ME 581 – C04 **Name** ________________________

1) Plot the crank torque required to drive the web cutter at constant crank speed 60 rpm CCW through one full revolution of the crank. Assume that the mechanism operates in a vertical plane, that there are no external forces on the mechanism other than the weight of the links, and that friction is negligible. Neglect cutting forces.

constant speed crank 6.28 rad/sec CCW

2) What are the bearing reactions and crank torque for part 1) when $\phi_2 = 85.5$ degrees?

T_{1on2} 0.377 N.cm CW F_{1on2} (0.113 ,0.391 $)^T$ N F_{2on3} (0.115 ,0.113 $)^T$ N

F_{3on4} (0.351 ,-1.338 $)^T$ N F_{1on4} (-0.543 ,2.820 $)^T$ N

3) Assume that the crank of the web cutter is rotating at constant crank speed 60 rpm CCW when the crank motor power is suddenly cut at $\phi_2 = 85.5°$. Determine the acceleration of the crank at the moment when crank torque is cut to zero, and predict crank angle and crank speed at h = 0.05 seconds later.

use $\{q\}_{NEW} = \{q\}_{OLD} + \{\dot{q}\}_{OLD} h + 0.5\{\ddot{q}\}_{OLD} h^2$ and $\{\dot{q}\}_{NEW} = \{\dot{q}\}_{OLD} + \{\ddot{q}\}_{OLD} h$

$\ddot{\phi}_2$ 21.72 rad/sec^2 CCW $\dot{\phi}_2$ at h later 7.37 rad/sec CCW ϕ_2 at h later 105.06 deg

4) Check the kinematic consistency of your $\{q\}_{NEW}$ and $\{\dot{q}\}_{NEW}$ predicted at h later.

max abs $\{\Phi\}$ 0.0177 cm max abs $[\Phi_q]\{\dot{q}\}$ 1.0490 cm/sec

EXTRA CREDIT

Determine the STATIC crank torque and bearing reactions at $\phi_2 = 85.5°$ required to produce 10 N vertical cutting force between cutters P and Q of the web cutter. Assume there are no external forces on the mechanism other than the vertical cutting forces. Assume that friction is negligible. Do not include the weight of the links. Do not include dynamic effects.

T_{1on2} 31.56 N.cm CCW F_{1on2} (-7.64 , 3.54 $)^T$ N F_{2on3} (-7.64 , 3.54 $)^T$ N

F_{3on4} (-7.64 , 13.54 $)^T$ N F_{1on4} (+7.64 ,-3.54 $)^T$ N

EXTRA EXTRA CREDIT

Use velocities and virtual work to determine STATIC crank torque at $\phi_2 = 85.5°$ per above.

T_{1on2} 31.57 N.cm CCW

ME 581 – C04 Name ________________________

Parts 1 and 2 - **INVERSE DYNAMICS**

$$\{Q_{on\ 2}\} = \begin{Bmatrix} 0 \\ -m_2 g \\ 0 \end{Bmatrix} \qquad \{Q_{on\ 3}\} = \begin{Bmatrix} 0 \\ -m_3 g \\ 0 \end{Bmatrix} \qquad \{Q_{on\ 4}\} = \begin{Bmatrix} 0 \\ -m_4 g \\ 0 \end{Bmatrix} \qquad \{Q\}_{APPLIED} = \begin{Bmatrix} \{Q_{on\ 2}\} \\ \{Q_{on\ 3}\} \\ \{Q_{on\ 4}\} \end{Bmatrix}$$

$$\underset{9x9}{[M]}\underset{9x1}{\{\ddot{q}\}} + \underset{9x9}{[\Phi_q]^T}\underset{9x1}{\{\lambda\}} = \underset{9x1}{\{Q\}_{APPLIED}} \qquad \{\lambda\} = \left([\Phi_q]^T\right)^{-1}\left(\{Q\}_{APPLIED} - [M]\{\ddot{q}\}\right)$$

$$\begin{bmatrix} \underset{9x9}{[M]} & \underset{9x9}{[\Phi_q]^T} \\ \underset{9x9}{[\Phi_q]} & \underset{9x9}{[0]} \end{bmatrix}_{18x18} \begin{Bmatrix} \underset{9x1}{\{\ddot{q}\}} \\ \underset{9x1}{\{\lambda\}} \end{Bmatrix}_{18x1} = \begin{Bmatrix} \underset{9x1}{\{Q\}_{APPLIED}} \\ \underset{9x1}{\{\gamma\}} \end{Bmatrix}_{18x1}$$

units $m = [kg]$ $\ddot{r} = [cm/sec^2]$ $J' = [kg.cm^2]$ $\ddot{\phi} = [rad/sec^2]$
$F = [kg.cm/sec^2]$ $T = [kg.cm^2/sec^2]$

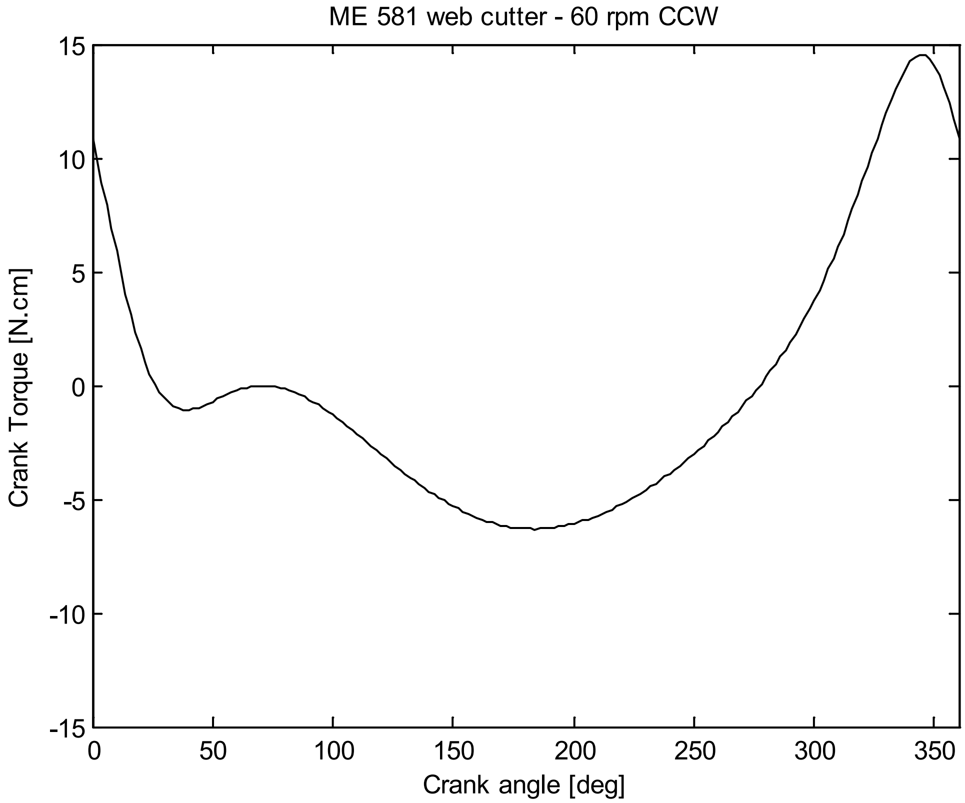
$$\{\Phi\}_{REV} = \{r_j\}^P - \{r_i\}^P \qquad \{\lambda\}_{REV} = -\{F_{on\ j}\} = -\{F_{i\ on\ j}\}$$

$$\Phi_{DRIVER} = \phi_j - \phi_i - C - f(t) \qquad \lambda_{DRIVER} = -T_{on\ j}$$

$$\{\Phi\} = \begin{Bmatrix} \{r_2\}^A - \{r_1\}^A \\ \{r_3\}^B - \{r_2\}^B \\ \{r_4\}^C - \{r_3\}^C \\ \{r_4\}^D - \{r_1\}^D \\ \phi_2 - \phi_{2\_0} - \omega_2 t \end{Bmatrix} \quad \begin{matrix} j = 2, i = 1 \\ j = 3, i = 2 \\ j = 4, i = 3 \\ j = 4, i = 1 \\ j = 2, i = 1 \end{matrix} \quad \{\lambda\} = \begin{Bmatrix} -\{F_{1\ on\ 2}\} \\ -\{F_{2\ on\ 3}\} \\ -\{F_{3\ on\ 4}\} \\ -\{F_{1\ on\ 4}\} \\ -T_{1\ on\ 2} \end{Bmatrix}$$

Same code for
wc_ini_c03.m wc_phi.m wc_kin.m

Verified same graphs as C03




```
% wc_main_c04_part1.m - web cutter four-bar for ME 581
%   main for C04 part 1
% HJSIII - 20.03.04

clear

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% initialize - centroidal coordinates
wc_ini_c03

% mass matrix - units [kg] and [kg*cm*cm]
m2 = 0.03082;
m3 = 0.15962;
m4 = 0.15180;
J2p = 0.091;
J3p = 6.448;
J4p = 6.808;

M = diag( [ m2  m2  J2p  m3  m3  J3p  m4  m4  J4p ] );

% applied external forces - units [kg*cm/s/s]
% weight due to gravity in negative y direction
Q2=[ 0  -981*m2  0 ]';
Q3=[ 0  -981*m3  0 ]';
Q4=[ 0  -981*m4  0 ]';
QA=[ Q2 ; Q3 ; Q4 ];

% starting position
phi2_start = 0 * d2r;  % start at zero
%phi2_start = 30 * d2r;  % position in line drawing
%phi2_start = 90 * d2r;  % check at 90 degrees
%phi2_start =  85.5 * d2r;  % cutter closing
%q = [ 0  0  4.4925  -0.8726  -3.9037  1.7233  -3.0341  10.1612  -0.6438]'; % q at opening
```

```
%phi2_start =  257.4 * d2r;  % cutter opening

% time loop
%tpr = 0;                       % single poisition
tpr = 2 * pi / w2;             % one revolution at constant speed
t_start = 0;                   % start
t_end = tpr;                   % end
nt = 180;                      % number of time steps
dt = (t_end - t_start) / nt;

keep_q = [];
keep = [];
for t = t_start : dt : t_end;

% kinematics
  wc_kin

% inverse dynamics - solve for Lagrange multipliers
  lambda = inv(JAC') * ( QA - M*qdd );

% revolute bearing forces - units [kg*cm/s/s]
% constraints = rj-ri for r2A-r1A, r3B-r2B, r4C-r3C, r4D-r1D, phi2-phi1
% lambda = -F i on j
  F1on2 = -lambda(1:2) / 100;     % convert to N, j=2 i=1
  F2on3 = -lambda(3:4) / 100;     % convert to N, j=3 i=2
  F3on4 = -lambda(5:6) / 100;     % convert to N, j=4 i=3
  F1on4 = -lambda(7:8) / 100;     % convert to N, j=4 i=1
  T1on2 = -lambda(9) / 100;       % convert to N.cm, j=2 i=1

% check crank torque by simple virtual work
  T1on2_vw = -qd' * ( QA - M*qdd ) / phi2d;
  T1on2_vw = T1on2_vw / 100;          % convert to N.cm

% save kinematics and forces
  ang2 = phi2 / d2r;
  detJAC = det(JAC);
  keep_q = [ keep_q ; q'  qd'  qdd' ];
  keep = [ keep ; detJAC T1on2 T1on2_vw ];

% bottom - for t
end

% plot
ang2 = keep_q(:,3) /d2r;
phi3dd = keep_q(:,24);
phi4dd = keep_q(:,27);
detJAC = keep(:,1);
T1on2 = keep(:,2);
T1on2_vw = keep(:,3);

figure( 1 )
  plot( ang2,phi3dd,'k--', ang2,phi4dd,'k-' )
  axis( [ 0 360 -40 40 ] )
  title( 'ME 581 web cutter - 60 rpm CCW' )
  xlabel( 'Crank angle [deg]' )
  ylabel( 'Angular acceleration [rad/s/s]' )
  legend( 'Link 3', 'Link 4' )

figure( 2 )
  plot( ang2,detJAC,'k' )
  axis( [ 0 360 -300 0 ] )
  title( 'ME 581 web cutter - 60 rpm CCW' )
  xlabel( 'Crank angle [deg]' )
  ylabel( 'Determinant of Jacobian [cm^2]' )

figure( 3 )
  plot( ang2,T1on2,'k' )
  axis( [ 0 360 -15 15 ] )
  title( 'ME 581 web cutter - 60 rpm CCW' )
  xlabel( 'Crank angle [deg]' )
  ylabel( 'Crank Torque [N.cm]' )

figure( 4 )
```

```
  plot( ang2,T1on2,'k', ang2,T1on2_vw,'ro' )
  axis( [ 0 360 -15 15 ] )
  title( 'ME 581 web cutter - 60 rpm CCW' )
  xlabel( 'Crank angle [deg]' )
  ylabel( 'Crank Torque [N.cm]' )

% bottom - wc_main_c04_part1
```

```
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>> wc_main_c04_part2

   F1on2_x    F1on2_y    F2on3_x    F2on3_y    F3on4_x    F3on4_y    F1on4_x    F1on4_y     T1on2
       [N]        [N]        [N]        [N]        [N]        [N]        [N]        [N]    [N.cm]
    0.1133     0.3905     0.1152     0.1125     0.3512    -1.3378    -0.5429     2.8204   -0.3767

T1on2_vw = -0.3767

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% wc_main_c04_part2.m - web cutter four-bar for ME 581
%   main for C04 part 2
% HJSIII - 20.03.04

clear

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% initialize - centroidal coordinates
wc_ini_c03

% mass matrix - units [kg] and [kg*cm*cm]
m2 = 0.03082;
m3 = 0.15962;
m4 = 0.15180;
J2p = 0.091;
J3p = 6.448;
J4p = 6.808;

M = diag( [ m2  m2  J2p  m3  m3  J3p  m4  m4  J4p ] );

% applied external forces - units [kg*cm/s/s]
% weight due to gravity in negative y direction
Q2=[ 0  -981*m2  0 ]';
Q3=[ 0  -981*m3  0 ]';
Q4=[ 0  -981*m4  0 ]';
QA=[ Q2 ; Q3 ; Q4 ];

% starting position
phi2_start = 85.5 * d2r;  % start when cutter is closing
t = 0;

% kinematics
  wc_kin

% inverse dynamics - solve for Lagrange multipliers
  lambda = inv(JAC') * ( QA - M*qdd );

% revolute bearing forces - units [kg*cm/s/s]
% constraints = rj-ri for r2A-r1A, r3B-r2B, r4C-r3C, r4D-r1D, phi2-phi1
% lambda = -F i on j
  F1on2 = -lambda(1:2) / 100;     % convert to N, j=2 i=1
  F2on3 = -lambda(3:4) / 100;     % convert to N, j=3 i=2
  F3on4 = -lambda(5:6) / 100;     % convert to N, j=4 i=3
  F1on4 = -lambda(7:8) / 100;     % convert to N, j=4 i=1
  T1on2 = -lambda(9) / 100;       % convert to N.cm, j=2 i=1

% check crank torque by simple virtual work
  T1on2_vw = -qd' * ( QA - M*qdd ) / phi2d;
  T1on2_vw = T1on2_vw / 100;          % convert to N.cm

% show results
disp( '   F1on2_x    F1on2_y    F2on3_x    F2on3_y    F3on4_x    F3on4_y    F1on4_x    F1on4_y     T1on2' )
disp( '       [N]        [N]        [N]        [N]        [N]        [N]        [N]        [N]    [N.cm]' )
disp( [ F1on2'  F2on3'  F3on4'  F1on4'  T1on2 ] )

T1on2_vw

% bottom - wc_main_c04_part2
```

ME 581 – C04 **Name** ________________________

Parts 3 and 4 - **FORWARD DYNAMICS**

remove driver constraint

remove row 9 of $\{\Phi\}$ $[\Phi_q]$ $\{\nu\}$ $\{\gamma\}$ $\{\lambda\}$

$$\begin{bmatrix} \underset{9x9}{[M]} & \underset{9x8}{[\Phi_q]^T} \\ \underset{8x9}{[\Phi_q]} & \underset{8x8}{[0]} \end{bmatrix}_{17x17} \begin{Bmatrix} \underset{9x1}{\{\ddot{q}\}} \\ \underset{8x1}{\{\lambda\}} \end{Bmatrix}_{17x1} = \begin{Bmatrix} \underset{9x1}{\{Q\}_{APPLIED}} \\ \underset{8x1}{\{\gamma\}} \end{Bmatrix}_{17x1}$$

```
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>> wc_main_c04_part3

phi2dd = 21.7197          phi2d_new = 7.3692          phi2_new = 105.0556

PHI_test =
   -0.0118    0.0177   -0.0159    0.0120    0.0050   -0.0076   -0.0002    0.0070

vel_test =
   -0.7702    1.0490   -1.0025    0.7281    0.3209   -0.4652   -0.0164    0.4515

+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
```

```
% wc_main_c04_part3.m - web cutter four-bar for ME 581
%   main for C04 part 3
% HJSIII - 20.03.04

clear

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% initialize - centroidal coordinates
wc_ini_c03

% mass matrix - units [kg] and [kg*cm*cm]
m2 = 0.03082;
m3 = 0.15962;
m4 = 0.15180;
J2p = 0.091;
J3p = 6.448;
J4p = 6.808;

M = diag( [ m2  m2  J2p  m3  m3  J3p  m4  m4  J4p ] );

% applied external forces - units [kg*cm/s/s]
% weight due to gravity in negative y direction
Q2=[ 0  -981*m2  0 ]';
Q3=[ 0  -981*m3  0 ]';
Q4=[ 0  -981*m4  0 ]';
QA=[ Q2 ; Q3 ; Q4 ];

% starting position
phi2_start = 85.5 * d2r;  % start when cutter is closing
t = 0;

% standard kinematics immediately before driver is cut
  wc_kin

% forward dynamics - solve for qdd and Lagrange multipliers simultaneously
% NOTE:  torque on link2 is cut to zero
  JAC89 = JAC(1:8,1:9);
```

```
  gamma8 = accrhs(1:8);
  EOMmat = [ M      JAC89'     ;
             JAC89  zeros(8,8) ];
  EOMrhs = [ QA     ;
             gamma8 ];
  EOMsol = inv(EOMmat) * EOMrhs;
  qdd = EOMsol(1:9);
  lambda = EOMsol(10:17);

% predict one time step forward
  h = 0.05;
  q_new = q + qd*h + qdd*h*h/2;
  qd_new = qd + qdd*h;

  phi2dd = qdd(3)
  phi2d_new = qd_new(3)
  phi2_new = q_new(3) /d2r

% check kinematic position consistency at predicted time step
% NOTE:  this will change JAC just a little
  q = q_new;
  wc_phi
  PHI_test = PHI(1:8)'

% check kinematic position consistency at predicted time step
  vel_test = ( JAC(1:8,:) * qd_new )'

% bottom - wc_main_c04_part3
```

ME 581 – C04 **Name** ________________________

Extra credit - **STATIC**

$$\{F\}^P = \begin{Bmatrix} 0 \\ +10\ \text{N} \end{Bmatrix} \qquad \{F\}^Q = \begin{Bmatrix} 0 \\ -10\ \text{N} \end{Bmatrix}$$

$$\{Q_{on\,2}\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \qquad \{Q_{on\,3}\} = \begin{Bmatrix} \{F\}^P \\ \{s_3\}'^{P\,T} [B_3]^T \{F\}^P \end{Bmatrix} \qquad \{Q_{on\,4}\} = \begin{Bmatrix} \{F\}^Q \\ \{s_4\}'^{Q\,T} [B_4]^T \{F\}^Q \end{Bmatrix}$$

$$\{Q\}_{APPLIED} = \begin{Bmatrix} \{Q_{on\,2}\} \\ \{Q_{on\,3}\} \\ \{Q_{on\,4}\} \end{Bmatrix}$$

$$[M]\{\ddot{q}\} + [\Phi_q]^T\{\lambda\} = \{Q\}_{APPLIED} \qquad \{\ddot{q}\} = \{0\} \qquad \{\lambda\} = \left([\Phi_q]^T\right)^{-1}\{Q\}_{APPLIED}$$

VIRTUAL WORK

$$T_{12} \circ \omega_2 + \{F\}^P \circ \{\dot{r}_3\}^P + \{F\}^Q \circ \{\dot{r}_4\}^Q = 0$$

units ω = [rad/sec] F = [N] $\dot{r}$ = [cm/sec] T = [N.cm]

```
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>> wc_main_c04_ec

   F1on2_x   F1on2_y   F2on3_x   F2on3_y   F3on4_x   F3on4_y   F1on4_x   F1on4_y     T1on2
       [N]       [N]       [N]       [N]       [N]       [N]       [N]       [N]    [N.cm]
   -7.6376    3.5386   -7.6376    3.5386   -7.6376   13.5386    7.6376   -3.5386   31.5666

T12_vw = 31.5666

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% wc_main_c04_ec.m - web cutter four-bar for ME 581
%   main for C04 extra credit
% HJSIII - 20.03.04

clear

% general constants
d2r = pi / 180;
R = [ 0 -1; 1 0 ];

% initialize - centroidal coordinates
wc_ini_c03

% starting position
phi2_start = 85.5 * d2r;  % start when cutter is closing
t = 0;

% kinematics - need postion, need Jacobian
%   use velocity for virtual work, do not need accleration
wc_kin

% applied external forces - units [N]
```

```
% 10 N vertical cutter forces between points P3 and Q4
FP = [ 0   10 ]';
FQ = [ 0  -10 ]';

Q2 = [ 0  0  0 ]';
Q3 = [ FP             ;
       s3pP'*B3'*FP ];
Q4 = [ FQ             ;
       s4pQ'*B4'*FQ ];
QA = [ Q2 ; Q3 ; Q4 ];

% inverse dynamics - solve for Lagrange multipliers
lambda = inv(JAC') * QA;

% revolute bearing forces - units [kg*cm/s/s]
% constraints = rj-ri for r2A-r1A, r3B-r2B, r4C-r3C, r4D-r1D, phi2-phi1
% lambda = -F i on j
F1on2 = -lambda(1:2);      % j=2 i=1    already [N], do not convert
F2on3 = -lambda(3:4);      % j=3 i=2
F3on4 = -lambda(5:6);      % j=4 i=3
F1on4 = -lambda(7:8);      % j=4 i=1
T1on2 = -lambda(9);        % j=2 i=1

% show results
disp( '   F1on2_x   F1on2_y   F2on3_x   F2on3_y   F3on4_x   F3on4_y   F1on4_x   F1on4_y
T1on2'  )
disp( '       [N]       [N]       [N]       [N]       [N]       [N]       [N]       [N]
[N.cm]'  )
disp( [ F1on2'  F2on3'  F3on4'  F1on4'  T1on2 ] )

% check crank torque by simple virtual work
T12_vw = -( FP'*r3Pd + FQ'*r4Qd ) / phi2d

% bottom - wc_main_c04_ec
```