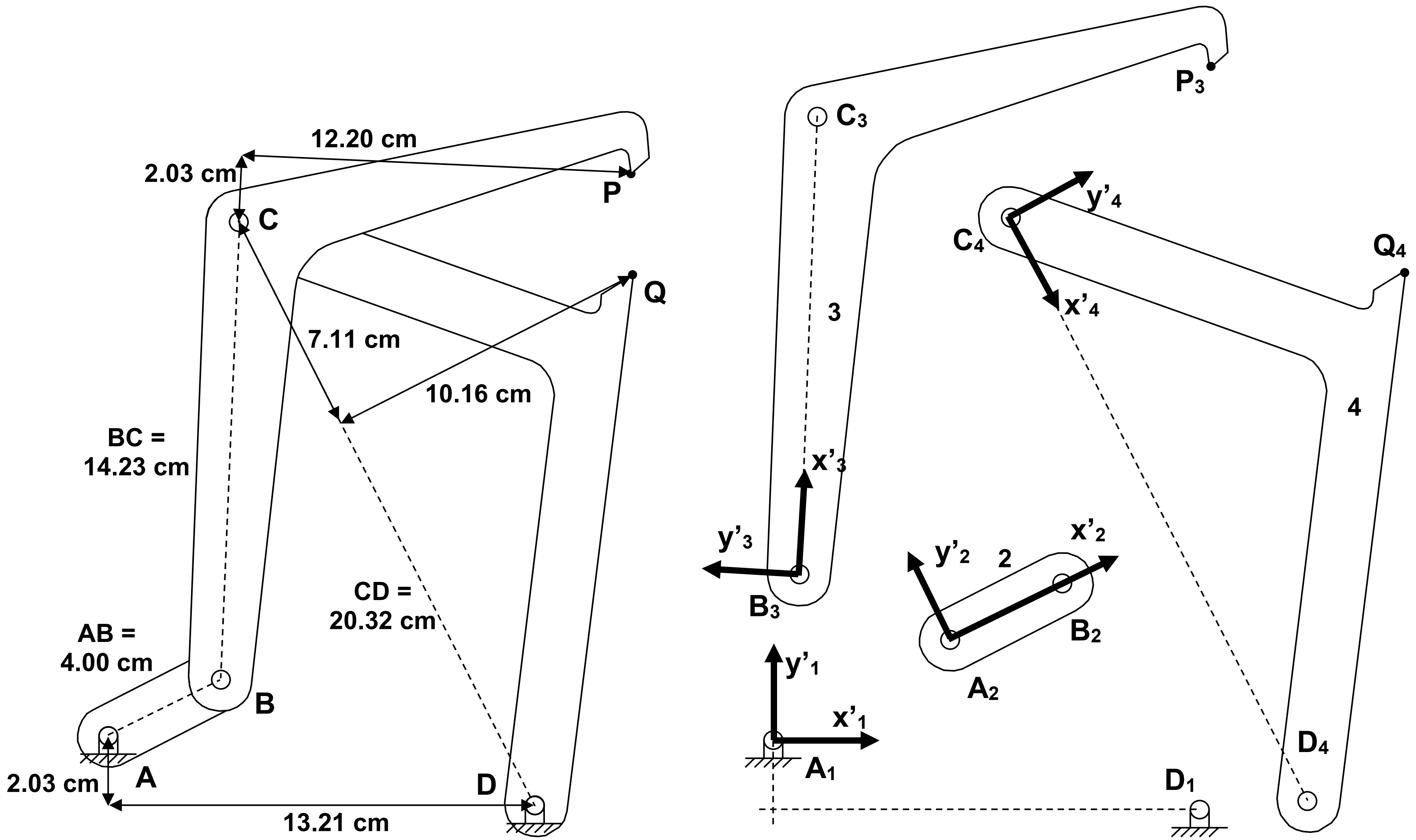
8) Replace the absolute angle driving constraint for ϕ_2 with a relative position driving constraint between y_3^P and y_4^Q that would cut the web. Reevaluate the last row of the constraint vector and the Jacobian both symbolically and numerically. Using values from parts 1) and 2) above, compute residuals for your new driving constraint and the determinant of the Jacobian.

maximum abs $\{\Phi\}$ 3.1074 cm

det $\lfloor\Phi_q\rfloor$ 382.6448 cm^3

units using new $\{s_i\}'^P = 10 *$ old $\{s_i\}'^P$

ME 581 – C01 Name ___________________________



ME 581 – C01 **Name _______________________**

```
% test_jac.m - evaluate Jacobian by numerical partial derivatives
%   used for ME 581 web cutter
% HJSIII, 20.02.19

% hold estimates for generalized coordinates
nq = length(q);
qhold = q;

% evaluate constraints
wc_phi

% hold constraints
phold = PHI;

% perturb one coordinate at a time
for iq = 1:nq,
  q = qhold;
  q(iq) = q(iq) + 0.01;

% change in constraints caused by coordinate perturbation is
% approximately equal to partial derivative
  wc_phi
  jtest(:,iq) = ( PHI - phold ) / 0.01;
end

% reset coordinates and constraints
q = qhold;
wc_phi

% bottom - test_jac
```

link 2 angular driver

$$\{\Phi\} = \begin{Bmatrix} \{\Phi\}_{KINEMATIC} \\ \{\Phi\}_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{\Phi\}_{REV\_A} \\ \{\Phi\}_{REV\_B} \\ \{\Phi\}_{REV\_C} \\ \{\Phi\}_{REV\_D} \\ \Phi_{DRIVER} \end{Bmatrix} = \begin{Bmatrix} \{r_2\}^A - \{r_1\}^A \\ \{r_3\}^B - \{r_2\}^B \\ \{r_4\}^C - \{r_3\}^C \\ \{r_4\}^D - \{r_1\}^D \\ \phi_2 - \phi_{2\_START} - \omega_2 t \end{Bmatrix} = \{0\}$$

$$[\Phi_q] = \begin{bmatrix} [I_2] & [B_2]\{s_2\}'^A & [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] \\ -[I_2] & -[B_2]\{s_2\}'^B & [I_2] & [B_3]\{s_3\}'^B & [0_{2x2}] & [0_{2x1}] \\ [0_{2x2}] & [0_{2x1}] & -[I_2] & -[B_3]\{s_3\}'^C & [I_2] & [B_4]\{s_4\}'^C \\ [0_{2x2}] & [0_{2x1}] & [0_{2x2}] & [0_{2x1}] & [I_2] & [B_4]\{s_4\}'^D \\ [0_{1x2}] & 1 & [0_{1x2}] & 0 & [0_{1x2}] & 0 \end{bmatrix}$$

++

driver using cutter points P and Q

$$\{\Phi\} \text{ row } 9 = \text{second row of } \{r_3\}^P - \{r_4\}^Q = 0$$

$$[\Phi_q] \text{ row } 9 =$$

$$\text{second row of } \big[[0_{2x2}] \quad [0_{2x1}] \quad +[I_2] \quad +[B_3]\{s_3\}'^P \quad -[I_2] \quad -[B_4]\{s_4\}'^Q\big]$$

++

INCORRECT DISTANCE DRIVER using cutter points P and Q

$$\{\Phi\}\text{ row }9=\{d_{ij}\}^T\{d_{ij}\}=0 \qquad \{d_{ij}\}=\{r_3\}^P-\{r_4\}^Q$$

same as double revolute Pj = P3 Pi = Q4 using L = 0

Double revolute

$$\Phi_{REV\_REV}=\{d_{ij}\}^T\{d_{ij}\}-L^2=0 \qquad \{d_{ij}\}=\{r_j\}^P-\{r_i\}^P$$

$$[\Phi_{qi}]_{REV\_REV}=2\{d_{ij}\}^T[\Phi_{qi}]_{REV}=-2\{d_{ij}\}^T\left[\,[I_2] \qquad [B_i]\{s_i\}'^P\,\right]$$

$$[\Phi_{qj}]_{REV\_REV}=2\{d_{ij}\}^T[\Phi_{qj}]_{REV}=2\{d_{ij}\}^T\left[\,[I_2] \qquad [B_j]\{s_j\}'^P\,\right]$$

$$\{\Phi\}_{REV}=\{r_j\}^P-\{r_i\}^P=\{0_{2x1}\}$$

$$[\Phi_{qi}]_{REV}=-\left[\,[I_2] \qquad [B_i]\{s_i\}'^P\,\right]$$

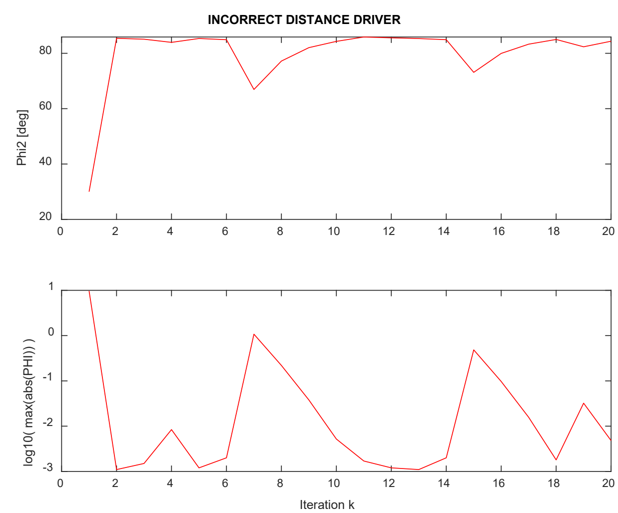
$$[\Phi_{qj}]_{REV}=\left[\,[I_2] \qquad [B_j]\{s_j\}'^P\,\right]$$

$$[\Phi_q]\text{ row }9=\left[[0_{1x3}] \qquad 2\{d_{ij}\}^T\left[\,[I_2] \qquad [B_3]\{s_3\}'^P\,\right] \qquad -2\{d_{ij}\}^T\left[\,[I_2] \qquad [B_4]\{s_4\}'^Q\,\right]\right]$$

ME 581 – C01 Name ________________________

```
>> wc_main_c01
Parts 4 and 6 ++++++++++++++++++++++++++++++++++++++++

PHI =
         0
         0
    0.0359
    0.0000
   -0.0000
         0
    0.4500
    0.6624
         0

JAC =
    1.0000         0         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0         0         0
   -1.0000         0    2.0000    1.0000         0         0         0         0         0
         0   -1.0000   -3.4641         0    1.0000         0         0         0         0
         0         0         0   -1.0000         0   14.2300    1.0000         0         0
         0         0         0         0   -1.0000   -0.0000         0    1.0000         0
         0         0         0         0         0         0    1.0000         0   17.5976
         0         0         0         0         0         0         0    1.0000   10.1600
         0         0    1.0000         0         0         0         0         0         0

detJAC =
 -144.5768

Parts 7 ++++++++++++++++++++++++++++++++++++++++

jtest =
    1.0000         0         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0         0         0
   -1.0000         0    2.0173    1.0000         0         0         0         0         0
         0   -1.0000   -3.4540         0    1.0000         0         0         0         0
         0         0         0   -1.0000         0   14.2298    1.0000         0         0
         0         0         0         0   -1.0000    0.0711         0    1.0000         0
         0         0         0         0         0         0    1.0000         0   17.5465
         0         0         0         0         0         0         0    1.0000   10.2478
         0         0    1.0000         0         0         0         0         0         0

det_jtest =
 -144.5756

Part 8 ++++++++++++++++++++++++++++++++++++++++

PHI =
         0
         0
    0.0359
    0.0000
   -0.0000
         0
    0.4500
    0.6624
    3.1074

JAC =
    1.0000         0         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0         0         0
   -1.0000         0    2.0000    1.0000         0         0         0         0         0
         0   -1.0000   -3.4641         0    1.0000         0         0         0         0
         0         0         0   -1.0000         0   14.2300    1.0000         0         0
         0         0         0         0   -1.0000   -0.0000         0    1.0000         0
         0         0         0         0         0         0    1.0000         0   17.5976
         0         0         0         0         0         0         0    1.0000   10.1600
         0         0         0         0    1.0000   12.2000         0   -1.0000  -12.3538

detJAC =
  382.6448
```

```
INCORRECT DISTANCE DRIVER ++++++++++++++++++++++++++++++++++++++++

PHI =

         0
         0
    0.0359
    0.0000
   -0.0000
         0
    0.4500
    0.6624
    9.6798


JAC =

    1.0000         0         0         0         0         0         0         0         0
         0    1.0000         0         0         0         0         0         0         0
   -1.0000         0    2.0000    1.0000         0         0         0         0         0
         0   -1.0000   -3.4641         0    1.0000         0         0         0         0
         0         0         0   -1.0000         0   14.2300    1.0000         0         0
         0         0         0         0   -1.0000   -0.0000         0    1.0000         0
         0         0         0         0         0         0    1.0000         0   17.5976
         0         0         0         0         0         0         0    1.0000   10.1600
         0         0         0   -0.3076    6.2149   80.8237   -0.3076    6.2149   76.4461


detJAC =

   4.2904e+03


convergenCe history
         k   phi2 [deg]   max(abs(PHI))
    1.0000     30.0000          9.6798
    2.0000     85.4229          0.0011
    3.0000     85.0988          0.0015
    4.0000     83.9512          0.0084
    5.0000     85.3516          0.0012
    6.0000     84.9132          0.0020
    7.0000     66.9333          1.0754
    8.0000     77.1973          0.2194
    9.0000     82.0474          0.0379
   10.0000     84.3116          0.0052
   11.0000     85.8923          0.0017
   12.0000     85.5904          0.0012
   13.0000     85.3305          0.0011
   14.0000     84.9222          0.0020
   15.0000     73.1215          0.4843
   16.0000     79.9616          0.0970
   17.0000     83.3029          0.0157
   18.0000     85.0030          0.0018
   19.0000     82.3523          0.0322
   20.0000     84.3688          0.0048
```



```
% wc_main_c01.m - web cutter four-bar for ME 581
%   main for Computer 1
% HJSIII, 22.02.16

clear

% initialize
wc_ini_c01

% evaluate constraints and Jacobian
t = 0;
wc_phi

% parts 4 and 6 - residuals and Jacobian
disp( 'Parts 4 and 6 ++++++++++++++++++++++++++++++++++++++++' )
PHI
JAC
detJAC = det( JAC )

% part 7 - numerical Jacobian
disp( 'Part 7 ++++++++++++++++++++++++++++++++++++++++' )
test_jac
jtest
det_jtest = det( jtest )

% part 8 - cutter = r3P(2) - r4Q(2);
disp( 'Part 8 ++++++++++++++++++++++++++++++++++++++++' )
PHI(9) = cutter;

JAC_P3 = [  eye(2)   B3*s3pP ];
JAC_Q4 = [ -eye(2)  -B4*s4pQ ];
JAC(9,:) = [ 0  0  0  JAC_P3(2,:)  JAC_Q4(2,:) ];
PHI
JAC
detJAC = det(JAC)

% INCORRECT DISTANCE DRIVER - cutter = trans(r3P - r4Q) * (r3P - r4Q);
disp( 'INCORRECT DISTANCE DRIVER ++++++++++++++++++++++++++++++++++++++++' )
dij = r3P - r4Q;
PHI(9) = dij' * dij;

JAC(9,:) = [ 0  0  0  2*dij'*[ eye(2)  B3*s3pP ]  -2*dij'*[ -eye(2)  -B4*s4pQ ] ];
PHI
JAC
detJAC = det(JAC)

% bottom - wc_main_c01
```

ME 581 – C01 **Name ________________________**

```
% wc_ini_c01.m - web cutter four-bar for ME 581
%   initialize constants and assembly guesses
% HJSIII, 22.02.16

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% mechanism constants
len2 = 4;
len3 = 14.23;
len4 = 20.32;

s1pA = [ 0    0 ]';
s1pD = [ 13.21  -2.03 ]';

s2pA = [ 0  0 ]';
s2pB = [ len2  0 ]';

s3pB = [ 0  0 ]';
s3pC = [ len3  0 ]';
s3pP = [ 16.26  -12.20 ]';

s4pC = [ 0  0 ]';
s4pD = [ len4  0 ]';
s4pQ = [ 7.11  10.16 ]';

% global locations
r1A = [ 0  0 ]';
r1D = [ 13.21  -2.03 ]';

% initial guesses
phi2 =  30 * d2r;
phi3 =  90 * d2r;
phi4 = -60 *  d2r;

q(1,1) = 0;
q(2,1) = 0;
q(3,1) = phi2;

q(4,1) = 3.5;
q(5,1) = 2;
q(6,1) = phi3;

q(7,1) = 3.5;
q(8,1) = 16.23;
q(9,1) = phi4;

% driver for crank - phi2 = phi2_start + w2*t
phi2_start = 30 * d2r;
w2 = +60 * 2 * pi / 60;   % 60 rpm CCW, convert to rad/sec

% bottom - wc_ini_c01
```

```matlab
% wc_phi.m - web cutter four-bar for ME 581
%   evaluate constraints and Jacobian for crank driving constraint
% HJSIII, 22.02.16

% global location of local frames and rotation matrices
r2 = q(1:2);
r3 = q(4:5);
r4 = q(7:8);
phi2 = q(3);
phi3 = q(6);
phi4 = q(9);
A2 = [ cos(phi2) -sin(phi2); sin(phi2) cos(phi2) ];
A3 = [ cos(phi3) -sin(phi3); sin(phi3) cos(phi3) ];
A4 = [ cos(phi4) -sin(phi4); sin(phi4) cos(phi4) ];
B2 = R * A2;
B3 = R * A3;
B4 = R * A4;

% global locations of points
r2A = r2 + A2*s2pA;
r2B = r2 + A2*s2pB;
r3B = r3 + A3*s3pB;
r3C = r3 + A3*s3pC;
r4C = r4 + A4*s4pC;
r4D = r4 + A4*s4pD;
r3P = r3 + A3*s3pP;
r4Q = r4 + A4*s4pQ;

% revolute constraints
PHI(1:2,1) = r2A - r1A;
PHI(3:4,1) = r3B - r2B;
PHI(5:6,1) = r4C - r3C;
PHI(7:8,1) = r4D - r1D;

% crank driver constraint
PHI(9,1) = phi2 - phi2_start - w2*t;

% Jacobian by rows
JAC = zeros(9,9);
JAC(1:2,1:3) = [  eye(2)   B2*s2pA ];

JAC(3:4,1:3) = [ -eye(2)  -B2*s2pB ];
JAC(3:4,4:6) = [  eye(2)   B3*s3pB ];

JAC(5:6,4:6) = [ -eye(2)  -B3*s3pC ];
JAC(5:6,7:9) = [  eye(2)   B4*s4pC ];

JAC(7:8,7:9) = [  eye(2)   B4*s4pD ];

% driver constraint in Jacobian
JAC(9,3) = 1;

% INCORRECT DISTANCE DRIVER
%dij = r3P - r4Q;
%PHI(9) = dij' * dij;
%JAC(9,:) = [ 0  0  0  2*dij'*[ eye(2)  B3*s3pP ]  -2*dij'*[ -eye(2)  -B4*s4pQ ] ];

% current results
current_crank = phi2 / d2r;
cutter = r3P(2) - r4Q(2);

% bottom - wc_phi
```