ME 581 – C05 **Name ________________________**

1) For a right McPherson strut, attach local coordinate frames to each link for a fixed position of the steering rack. Clearly label all auxiliary points and vectors needed to form constraints.

2) Write a corresponding set of generalized coordinates $\{q\}$.

3) Symbolically write a constraint vector $\{\Phi\}$ for this mechanism using the vertical absolute translation driver for the strut-spindle assembly $y_3 - y_{3\_START} - y_{3\_VEL}\ t = 0$ for $y_{3\_VEL} = 4\ \mathrm{cm/sec}$.

4) Numerically check the residuals of $\{\Phi\}$ using measurements from model hardware.

5) Symbolically write the corresponding Jacobian matrix $\lfloor \Phi_q \rfloor$.

6) Numerically evaluate the elements of the Jacobian and compute the determinant.

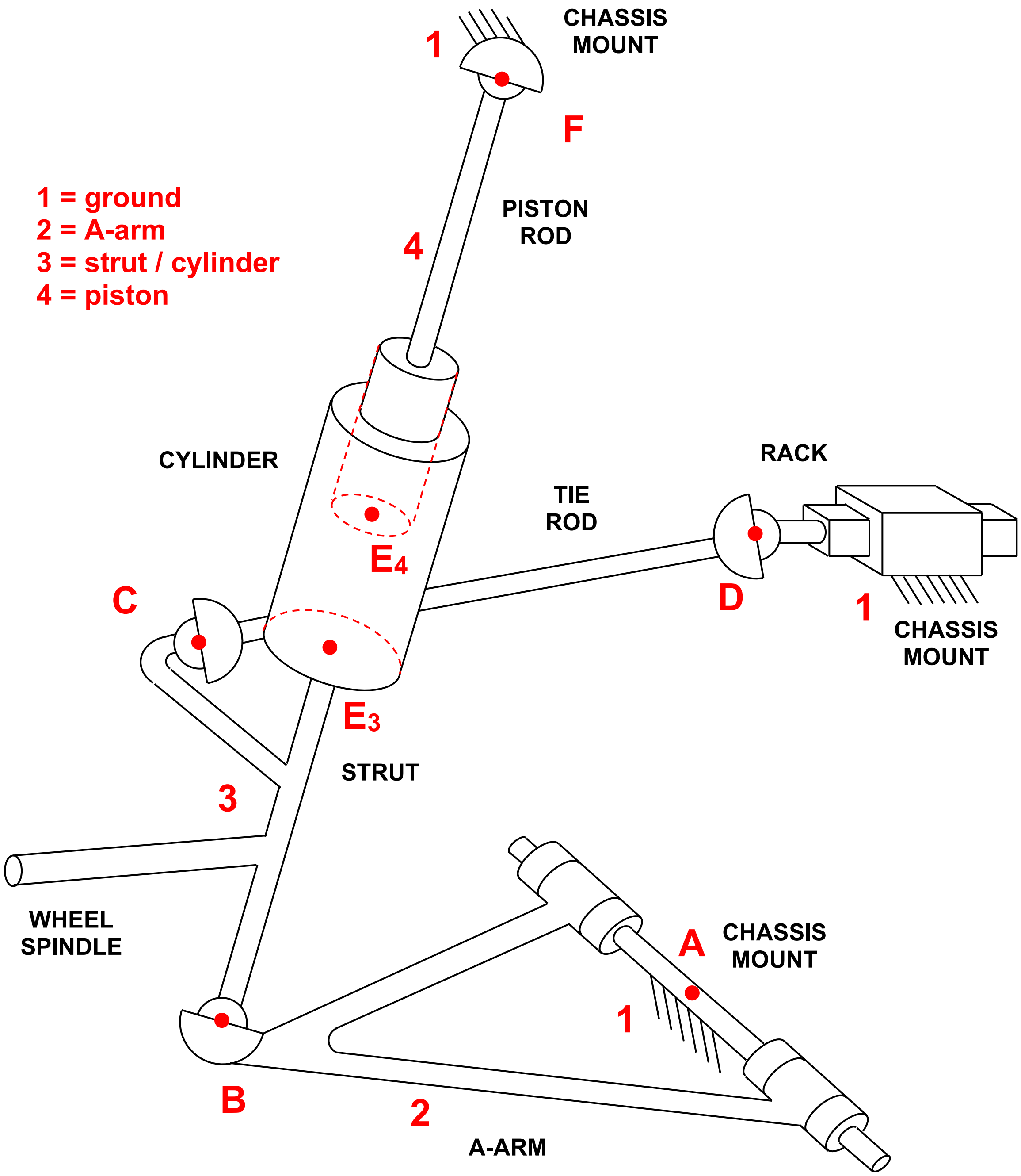
7) Program your constraints and Jacobian into a Newton-Raphson iterative algorithm to solve position kinematics at any desired time.

8) Symbolically write the velocity right-hand-side vector and program to solve for generalized coordinate velocities at any desired time.

9) Place your position and velocity algorithms within an outer loop to drive the strut-spindle assembly over $0 \le t \le 1$ sec. Plot toe-in angle versus vertical position of the strut-spindle assembly. Additionally plot all three components of global angular velocity of the strut-spindle assembly versus time.

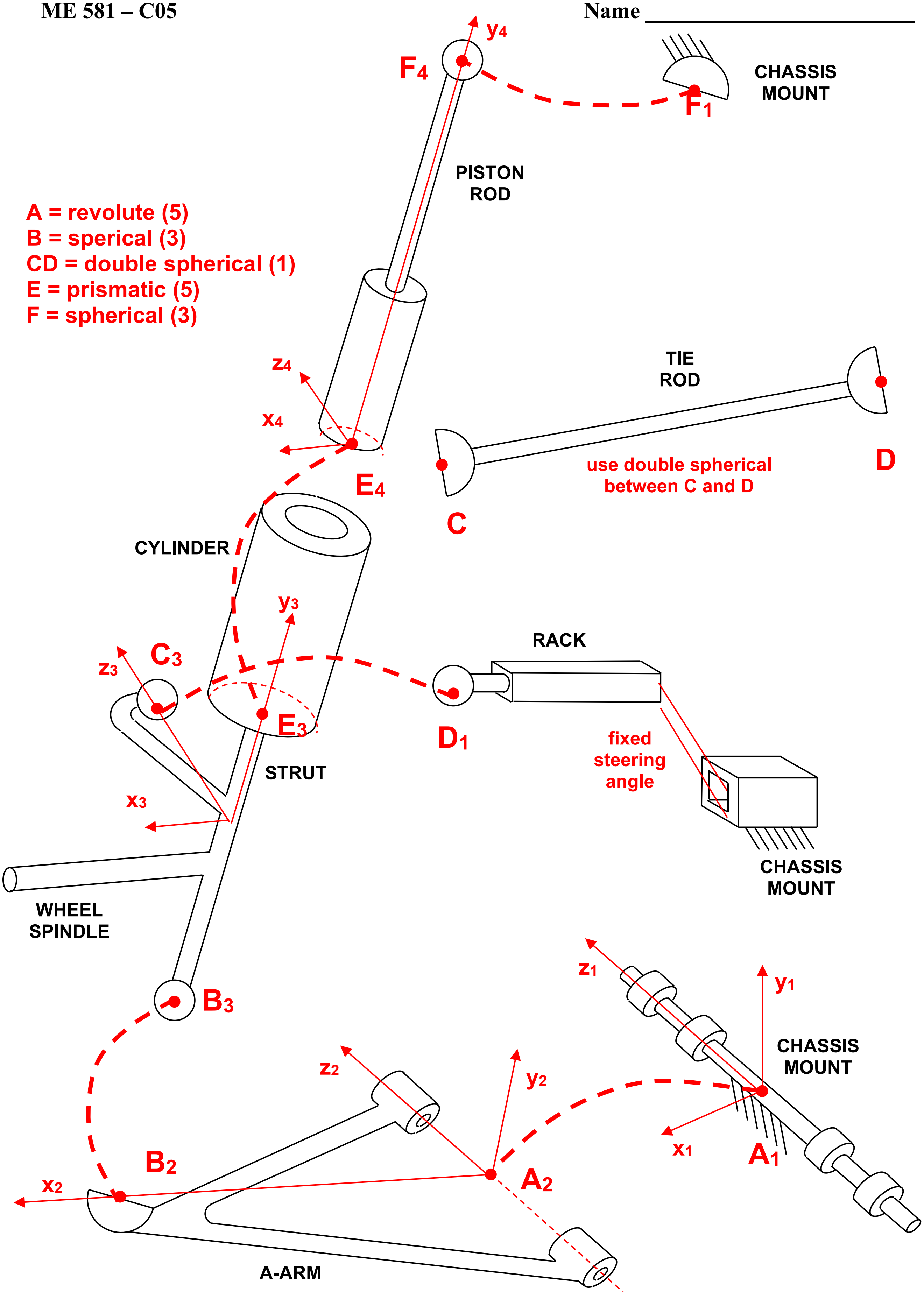
EXTRA CREDIT
Symbolically write the acceleration right-hand-side vector and program to solve for generalized coordinate accelerations. Plot all three components of global angular acceleration of the strut-spindle assembly versus time.

Name ________________________



1 = ground
2 = A-arm
3 = strut / cylinder
4 = piston

Name ____________________________



y4
F4
CHASSIS MOUNT
F1
PISTON ROD
A = revolute (5)
B = sperical (3)
CD = double spherical (1)
E = prismatic (5)
F = spherical (3)
z4
x4
TIE ROD
E4
use double spherical between C and D
D
C
CYLINDER
y3
RACK
z3
C3
E3
D1
fixed steering angle
STRUT
x3
CHASSIS MOUNT
WHEEL SPINDLE
B3
z1
y1
CHASSIS MOUNT
z2
y2
x1
A1
B2
A2
x2
A-ARM

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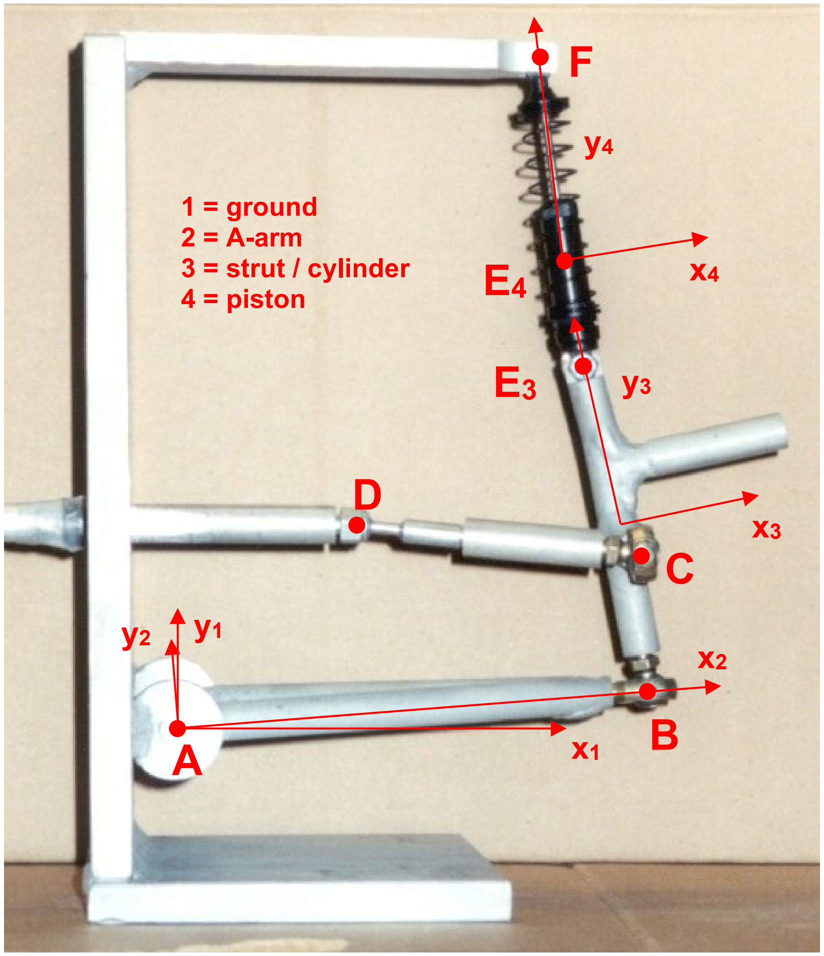
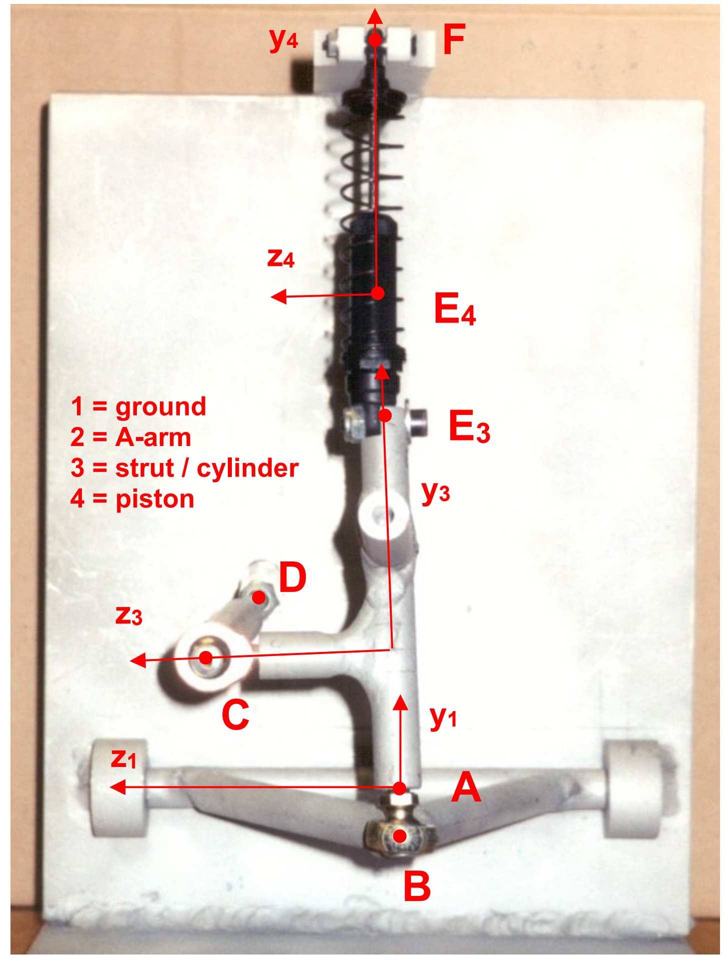
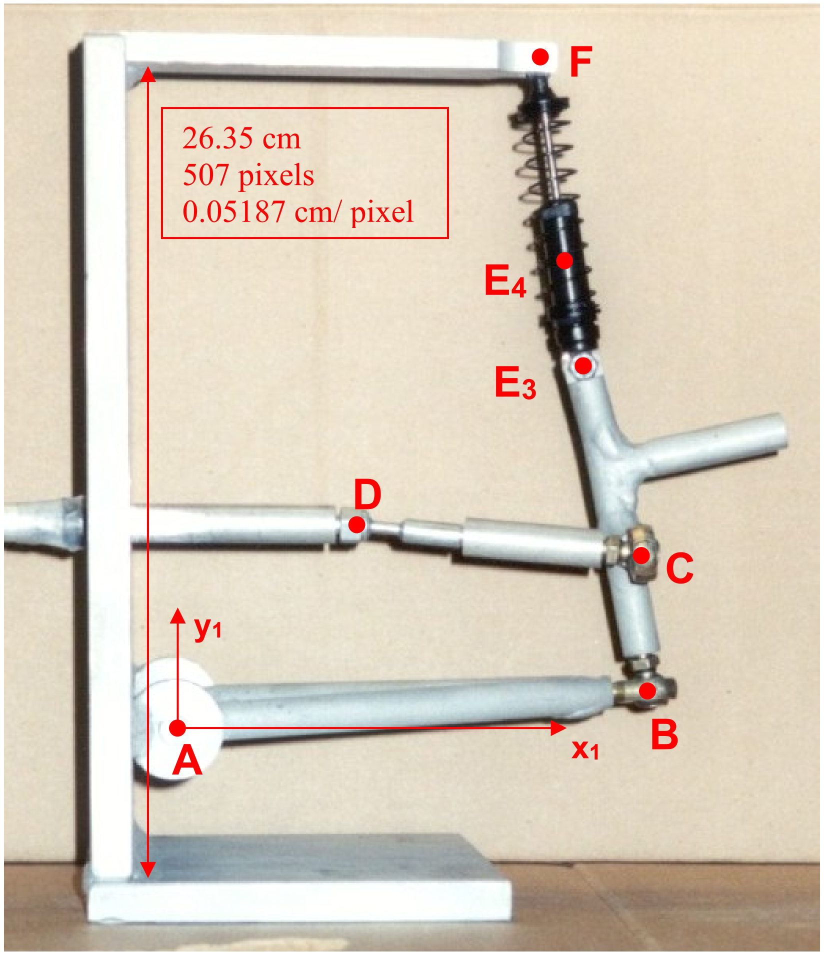
McPherson strut model - rear view

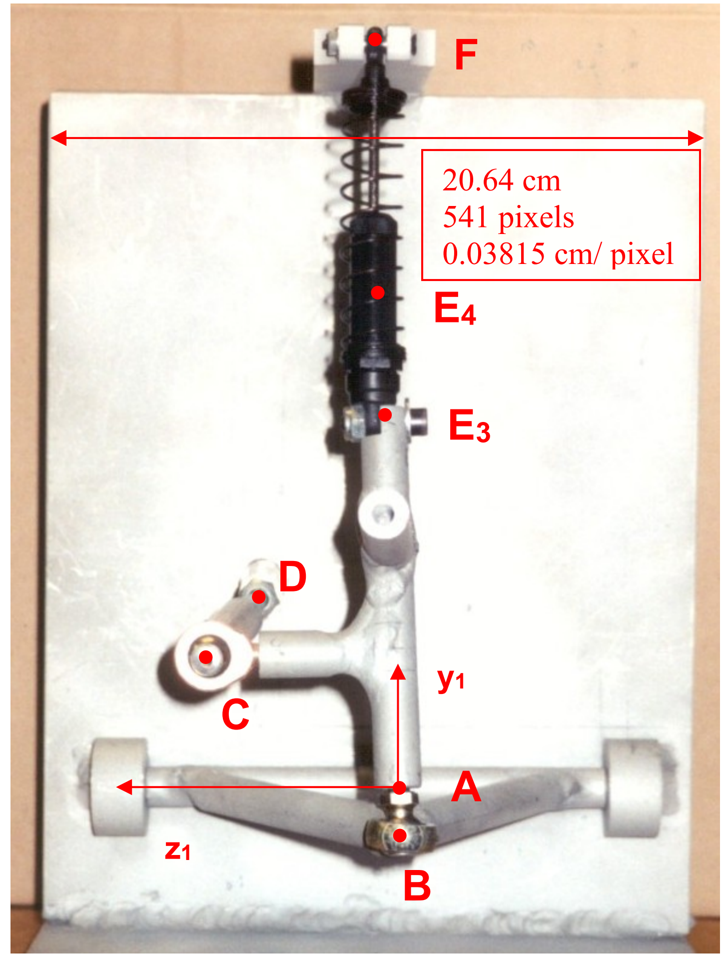


McPherson strut model - side view



ME 581 – C05 Name ________________________





scaled from photographs on class web page

cm	x1	y1	z1
A	0	0	0
B	15.70	1.40	0
C	15.49	5.87	5.77
D	6.03	6.81	4.60
E3	13.51	12.16	0
E4	13.03	15.56	0
F	12.06	22.35	0

ME 581 – C05 **Name ________________________**

Constraints

$$\underset{21\times1}{\{q\}} = \begin{Bmatrix} \{r_2\} \\ \{p_2\} \\ \{r_3\} \\ \{p_3\} \\ \{r_4\} \\ \{p_4\} \end{Bmatrix} \qquad \underset{21\times1}{\{\Phi\}} = \begin{Bmatrix} \{r_1\}^A - \{r_2\}^A \\ \{\hat{f}_2\}^T\{\hat{h}_1\} \\ \{\hat{g}_2\}^T\{\hat{h}_1\} \\ \\ \{r_2\}^B - \{r_3\}^B \\ \\ \{d_{ij}\}^T\{d_{ij}\} - CD^2 \\ \\ \{\hat{f}_3\}^T\{\hat{g}_4\} \\ \{\hat{h}_3\}^T\{\hat{g}_4\} \\ \{\hat{f}_3\}^T\{d_{ij}\} \\ \{\hat{h}_3\}^T\{d_{ij}\} \\ \{\hat{f}_3\}^T\{\hat{h}_4\} \\ \\ \{r_1\}^F - \{r_4\}^F \\ \\ \{p_2\}^T\{p_2\} - 1 \\ \{p_3\}^T\{p_3\} - 1 \\ \{p_4\}^T\{p_4\} - 1 \\ \\ y_3 - y_{3\_START} - y_{3\_VEL}\ t \end{Bmatrix}$$

revolute A		revA, $i = 2, j = 1$
		$f_2h_1, i = 2, j = 1$
		$g_2h_1, i = 2, j = 1$
spherical B		sphB, $i = 3, j = 2$
	$\{d_{ij}\} = \{r_1\}^D - \{r_3\}^C$	1D – 3C, $i = 3, j = 1$
prismatic E		$f_3g_4, i = 3, j = 4$
		$h_3g_4, i = 3, j = 4$
	$\{d_{ij}\} = \{r_4\}^E - \{r_3\}^E$	$f_3, i = 3, j = 4$
		$h_3, i = 3, j = 4$
		$f_3h_4, i = 3, j = 4$
spherical F		sphF, $i = 4, j = 1$
Euler parameters		p2
		p3
		p4
driver		driver

Velocity

$$\underset{21\times1}{\{\nu\}} = \begin{Bmatrix} 0_{20\times1} \\ \\ y_{3\_VEL} \end{Bmatrix}$$

Constants

$$\{r_1\}^A = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{r_1\}^D = \begin{Bmatrix} 6.03 \\ 6.81 \\ 4.60 \end{Bmatrix} \quad \{r_1\}^F = \begin{Bmatrix} 12.06 \\ 22.35 \\ 1.06 \end{Bmatrix}$$

$$\{s_2\}'^A = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{s_2\}'^B = \begin{Bmatrix} 15.76 \\ 0 \\ 0 \end{Bmatrix}$$

$$\{s_3\}'^B = \begin{Bmatrix} 0 \\ -4.47 \\ 0 \end{Bmatrix} \quad \{s_3\}'^C = \begin{Bmatrix} 0 \\ 0 \\ 5.77 \end{Bmatrix} \quad \{s_3\}'^E = \begin{Bmatrix} 0 \\ 6.46 \\ 0 \end{Bmatrix}$$

$$\{s_4\}'^E = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \{s_4\}'^F = \begin{Bmatrix} 0 \\ 5.32 \\ 0 \end{Bmatrix} \qquad CD = 9.64$$

Initial Estimates

$$\{r_2\} = \begin{Bmatrix} 0 \\ 0 \\ 0 \end{Bmatrix} \quad \chi_2 = 5^\circ \quad \{\hat{u}_2\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\{r_3\} = \begin{Bmatrix} 15.49 \\ 5.87 \\ 0 \end{Bmatrix} \quad \chi_3 = 11.5^\circ \quad \{\hat{u}_3\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

$$\{r_4\} = \begin{Bmatrix} 13.51 \\ 17.16 \\ 1.06 \end{Bmatrix} \quad \chi_4 = 11.5^\circ \quad \{\hat{u}_4\} = \begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$$

Absolute Vertical Position Driver

$$y_3 = y_{3\_START} - y_{3\_VEL}\ t$$

ME 581 – C05 **Name** ________________________

Jacobian

$$
\begin{array}{cc}
 & \begin{array}{cccccc} \partial/\partial\{r_2\} & \partial/\partial\{p_2\} & \partial/\partial\{r_3\} & \partial/\partial\{p_3\} & \partial/\partial\{r_4\} & \partial/\partial\{p_4\} \end{array} \\
[\Phi_q] = & \left[\begin{array}{cccccc}
-[I_3] & 2[A_2][\tilde{s}_2]'^{A}[G_2] & 0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x4} \\
0_{1x3} & -2\{h_1\}'^{T}[A_1]^{T}[A_2][\tilde{f}_2]'[G_2] & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & -2\{h_1\}'^{T}[A_1]^{T}[A_2][\tilde{g}_2]'[G_2] & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
[I_3] & -2[A_2][\tilde{s}_2]'^{B}[G_2] & -[I_3] & 2[A_3][\tilde{s}_3]'^{B}[G_3] & 0_{3x3} & 0_{3x4} \\
0_{1x3} & 0_{1x4} & -2\{d_{ij}\}^{T} & 4\{d_{ij}\}^{T}[A_3][\tilde{s}_3]'^{C}[G_3] & 0_{1x3} & 0_{1x4} \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{g_4\}'^{T}[A_4]^{T}[A_3][\tilde{f}_3]'[G_3] & 0_{1x3} & -2\{f_3\}'^{T}[A_3]^{T}[A_4][\tilde{g}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{g_4\}'^{T}[A_4]^{T}[A_3][\tilde{h}_3]'[G_3] & 0_{1x3} & -2\{h_3\}'^{T}[A_3]^{T}[A_4][\tilde{g}_4]'[G_4] \\
0_{1x3} & 0_{1x4} & -\{f_3\}'^{T}[A_3]^{T} & 2(\{f_3\}'^{T}[\tilde{s}_3]'^{E}-\{d_{ij}\}^{T}[A_3][\tilde{f}_3]')[G_3] & \{f_3\}'^{T}[A_3]^{T} & -2\{f_3\}'^{T}[A_3]^{T}[A_4][\tilde{s}_4]'^{E}[G_4] \\
0_{1x3} & 0_{1x4} & -\{h_3\}'^{T}[A_3]^{T} & 2(\{h_3\}'^{T}[\tilde{s}_3]'^{E}-\{d_{ij}\}^{T}[A_3][\tilde{h}_3]')[G_3] & \{h_3\}'^{T}[A_3]^{T} & -2\{h_3\}'^{T}[A_3]^{T}[A_4][\tilde{s}_4]'^{E}[G_4] \\
0_{1x3} & 0_{1x4} & 0_{1x3} & -2\{h_4\}'^{T}[A_4]^{T}[A_3][\tilde{f}_3]'[G_3] & 0_{1x3} & -2\{f_3\}'^{T}[A_3]^{T}[A_4][\tilde{h}_4]'[G_4] \\
0_{3x3} & 0_{3x4} & 0_{3x3} & 0_{3x4} & -[I_3] & 2[A_4][\tilde{s}_4]'^{F}[G_4] \\
0_{1x3} & 2\{p_2\}^{T} & 0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & 0_{1x4} & 0_{1x3} & 2\{p_3\}^{T} & 0_{1x3} & 0_{1x4} \\
0_{1x3} & 0_{1x4} & 0_{1x3} & 0_{1x4} & 0_{1x3} & 2\{p_4\}^{T} \\
0_{1x3} & 0_{1x4} & \begin{matrix} 0 & 1 & 0 \end{matrix} & 0_{1x4} & 0_{1x3} & 0_{1x4}
\end{array}\right]
\begin{array}{l}
\text{revA}, i=2, j=1 \\
f_2 h_1, i=2, j=1 \\
g_2 h_1, i=2, j=1 \\
\text{sphB}, i=3, j=2 \\
\text{1D}-\text{3C}, i=3, j=1 \\
f_3 g_4, i=3, j=4 \\
h_3 g_4, i=3, j=4 \\
f_3, i=3, j=4 \\
h_3, i=3, j=4 \\
f_3 h_4, i=3, j=4 \\
\text{sphF}, i=4, j=1 \\
\text{p2} \\
\text{p3} \\
\text{p4} \\
\text{driver}
\end{array}
\end{array}
$$

$$
\frac{\partial}{\partial\{p_i\}} = 2\left(\frac{\partial}{\partial\{\pi_i\}'}\right)[G_i]
$$

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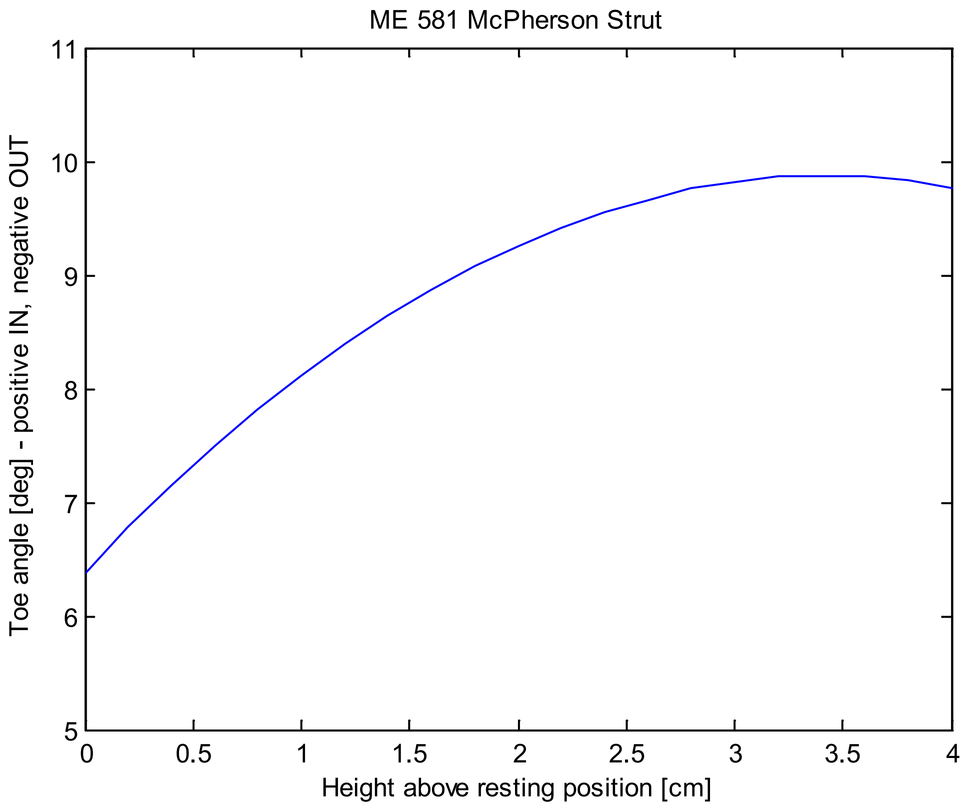
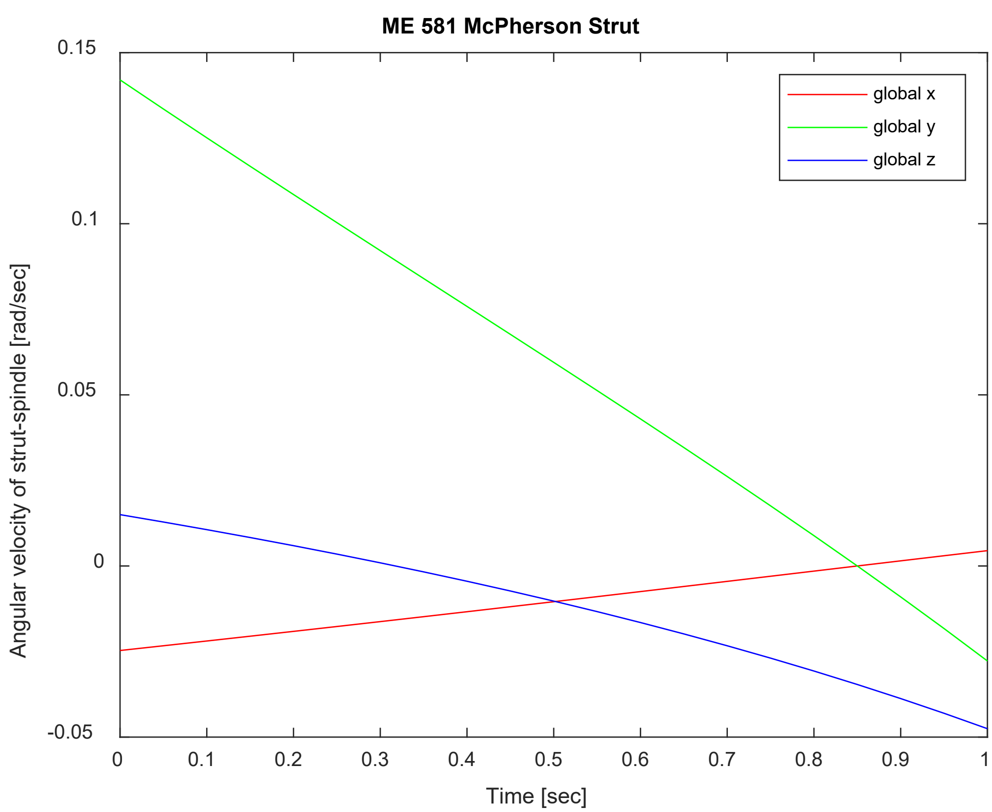
Acceleration - double spherical and prismatic not ready

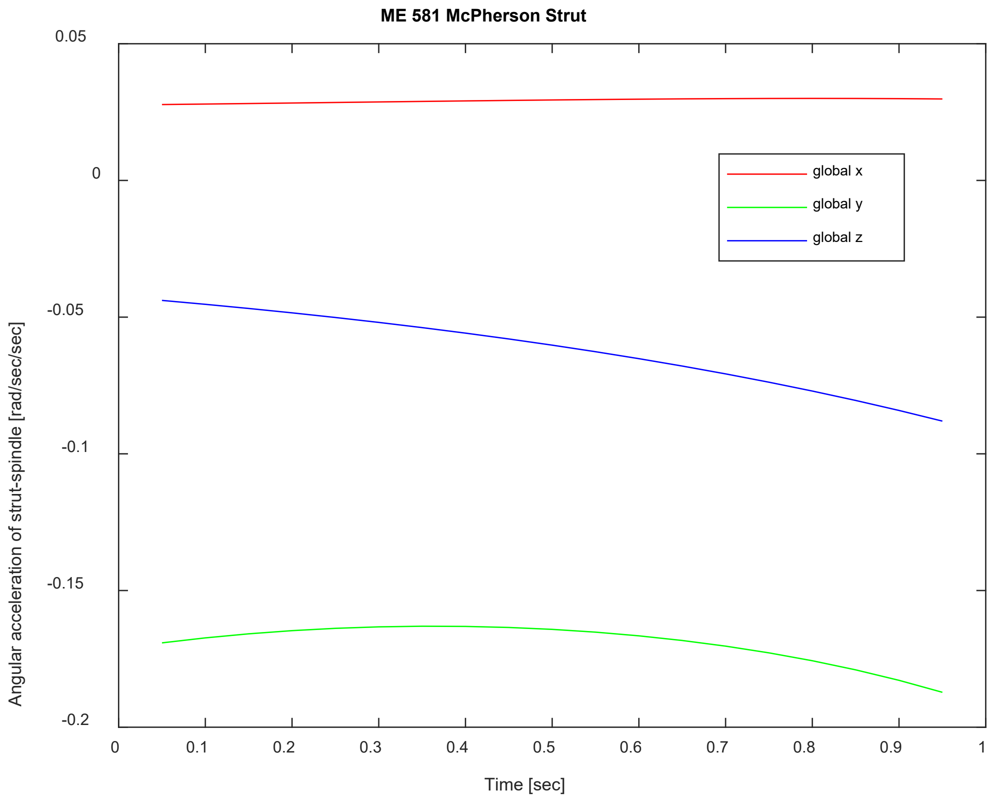
$$
\underset{21x1}{\{\gamma\}} = \left\{ \begin{array}{c}
[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]'\{s_2\}'^{A} \\
-\{h_1\}'^{T}([A_1]^{T}[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{f_2\}' \\
-\{h_1\}'^{T}([A_1]^{T}[A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]')\{g_2\}' \\
\\
[A_3][\tilde{\omega}_3]'[\tilde{\omega}_3]'\{s_3\}'^{B} - [A_2][\tilde{\omega}_2]'[\tilde{\omega}_2]'\{s_2\}'^{B} \\
\\
\{d_{ij}\}^{T}\{d_{ij}\} - CD^2 \\
\\
\{\hat{f}_3\}^{T}\{\hat{g}_4\} \\
\{\hat{h}_3\}^{T}\{\hat{g}_4\} \\
\{\hat{f}_3\}^{T}\{d_{ij}\} \\
\{\hat{h}_3\}^{T}\{d_{ij}\} \\
\{\hat{f}_3\}^{T}\{\hat{h}_4\} \\
\\
[A_4][\tilde{\omega}_4]'[\tilde{\omega}_4]'\{s_4\}'^{F} \\
\\
-2\{\dot{p}_2\}^{T}\{\dot{p}_2\} \\
-2\{\dot{p}_3\}^{T}\{\dot{p}_3\} \\
-2\{\dot{p}_4\}^{T}\{\dot{p}_4\} \\
\\
0
\end{array} \right\}
$$

revolute A	$revA, i = 2, j = 1$ $f_2h_1, i = 2, j = 1$ $g_2h_1, i = 2, j = 1$
spherical B	$sphB, i = 3, j = 2$
$\{d_{ij}\} = \{r_1\}^{D} - \{r_3\}^{C}$	$1D - 3C, i = 3, j = 1$
prismatic E $\{d_{ij}\} = \{r_4\}^{E} - \{r_3\}^{E}$	$f_3g_4, i = 3, j = 4$ $h_3g_4, i = 3, j = 4$ $f_3, i = 3, j = 4$ $h_3, i = 3, j = 4$ $f_3h_4, i = 3, j = 4$
spherical F	$sphF, i = 4, j = 1$
Euler parameters	p2 p3 p4
driver	driver


ME 581 McPherson Strut
Toe angle [deg] - positive IN, negative OUT
Height above resting position [cm]
11
10
9
8
7
6
5
0
0.5
1
1.5
2
2.5
3
3.5
4


ME 581 McPherson Strut
Angular velocity of strut-spindle [rad/sec]
Time [sec]
global x
global y
global z
0.15
0.1
0.05
0
-0.05
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

ME 581 – C05 Name ________________________




```
% mcp_main.m - McPherson strut for ME 581
%   main for C05 - time loop
% HJSIII - 20.04.23

% general constants
d2r = pi / 180;

% local coordinate axes
fp = [ 1  0  0]';
gp = [ 0  1  0]';
hp = [ 0  0  1]';

% nominal initial position
mcp_ini

% time loop
% 4 cm of vertical motion over 1 second
t_start = 0;  % start
t_end = 1;    % end
nt = 20;     % number of time steps
dt = (t_end - t_start) / nt;

keep_q = [];
keep = [];
for t = t_start : dt : t_end,

% kinematics
  mcp_kin

% save kinematics
  height = y3 - y3_start;
  toe = asin( h3(1) );
  keep_q = [ keep_q ; q'  qd' ];
  keep = [ keep ; t  height  toe  w3' ];

% bottom - for t
end

% plot
time = keep(:,1);
height = keep(:,2);
toe = keep(:,3) /d2r;
```

```
w3x = keep(:,4);
w3y = keep(:,5);
w3z = keep(:,6);

figure( 1 )
  plot( height, toe )
  xlabel( 'Height above resting position [cm]' )
  ylabel( 'Toe angle [deg] - positive IN, negative OUT' )
  title( 'ME 581 McPherson Strut' )
  axis( [ 0 4 5 11 ] )

%{
figure( 2 )
  plot( chi2, toe )
  xlabel( 'Angle of A-arm above horizontal [deg]' )
  ylabel( 'Toe angle [deg] - positive IN, negative OUT' )
  title( 'ME 581 McPherson Strut' )
%}

figure( 2 )
  plot( time,w3x,'r', time,w3y,'g', time,w3z,'b' )
  xlabel( 'Time [sec]' )
  ylabel( 'Angular velocity of strut-spindle [rad/sec]' )
  legend( 'global x', 'global y', 'global z' )
  title( 'ME 581 McPherson Strut' )

% finite difference
w3xd_fd = ( [ diff(w3x) ; NaN ] + [ NaN ; diff(w3x) ] ) /2 /dt;
w3yd_fd = ( [ diff(w3y) ; NaN ] + [ NaN ; diff(w3y) ] ) /2 /dt;
w3zd_fd = ( [ diff(w3z) ; NaN ] + [ NaN ; diff(w3z) ] ) /2 /dt;

figure( 3 )
  plot( time,w3xd_fd,'r', time,w3yd_fd,'g', time,w3zd_fd,'b' )
  xlabel( 'Time [sec]' )
  ylabel( 'Angular acceleration of strut-spindle [rad/sec/sec]' )
  legend( 'global x', 'global y', 'global z' )
  title( 'ME 581 McPherson Strut' )

% bottom - mcp_main
```

```
% mcp_ini.m - McPherson strut for ME 581
%   initialize constants and assembly guesses
% HJSIII - 20.03.04

% global and local joint locations - units = cm
s1pA = [  0.00   0.00  0.00 ]';
s1pD = [  6.03   6.81  4.60 ]';
s1pF = [ 12.06  22.35  0.00 ]';

s2pA = [  0.00   0.00  0.00 ]';
s2pB = [ 15.76   0.00  0.00 ]';

s3pB = [ 0.00  -4.47  0.00 ]';
s3pC = [ 0.00   0.00  5.77 ]';
s3pE = [ 0.00   6.46  0.00 ]';

s4pE = [ 0.00   6.00  0.00 ]';
s4pF = [ 0.00  10.32  0.00 ]';

lenCD = 9.64;

% initial guesses
r2 = [ 0.00  0.00  0.00 ]';
chi2 = 5 * d2r;
p2 = [ cos(chi2/2)  0.000  0.000  sin(chi2/2) ]';

r3 = [ 15.49  5.87  0.00 ]';
chi3 = 11.5 * d2r;
p3 = [ cos(chi3/2)  0.000  0.000  sin(chi3/2) ]';

r4 = [ 13.51  12.16  0.00 ]';
p4 = p3;

% simple guesses for Euler parameters - converges well
% p2 = [ 1  0  0  0 ]';
% p3 = p2;
% p4 = p2;

% generalized coordinates
q = [ r2 ; p2 ; r3 ; p3 ; r4 ; p4 ];

% translation driver for origin of frame 3 - y3 = y3_start + y3_vel * t
%   use 0 <= t <= 1 for y3_vel = 4 cm/sec
y3_start = 5.87;
y3_vel = 4;

%{
% rotation driver for A-arm link 2 - chi2 = chi2_start + w2 * t
%   use 0 <= t <= 1 for total chi2_delta = 15 deg
chi2_start = 5 * d2r;
w2 = 15 * d2r;
%}

% bottom - mcp_ini
```

ME 581 – C05 Name ________________________

```
% mcp_phi.m - McPherson strut for ME 581
%   evaluate constraints and Jacobian for translation driver frame 3
% HJSIII - 20.03.04

% global location of local frames
r1 = [ 0  0  0 ]';
p1 = [ 1  0  0  0 ]';
[ E1,G1,A1,f1,g1,h1 ] = make_ega(p1);

r2 = q(1:3);
p2 = q(4:7);
[ E2,G2,A2,f2,g2,h2 ] = make_ega(p2);

r3 = q(8:10);
p3 = q(11:14);
[ E3,G3,A3,f3,g3,h3 ] = make_ega(p3);

r4 = q(15:17);
p4 = q(18:21);
[ E4,G4,A4,f4,g4,h4 ] = make_ega(p4);

% global locations of joint centers
r1A = r1 + A1*s1pA;
r1D = r1 + A1*s1pD;
r1F = r1 + A1*s1pF;

r2A = r2 + A2*s2pA;
r2B = r2 + A2*s2pB;

r3B = r3 + A3*s3pB;
r3C = r3 + A3*s3pC;
r3E = r3 + A3*s3pE;

r4E = r4 + A4*s4pE;
r4F = r4 + A4*s4pF;

% constraints and Jacobian
PHI = zeros(21,1);
JAC = zeros(21,21);

% revolute A
PHI(1:3) = r1A - r2A;
PHI(4) = f2' * h1;
PHI(5) = g2' * h1;

JAC(1:3,1:3) = -eye(3);
JAC(1:3,4:7) = 2 * A2 * skew_sym(s2pA) * G2;
JAC(4,4:7) = -2 * hp' * A1' * A2 * skew_sym(fp) * G2;
JAC(5,4:7) = -2 * hp' * A1' * A2 * skew_sym(gp) * G2;

% spherical B
PHI(6:8) = r2B - r3B;

JAC(6:8,1:3) = eye(3);
JAC(6:8,4:7) = -2 * A2 * skew_sym(s2pB) * G2;
JAC(6:8,8:10) = -eye(3);
JAC(6:8,11:14) = 2 * A3 * skew_sym(s3pB) * G3;

% double spherical C
dij = r1D - r3C;
PHI(9) = dij' * dij - lenCD * lenCD;

JAC(9,8:10) = -2 * dij';
JAC(9,11:14) = 4 * dij' * A3 * skew_sym(s3pC) * G3;

% prismatic E
dij = r4E - r3E;
PHI(10) = f3' * g4;
PHI(11) = h3' * g4;
PHI(12) = f3' * dij;
PHI(13) = h3' * dij;
PHI(14) = f3' * h4;

JAC(10,11:14) = -2 * gp' * A4' * A3 * skew_sym(fp) * G3;
JAC(10,18:21) = -2 * fp' * A3' * A4 * skew_sym(gp) * G4;

JAC(11,11:14) = -2 * gp' * A4' * A3 * skew_sym(hp) * G3;
JAC(11,18:21) = -2 * hp' * A3' * A4 * skew_sym(gp) * G4;
```

```
JAC(12,8:10) = -fp' * A3';
JAC(12,11:14) = 2 * (fp' * skew_sym(s3pE) - dij' * A3 * skew_sym(fp)) * G3;
JAC(12,15:17) = fp' * A3';
JAC(12,18:21) = -2 * fp' * A3' * A4 * skew_sym(s4pE) * G4;

JAC(13,8:10) = -hp' * A3';
JAC(13,11:14) = 2 * (hp' * skew_sym(s3pE) - dij' * A3 * skew_sym(hp)) * G3;
JAC(13,15:17) = hp' * A3';
JAC(13,18:21) = -2 * hp' * A3' * A4 * skew_sym(s4pE) * G4;

JAC(14,11:14) = -2 * hp' * A4' * A3 * skew_sym(fp) * G3;
JAC(14,18:21) = -2 * fp' * A3' * A4 * skew_sym(hp) * G4;

% spherical F
PHI(15:17) = r1F - r4F;

JAC(15:17,15:17) = -eye(3);
JAC(15:17,18:21) = 2 * A4 * skew_sym(s4pF) * G4;

% Euler parameters
PHI(18) =  p2' * p2 - 1;
PHI(19) =  p3' * p3 - 1;
PHI(20) =  p4' * p4 - 1;

JAC(18,4:7)   =  2 * p2';
JAC(19,11:14) =  2 * p3';
JAC(20,18:21) =  2 * p4';

% vertical translation driver
y3 = r3(2);
PHI(21) = y3 - y3_start - y3_vel * t;
JAC(21,8:10) = [ 0 1 0 ];

%{
% rotation driver for A-arm link 2 - chi2 = chi2_start + w2 * t
%   use 0 <= t <= 1 for total chi2_delta = 15 deg
e0 = p2(1);
chi2 = 2*acos(e0);
PHI(21) = chi2 - chi2_start - w2*t;
JAC(21,4) = -2 / sqrt( 1 - e0*e0 );
%}

% bottom - mcp_phi
```

```
% mcp_kin.m - McPherson strut for ME 581
%   position solution
% HJSIII - 20.03.04

% Newton-Raphson position solution
assy_tol = 0.00001;
mcp_phi
%  test_jac;  % numerical Jacobian
%  JAC = jtest; % numerical Jacobian

while max(abs(PHI)) > assy_tol,
  q = q - inv(JAC) * PHI;
  mcp_phi
%  test_jac;  % numerical Jacobian
%  JAC = jtest; % numerical Jacobian
end

% velocity
velrhs = zeros(21,1);
velrhs(21) = y3_vel;
qd = inv(JAC) * velrhs;

% generalized velocities
r2d = qd(1:3);
p2d = qd(4:7);
r3d = qd(8:10);
p3d = qd(11:14);
r4d = qd(15:17);
p4d = qd(18:21);

% local angular velocities
w2p = 2 * G2 * p2d;
w3p = 2 * G3 * p3d;
w4p = 2 * G4 * p4d;

w2psk = skew_sym( w2p );
w3psk = skew_sym( w3p );
w4psk = skew_sym( w4p );

% global angular velocities
w2 = A2 * w2p;
w3 = A3 * w3p;
w4 = A4 * w4p;

% bottom - mcp_kin
```

```
% test_jac.m - McPherson strut for ME 581
%   evaluate Jacobian by numerical partial derivatives
% HJSIII - 14.04.28

% hold estimates for generalized Cartesian coordinates
[nq,nc] = size(q);
qhold = q;

% evaluate constraints
mcp_phi;

% hold constraints
phold = PHI;

% perturb one coordinate at a time
for iq=1:nq,
  q = qhold;
  q(iq) = q(iq) + 0.0001;

% change in constraints caused by coordinate perturbation is
% approximately equal to partial derivative
  mcp_phi;
  jtest(:,iq) = (PHI-phold) / 0.0001;
end

% reset coordinates and constraints
q = qhold;
mcp_phi;

% bottom - test_jac
```

Name ________________________

Analytical Jacobian - determinant 2.9828e+009

	r2			p2				r3			p3				r4			p4			
rev	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.09	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	2.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	1.00	0.00	0.00	0.14	0.00	0.00	-2.94	-1.00	0.00	0.00	0.74	0.47	0.26	-8.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	1.00	0.00	-1.47	0.00	0.00	31.35	0.00	-1.00	0.00	0.14	-0.37	0.01	-1.54	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	1.00	0.00	1.47	-31.49	0.00	0.00	0.00	-1.00	-0.26	8.88	0.76	0.40	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.95	-2.26	2.72	-11.36	43.21	211.14	1.32	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pris	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.17	0.10	-0.04	1.99	0.00	0.00	0.00	0.17	-0.10	0.04	-1.99
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	-1.99	-0.17	0.10	0.00	0.00	0.00	-0.04	1.99	0.17	-0.10
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.98	-0.18	0.09	-2.16	1.21	-0.52	24.73	0.98	0.18	-0.09	1.04	-0.58	0.25	-11.94
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	0.03	-0.99	0.52	-24.73	-2.16	1.21	0.10	-0.03	0.99	-0.25	11.94	1.04	-0.58
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.17	-1.99	-0.04	0.00	0.00	0.00	-0.10	-0.17	1.99	0.04
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	-1.72	-1.08	-0.60	20.23
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	-0.33	0.85	-0.02	3.56
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.60	-20.51	-1.75	-0.92
Eul2	0.00	0.00	0.00	2.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17
driv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Numerical Jacobian - A=E*trans(G) - determinant 2.9835e+009

	r2			p2				r3			p3				r4			p4			
rev	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.09	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	2.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	1.00	0.00	0.00	31.49	0.00	0.00	-1.47	-1.00	0.00	0.00	-0.78	0.44	0.19	-8.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	1.00	0.00	1.47	0.00	0.00	31.49	0.00	-1.00	0.00	8.89	-0.19	0.44	-0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	1.00	0.00	1.47	-31.49	0.00	0.00	0.00	-1.00	0.19	8.89	0.78	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.95	-2.26	2.72	42.46	44.35	213.79	6.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pris	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.17	0.10	-0.04	1.99	0.00	0.00	0.00	0.17	-0.10	0.04	-1.99
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	-1.99	-0.17	0.10	0.00	0.00	0.00	-0.04	1.99	0.17	-0.10
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.98	-0.18	0.09	-2.16	1.21	-0.52	24.73	0.98	0.18	-0.09	1.04	-0.58	0.25	-11.94
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	0.03	-0.99	0.52	-24.73	-2.16	1.21	0.10	-0.03	0.99	-0.25	11.94	1.04	-0.58
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.17	-1.99	-0.04	0.00	0.00	0.00	-0.10	-0.17	1.99	0.04
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	1.80	-1.01	-0.43	20.53
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	-20.53	0.43	-1.01	1.80
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-0.43	-20.53	-1.80	-1.01
Eul2	0.00	0.00	0.00	2.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17
driv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

ME 581 – C05 **Name ________________________**

Numerical Jacobian - A=E*trans(G) - determinant 2.9835e+009

	r2			p2				r3			p3				r4			p4			
rev	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.09	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	2.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	1.00	0.00	0.00	31.49	0.00	0.00	-1.47	-1.00	0.00	0.00	-0.78	0.44	0.19	-8.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	1.00	0.00	1.47	0.00	0.00	31.49	0.00	-1.00	0.00	8.89	-0.19	0.44	-0.78	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	1.00	0.00	1.47	-31.49	0.00	0.00	0.00	-1.00	0.19	8.89	0.78	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.95	-2.26	2.72	42.46	44.35	213.79	6.02	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pris	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.17	0.10	-0.04	1.99	0.00	0.00	0.00	0.17	-0.10	0.04	-1.99
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.04	-1.99	-0.17	0.10	0.00	0.00	0.00	-0.04	1.99	0.17	-0.10
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.98	-0.18	0.09	-2.16	1.21	-0.52	24.73	0.98	0.18	-0.09	1.04	-0.58	0.25	-11.94
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	0.03	-0.99	0.52	-24.73	-2.16	1.21	0.10	-0.03	0.99	-0.25	11.94	1.04	-0.58
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.10	0.17	-1.99	-0.04	0.00	0.00	0.00	-0.10	-0.17	1.99	0.04
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	1.80	-1.01	-0.43	20.53
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	-20.53	0.43	-1.01	1.80
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-0.43	-20.53	-1.80	-1.01
Eul2	0.00	0.00	0.00	2.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17
driv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Numerical Jacobian - A explicit - determinant 2.9834e+009

	r2			p2				r3			p3				r4			p4			
rev	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	-1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.09	-2.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	2.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	1.00	0.00	0.00	62.97	0.00	0.00	0.00	-1.00	0.00	0.00	-0.78	0.44	0.19	-8.89	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	1.00	0.00	1.47	0.00	0.00	31.49	0.00	-1.00	0.00	17.79	0.00	0.87	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	1.00	0.00	1.47	-31.49	0.00	0.00	0.00	-1.00	0.19	8.89	0.78	0.44	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	18.95	-2.26	2.72	73.75	45.00	215.32	8.76	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pris	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.51	0.09	-0.06	1.96	0.00	0.00	0.00	0.52	-0.09	0.06	-1.96
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.14	-1.99	-0.17	0.11	0.00	0.00	0.00	-0.11	1.99	0.17	-0.10
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.98	-0.18	0.09	-6.45	1.12	-0.73	24.35	0.98	0.18	-0.09	3.14	-0.54	0.35	-11.75
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.10	0.03	-0.99	1.54	-24.71	-2.11	1.30	0.10	-0.03	0.99	-0.64	11.93	1.03	-0.62
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.30	0.18	-1.98	-0.02	0.00	0.00	0.00	-0.28	-0.18	1.98	0.03
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	0.00	1.80	-1.01	-0.43	20.53
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	0.00	-41.07	0.00	-2.01	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-1.00	-0.43	-20.53	-1.80	-1.01
Eul2	0.00	0.00	0.00	2.00	0.00	0.00	0.09	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.99	0.04	0.10	0.17
driv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

ME 581 – C05 Name ________________________

Analytical minus Numerical - A=E*trans(G)

	r2			p2				r3			p3				r4			p4			
rev	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
B	0.00	0.00	0.00	-31.35	0.00	0.00	-1.47	0.00	0.00	0.00	1.52	0.03	0.07	0.13	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	-2.94	0.00	0.00	-0.14	0.00	0.00	0.00	-8.75	-0.18	-0.43	-0.77	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.44	-0.01	-0.02	-0.04	0.00	0.00	0.00	0.00	0.00	0.00	0.00
CD	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-53.82	-1.14	-2.65	-4.71	0.00	0.00	0.00	0.00	0.00	0.00	0.00
pris	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
F	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-3.51	-0.07	-0.17	-0.31
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	20.20	0.42	0.99	1.77
	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	1.03	0.02	0.05	0.09
Eul2	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul3	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
Eul4	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00
driv	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00	0.00

Difference divided by Euler 2

B	#DIV/0!	#DIV/0!	#DIV/0!	-15.69	-2E+20	-8E+18	-15.67	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
	#DIV/0!	#DIV/0!	#DIV/0!	-1.472	-307.2	-2.284	-1.472	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
	#DIV/0!	#DIV/0!	#DIV/0!	-2E-21	0	0	-1E-21	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
CD	#DIV/0!	#DIV/0!	#DIV/0!	0	0	0	0	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!

Difference divided by Euler 3

B	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	0.765	0.765	0.765	0.765	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	-4.399	-4.388	-4.403	-4.396	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	-0.223	-0.223	-0.223	-0.223	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!
CD	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	-27.05	-27.32	-27.16	-27.05	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!

Difference divided by Euler 4

F	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	-1.766	-1.766	-1.766	-1.766
	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	10.16	10.13	10.17	10.15
	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	#DIV/0!	0.516	0.516	0.516	0.516

```
% mcp_autofill_main.m - McPherson strut for ME 581
%   test to automatically fill constraints and Jacobian
%   does NOT handle drivers
% HJSIII - 14.04.29

% exact constraints and Jacobian
mcp_ini;
mcp_phi;
PHI_exact = PHI;
JAC_exact = JAC;

% specify number of bodies and number of constraints
% assumes body 1 is fixed at global origin
nb = 4;
nq = 7 * (nb-1);
nc = nq;

% specify bodies and vectors for kinematic constraints
%   does NOT handle drivers
% ctype=1  spherical        [ ctype  body_i  body_j  sipP'  sjpP'  0 0 0 ]
% ctype=2  double spherical [ ctype  body_i  body_j  sipP'  sjpP'  C 0 0 ] note: must provide C
% ctype=3  dot-1            [ ctype  body_i  body_j  aip'   ajp'   0 0 0 ]
% ctype=4  dot-2            [ ctype  body_i  body_j  aip'   sipP'  sjpP' ]
% ctype=5  Euler parameter  [ ctype  body_i  0       0 0 0  0 0 0  0 0 0 ]
constraints = [ 1  2  1  s2pA'  s1pA'  0 0 0 ;      % sph 1A-2A
                3  2  1  fp'    hp'    0 0 0 ;      % f2.h1
                3  2  1  gp'    hp'    0 0 0 ;      % g2.h1
                1  3  2  s3pB'  s2pB'  0 0 0 ;      % sph B
                2  3  1  s3pC'  s1pD'  lenCD 0 0 ;  % double sph 1D-3C
                3  3  4  fp'    gp'    0 0 0 ;      % f3.g4
                3  3  4  hp'    gp'    0 0 0 ;      % h3.g4
                4  3  4  fp'    s3pE'  s4pE' ;      % f3.dij  4E-3E
                4  3  4  hp'    s3pE'  s4pE' ;      % h3.dij  4E-3E
                3  3  4  fp'    hp'    0 0 0 ;      % f3.h4
                1  4  1  s4pF'  s1pF'  0 0 0 ;      % sph 1F-4F
                5  2  0  0 0 0  0 0 0  0 0 0 ;      % p2
                5  3  0  0 0 0  0 0 0  0 0 0 ;      % p3
                5  4  0  0 0 0  0 0 0  0 0 0 ];     % p4

% call autofill - does NOT handle drivers
mcp_autofill_phi;

% vertical translation driver
y3 = r3(2);
PHI( 21 ) = y3 - y3_start - y3_vel * t;
JAC( 21, : ) = zeros(1,nq);
JAC( 21, 8:10 ) = [ 0 1 0 ];

check_PHI = (PHI - PHI_exact)'

check_JAC = JAC - JAC_exact

% bottom of mcp_autofill_main
```

```
% mcp_autofill_phi.m
%   automatically fill constraints and Jacobian
%   does NOT handle drivers
% HJSIII - 14.04.29

% provided by main
% nb, nc, nq, q, constraints

% tables for positions and Euler parameters
r_all = zeros( 3,nb );
p_all = zeros( 4,nb );
r_all(:,1) = [ 0 0 0 ]';
p_all(:,1) = [ 1 0 0 0 ]';
for bi = 2 : nb,
  ci = 7*(bi-2) + 1;
  r_all(:,bi) = q( ci   : ci+2 );
  p_all(:,bi) = q( ci+3 : ci+6 );
end

% tables for rotation matrices and G
A_all = zeros( 3,3,nb );
G_all = zeros( 3,4,nb );
for bi = 1 : nb;
  p = p_all(:,bi);
  E = [ -p(2)  +p(1)  -p(4)  +p(3);
        -p(3)  +p(4)  +p(1)  -p(2);
        -p(4)  -p(3)  +p(2)  +p(1)];

  G = [ -p(2)  +p(1)  +p(4)  -p(3);
        -p(3)  -p(4)  +p(1)  +p(2);
        -p(4)  +p(3)  -p(2)  +p(1)];

  A_all(:,:,bi) = E * G';
  G_all(:,:,bi) = G;
end

% proceed through constraint table
[ ntable, ncol ] = size( constraints );
i2 = 0;
for itable = 1 : ntable,
  i1 = i2 + 1;                 % increment row in constraints and Jacobian
  ctype = constraints( itable, 1 );

  bi = constraints( itable, 2 );
  ri = r_all(:,bi);
  p_i = p_all(:,bi);   % note:  use p_i to prevent problems with "pi"
  Ai = A_all(:,:,bi);
  Gi = G_all(:,:,bi);
  ci = 7*(bi-2) + 1;

  bj = constraints( itable, 3 );
  if bj == 0;
    bj = 1;
  end
  rj = r_all(:,bj);
  Aj = A_all(:,:,bj);
  Gj = G_all(:,:,bj);
  cj = 7*(bj-2) + 1;

% spherical
  if ctype == 1,
    i2 = i1 + 2;
    sipP = constraints( itable, 4:6 )';
    sjpP = constraints( itable, 7:9 )';

    riP = ri + Ai * sipP;
    rjP = rj + Aj * sjpP;
    PHI( i1:i2 ) = rjP - riP;

    JAC( i1:i2, : ) = zeros(3,nq);
    if ci > 0,
```

```
      JAC( i1:i2, ci : ci+2 ) = -eye(3);
      JAC( i1:i2, ci+3:ci+6 ) = 2 * Ai * skew_sym(sipP) * Gi;
    end
    if cj > 0,
      JAC( i1:i2, cj : cj+2 ) = eye(3);
      JAC( i1:i2, cj+3:cj+6 ) = -2 * Aj * skew_sym(sjpP) * Gj;
    end
  end

% double spherical
  if ctype == 2,
    i2 = i1;
    sipP = constraints( itable, 4:6 )';
    sjpP = constraints( itable, 7:9 )';
    C    = constraints( itable, 10 );

    riP = ri + Ai * sipP;
    rjP = rj + Aj * sjpP;
    dij = rjP - riP;
    PHI( i1 ) = dij'*dij - C*C;

    JAC( i1, : ) = zeros(1,nq);
    if ci > 0,
      JAC( i1, ci : ci+2 ) = -2 * dij';
      JAC( i1, ci+3:ci+6 ) = 4 * dij' * Ai * skew_sym(sipP) * Gi;
    end
    if cj > 0,
      JAC( i1, cj : cj+2 ) = 2 * dij';
      JAC( i1, cj+3:cj+6 ) = -4 * dij' * Aj * skew_sym(sjpP) * Gj;
    end
  end

% dot-1
  if ctype == 3,
    i2 = i1;
    aip = constraints( itable, 4:6 )';
    ajp = constraints( itable, 7:9 )';

    ai = Ai * aip;
    aj = Aj * ajp;
    PHI( i1 ) = ai' * aj;

    JAC( i1, : ) = zeros(1,nq);
    if ci > 0,
      JAC( i1, ci+3:ci+6 ) = -2 * aj' * Ai * skew_sym(aip) * Gi;
    end
    if cj > 0,
      JAC( i1, cj+3:cj+6 ) = -2 * ai' * Aj * skew_sym(ajp) * Gj;
    end
  end

% dot-2
  if ctype == 4,
    i2 = i1;
    aip = constraints( itable, 4:6 )';
    sipP = constraints( itable, 7:9 )';
    sjpP = constraints( itable, 10:12 )';

    ai = Ai * aip;
    riP = ri + Ai * sipP;
    rjP = rj + Aj * sjpP;
    dij = rjP - riP;
    PHI( i1 ) = ai' * dij;

    JAC( i1, : ) = zeros(1,nq);
    if ci > 0,
      JAC( i1, ci : ci+2 ) = -ai';
      JAC( i1, ci+3:ci+6 ) = 2 * ( aip'*skew_sym(sipP) - dij'*Ai* skew_sym(aip) ) * Gi;
    end
    if cj > 0,
      JAC( i1, cj : cj+2 ) = ai';
```

```
      JAC( i1, cj+3:cj+6 ) = -2 * ai' * Aj * skew_sym(sjpP) * Gj;
    end
  end

% Euler parameters
  if ctype == 5,
    i2 = i1;
    PHI( i1 ) = p_i'*p_i - 1;   % note:  use p_i to prevent problems with "pi"

    JAC( i1, : ) = zeros(1,nq);
    JAC( i1, ci+3:ci+6 ) = 2 * p_i';
  end

% bottom of for itable
end

% bottom of mcp_autofill_phi
```