Additional Notes for C04

Please refer to Notes_08_11 as well as C01_solution, C02_solution and C03_solution

Interpretting Lagrange multipliers

Each row in the constraint vector $\{\Phi\}$ has a corresponding row in the Jacobian $\left[\Phi_q\right]$
Each row in the constraint vector $\{\Phi\}$ has a corresponding row in the velocity RHS $\{\nu\}$
Each row in the constraint vector $\{\Phi\}$ has a corresponding row in the acceleration RHS $\{\gamma\}$
Each row in the constraint vector $\{\Phi\}$ has a corresponding row in the Lagrange multipliers $\{\lambda\}$

Lagrange multipliers $\{\lambda\}$ are directly related to reactions at joint contraints $\{\Phi\}_{\text{KINEMATIC}}$ and forces/torques required to provide driver motion $\{\Phi\}_{\text{DRIVER}}$.

For example, C01_solution uses $\{r_3\}^B - \{r_2\}^B = \{0_{2x1}\}$ for revolute B in the third and fourth rows of the constraint vector.

Consequently, the third and fourth rows of $\{\lambda\}$ will correspond to revolute B

Following Notes_08_11 page 7, Lagrange multipliers for revolute joints $\{\lambda\}_{\text{REV}}$ are exactly equal to the reaction force at the joint.

However, you must inspect the subscripts for each your revolute constraints $\{\Phi\}_{\text{REV}}$ to ascertain if the corresponding $\{\lambda\}_{\text{REV}}$ are the force on body i or the force on body j.

Following Notes_08_11 page 7, the constraint $\{r_3\}^B - \{r_2\}^B = \{0_{2x1}\}$ for revolute B cited above uses j=3 and i=2.

Consequently $\left\{F_{3\text{ on }2}\right\} = \left\{F_{\text{on }2}\right\}_{\text{REV}} = +\{\lambda\}_{\text{REV}}$ and $\left\{F_{2\text{ on }3}\right\} = \left\{F_{\text{on }3}\right\}_{\text{REV}} = -\{\lambda\}_{\text{REV}}$

Similarly, the kinematic driver $\phi_2 - \phi_{2\_\text{START}} - \omega_2 t = 0$ which appears in the ninth row of the constraint vector $\{\Phi\}$ for C01_solution will have a corresponding λ_{DRIVER} in the ninth row of the Lagrange multipliers.

This λ_{DRIVER} is the force/torque required to cause the specifed driver motion.

Following Notes_08_11 page 9, the λ_{ANGLE} for a relative angle driver constraint is exactly equal to the torque required to cause the specified rotational motion.

However, you must again inspect the subscripts to ascertain ± directions.

Following Notes_08_11 page 9, absolute angle driver $\phi_2 - \phi_{2\_START} - \omega_2 t = 0$ may be rewritten as a relative angle driver $\phi_2 - \phi_1 - \phi_{2\_START} - \omega_2 t = 0$ using j=2 and i=1.

Consequently, $T_{1\text{ on }2} = \left(T_{\text{on }j}\right)_{ANGLE} = -\lambda_{ANGLE}$

Verbal description of differences between parts 1) and part 3)

Part 1) is kinematically driven. Crank speed $\dot{\phi}_2$ is constant. The other $\dot{\phi}$ and $\ddot{\phi}$ are not constant as the posture of the mechanism changes but at each posture they are proportional to $\dot{\phi}_2$ and $\dot{\phi}_2^{\,2}$ respectively. This is an inverse dynamic problem where we know motion at any time t and can then calculate joint forces and driver torque for ϕ_2 at any time t.

Part 1) is like you riding a bicycle at constant speed where $\dot{\phi}_2$ is the speed of the rear wheel (constant) and $\ddot{\phi}_2$=0 at all times t.

Part 3) requires forward dynamics because the driver for the crank is suddenly removed. This is like riding the bike and your foot slips off the pedal. The speed of the rear wheel $\dot{\phi}_2$ will decrease slightly as the bike coasts if you are on level ground ($\ddot{\phi}_2$ small negative). The speed of the rear wheel $\dot{\phi}_2$ will decrease much more quickly if your foot slips off the pedal and you are headed uphill ($\ddot{\phi}_2$ negative). The speed of the rear wheel $\dot{\phi}_2$ will increase if your foot slips off the pedal and you are headed downhill ($\ddot{\phi}_2$ positive).

You need to know ϕ_2 and $\dot{\phi}_2$ at the moment when your foot slips off the pedal (OLD) so that you can use friction, wind resistance and hill angle to find acceleration/deceleration $\ddot{\phi}_2$ (OLD but not zero) and calculate ϕ_2 and $\dot{\phi}_2$ at some time t=0.05 sec in the future (NEW).

For part 3) run your code at ϕ_2=85.5° and $\dot{\phi}_2$=60 rpm CCW to find all $\{q\}_{OLD}$ and $\{\dot{q}\}_{OLD}$ at the moment t=0 when the driver for link 2 is cut (OLD). The speed of the crank $\dot{\phi}_{2\_NEW}$ will no longer be constant and $\ddot{\phi}_{2\_NEW}$ will no longer be zero because that driver is now gone. The speed of the crank may increase or decrease just like your bicycle. You need to use the 17x17 forward dynamic equation to calculate $\{\ddot{q}\}_{OLD}$ at time t=0 using current values for $\{q\}_{OLD}$ and $\{\dot{q}\}_{OLD}$ just like the bike. Then you can predict the future for $\{q\}_{NEW}$ and $\{\dot{q}\}_{NEW}$ at time t=0.05 sec.

<u>C04 part 1)</u>

This is a kinematically driven problem that will use inverse dynamics.

Copy your code for C03 into a new set of routines for C04 and run them to obtain values for positions $\{q\}$, velocities $\{\dot{q}\}$ and accelerations $\{\ddot{q}\}$ using $\phi_{2\_START} = 0$ deg and $\omega_2 = 60$ rpm CCW **<u>WITHOUT ANY DYNAMICS</u>**.

Then provide values for m_2, J_{G2}, m_3, J_{G3}, m_4 and J_{G4} using either your values for C03 or values from C03_solution,

You may also use your centroid values $\{s_i\}'^P$ or values from C03_solution in your new code.

Create $[M] = \text{diag}(\; m_2 \;\; m_2 \;\; J_{G2} \;\; m_3 \;\; m_3 \;\; J_{G3} \;\; m_4 \;\; m_4 \;\; J_{G4}\;)$

Create $\{Q\}_{APPLIED} = \text{transpose}(\; 0 \;\; -m_2*g \;\; 0 \;\; 0 \;\; -m_3*g \;\; 0 \;\; 0 \;\; -m_4*g \;\; 0\;)$ to provide gravity loading on each link.

Be certain to use units for g that are the same as translational accelerations in your code.

Following Notes_08_11 page 5, use inverse dynamics to calculate accelerations $\{\ddot{q}\}$ and Langrange multipliers $\{\lambda\}$ with -

$$\begin{Bmatrix} \{\ddot{q}\} \\ \{\lambda\} \end{Bmatrix} = \begin{bmatrix} [M] & [\Phi_q]^T \\ [\Phi_q] & [0] \end{bmatrix}^{-1} \begin{Bmatrix} \{Q\}_{APPLIED} \\ \{\gamma\} \end{Bmatrix}$$

Accelerations $\{\ddot{q}\}$ from kinematically driven inverse dynamics should match accelerations $\{\ddot{q}\}$ from pure kinematics exactly.

Interpret Langrange multipliers $\{\lambda\}$ per above.

<u>C04 part 2)</u>

Repeat part 1) using $\phi_{2\_START} = 85.5$ deg and retain the values for positions $\{q\}$, velocities $\{\dot{q}\}$ at this posture for part 3) and EXTRA CREDIT.

<u>C04 part 3)</u>

This is a dynamically driven problem that will use forward dynamics.

It will become forward dynamics if the driver constraint is removed.

Use the positions $\{q\}_{OLD}$ and velocities $\{\dot{q}\}_{OLD}$ from part 2) as initial conditions immediately before the driver is removed.

Immediately after the driver is removed, the constraint vector $\{\Phi\}$ will become 8x1, the Jacobian $\left[\Phi_q\right]$ will become 8x9 and the acceleration RHS $\{\gamma\}$ will also become 8x1.

You will not be able to solve the purely kinematic acceleration equation $\{\ddot{q}\} = \left[\Phi_q\right]^{-1}\{\gamma\}$ because the mechanism is no longer kinematically driven and the Jacobian has insufficient row rank.

However, you will still be able to solve the full dynamic equation.

$$\left\{\begin{matrix} \underset{9\times1}{\{\ddot{q}\}} \\ \\ \underset{8\times1}{\{\lambda\}} \end{matrix}\right\} = \begin{bmatrix} \underset{9\times9}{[M]} & \underset{9\times8}{\left[\Phi_q\right]^T} \\ \\ \underset{8\times9}{\left[\Phi_q\right]} & \underset{8\times8}{[0]} \end{bmatrix}^{-1} \left\{\begin{matrix} \underset{9\times1}{\{Q\}_{APPLIED}} \\ \\ \underset{8\times1}{\{\gamma\}} \end{matrix}\right\}$$

Note the Lagrange multipliers $\{\lambda\}$ will become 8x1 because there is no corresponding driver constratint.

Accelerations $\{\ddot{q}\}_{PART\_2}$ correspond to constant speed crank rotation and will not match $\{\ddot{q}\}_{PART\_3}$ where there is no driver to maintain crank speed.

Use initial conditions $\{q\}_{OLD}$ and $\{\dot{q}\}_{OLD}$ from part 2) along with the dynamically driven accelerations $\{\ddot{q}\}_{OLD} = \{\ddot{q}\}_{PART\_3}$ from part 3) to integrate one time step h forward in time using the simplistic integrators.

<u>C04 part 4)</u>

Check kinematic consistency for the new positions $\{q\}_{NEW}$ and velocities $\{\dot{q}\}_{NEW}$ at one time step into the future.

They will not be perfectly consistent because of the large stime step and rudimentary integrators.