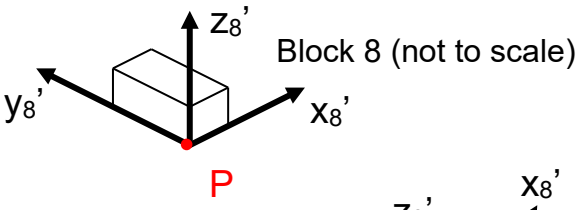
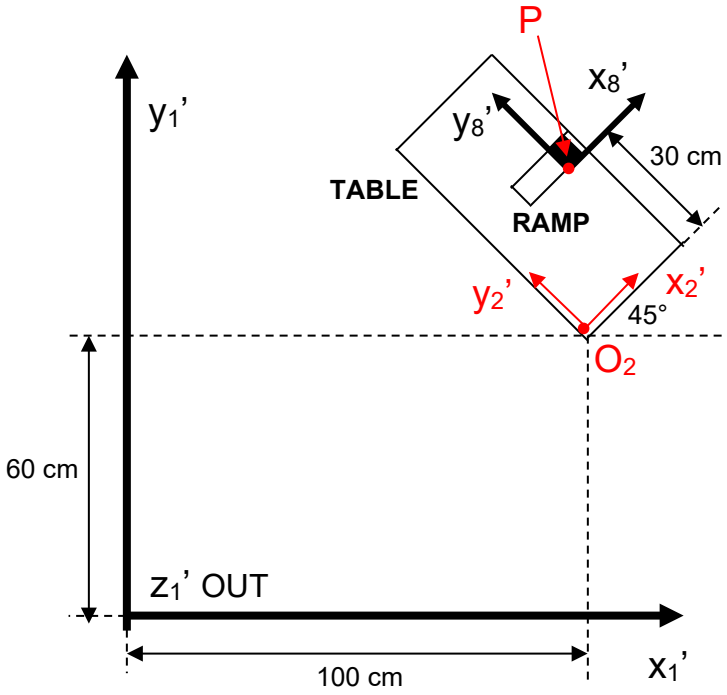
ME 581 – H04 **Name** _____

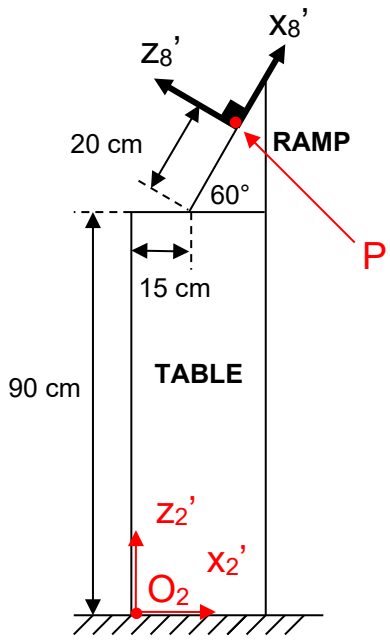
!) A machined block is resting on a ramp feeder on a small table in a robot work cell as shown below. Determine the location and attitude of block 8 with respect to global coordinate frame 1. Ensure that your attitude matrix is positive orthonormal.

$$\{r_8\} = \begin{Bmatrix} 96.46\text{ cm} \\ 98.89\text{ cm} \\ 107.32\text{ cm} \end{Bmatrix} \qquad [A_8] = \begin{bmatrix} 0.3536 & -0.7071 & -0.6124 \\ 0.3536 & 0.7071 & -0.6124 \\ 0.8660 & 0 & 0.5000 \end{bmatrix} \qquad \det[A_8] = +1.0000$$





Top view of work cell (not to scale)



Side view of table (not to scale)

$$\{r_2\} = \begin{Bmatrix} 100 \\ 60 \\ 0 \end{Bmatrix} \text{cm} \qquad [A_2] = \begin{bmatrix} C45° & -S45° & 0 \\ S45° & C45° & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

rotation $+45°$ about z_1'

$$\{s_2\}^P = \begin{Bmatrix} 15 + 20\,C60° \\ 30 \\ 90 + 20\,S60° \end{Bmatrix} \text{cm} = \begin{Bmatrix} 25 \\ 30 \\ 107.32 \end{Bmatrix} \text{cm} \qquad [C] = \begin{bmatrix} C(-60°) & 0 & +S(-60°) \\ 0 & 1 & 0 \\ -S(-60°) & 0 & C(-60°) \end{bmatrix}$$

rotation $-60°$ about y_2'

$$\{r_2\}^P = \{r_2\} + [A_2]\{s_2\}^P$$

$$[A_8] = [A_2][C]$$

ME 581 – H04 **Name** _____

```
% h04.m - homework 4 for ME 581
% HJSIII, 12.04.02

clear

% constants
d2r = pi / 180;

% ground to lower corner of table
r2 = [ 100  60  0 ]'

thz = 45 * d2r;
A2 = [ cos(thz)  -sin(thz)  0 ;
       sin(thz)   cos(thz)  0 ;
       0          0         1 ]

% lower corner table to feeder
s2pP = [ 15+20*cos(60*d2r)  30  90+20*sin(60*d2r) ]'

thy = -60 * d2r;
C = [ cos(thy)   0  sin(thy) ;
      0          1  0        ;
     -sin(thy)   0  cos(thy) ]

% global location of point P
r2P = r2 + A2 * s2pP

% global attitude of x8p
A8 = A2 * C
detA8 = det( A8 )

% bottom of h04.m
```

ME 581 – H04 **Name** _____

2) Extract Euler parameters from $[A] = \begin{bmatrix} 0.6538 & 0.3609 & 0.6650 \\ -0.7507 & 0.1998 & 0.6297 \\ 0.0944 & -0.9109 & 0.4016 \end{bmatrix}$

$\{p\} = \{$ 0.7509 -0.5129 0.1900 -0.3701 $\}^T$

Download and run "make_ega.m" from our class web page to check your work.

$e_0^2 = (\mathrm{tr}[A]+1)/4 \qquad \{e\} = \begin{Bmatrix} e_1 \\ e_2 \\ e_3 \end{Bmatrix} = \frac{1}{4e_0} \begin{Bmatrix} a_{32}-a_{23} \\ a_{13}-a_{31} \\ a_{21}-a_{12} \end{Bmatrix}$ check $e_0^2 + e_1^2 + e_2^2 + e_3^2 = 1$

```
>> p'
    0.7509   -0.5129    0.1900   -0.3701

>> [E,G,A,f,g,h] = make_ega(p);

>> A
    0.6538    0.3609    0.6650
   -0.7507    0.1999    0.6296
    0.0943   -0.9109    0.4017
```

3) Perform the following cross product by hand.

$\{a\} = \{0.6538 \quad -0.7507 \quad 0.0944\}^T \qquad \{b\} = \{0.3609 \quad 0.1998 \quad -0.9109\}^T$

$\{a\} \times \{b\} = \{$ 0.6650 0.6296 0.4016 $\}^T$

Download "skew_sym.m" from our class web page and calculate $[\tilde{a}]\{b\}$ to check your work.

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} & \hat{i} & \hat{j} \\ 0.6538 & -0.7507 & 0.0944 & 0.6538 & -0.7507 \\ 0.3609 & 0.1998 & -0.9109 & 0.3609 & 0.1998 \end{vmatrix} = \begin{Bmatrix} (-0.7507)(-0.9109)-(0.1998)(0.0944) \\ (0.0944)(0.3609)-(-0.9109)(0.6538) \\ (0.6538)(0.1998)-(0.3609)(-0.7507) \end{Bmatrix}$$

```
>> a'
    0.6538   -0.7507    0.0944

>> b'
    0.3609    0.1998   -0.9109

>> ( skew_sym(a)*b )'
    0.6650    0.6297    0.4016
```

ME 581 – H04 **Name** _____

EXTRA CREDIT

Local coordinates of five landmarks A,B,C,D,E on rigid body 7 are given below. Unfortunately, landmark labeling was scrambled when global pose of this object was measured, and the five global locations 1,2,3,4,5 shown below cannot be associated sequentially with the landmarks (i.e. global location 2 may refer to landmark D). Further, there was a measurement error, and one of the global locations is completely wrong.

LANDMARK	local x_7'	local y_7'	local z_7'
A	0	0	0
B	0	0	3
C	2	0	0
D	0	1	0
E	0	0	-4

LOCATION	global x_1'	global y_1'	global z_1'
1	2.574	-3.482	2.146
2	2.000	-3.000	4.000
3	2.510	-2.142	3.935
4	-1.244	-1.184	2.524
5	4.433	-4.362	1.107

a) Match the global locations to their respective landmarks.

local landmark	A	B	C	D	E
global location	2	5	1	3	4

b) Identify the incorrect global location and state why.

z for global location 5 should be 5.107 not 1.107

c) Using the correct global locations, describe the pose of object 7 in global coordinates.

$$\{r_7\} = \begin{Bmatrix} 2 \\ -3 \\ 4 \end{Bmatrix} \qquad [A_7] = \begin{bmatrix} 0.2870 & 0.5100 & 0.8110 \\ -0.2410 & 0.8580 & -0.4540 \\ -0.9270 & -0.0650 & 0.3690 \end{bmatrix}$$

ME 581 – H04 **Name** _____

```
% h04_ec.m - ME 581 H04 extra credit
% HJSIII, 19.04.05

clear

% constants
d2r = pi /180;

% local and global coordinates
loc = [  0   0   0 ;
         0   0   3 ;
         2   0   0 ;
         0   1   0 ;
         0   0  -4 ]';

glo = [  2.574  -3.482  2.146 ;
         2.000  -3.000  4.000 ;
         2.510  -2.142  3.935 ;
        -1.244  -1.184  2.524 ;
         4.433  -4.362  1.107 ]';

% interlandmark distances
for i=1:5;
  for j=1:5;
    dloc(i,j) = norm( loc(:,i) - loc(:,j) );
    dglo(i,j) = norm( glo(:,i) - glo(:,j) );
  end
end
dloc
dglo

% sort       A2        B5 - bad  C1        D3        E4
glo_sort = [ glo(:,2)  glo(:,5)  glo(:,1)  glo(:,3)  glo(:,4) ];

% zap bad location
loc_ok = [ loc(:,1)       loc(:,3:5)      ];
glo_ok = [ glo_sort(:,1)  glo_sort(:,3:5) ];
for i=1:4;
  for j=1:4;
    dloc_ok(i,j) = norm( loc_ok(:,i) - loc_ok(:,j) );
    dglo_ok(i,j) = norm( glo_ok(:,i) - glo_ok(:,j) );
  end
end
dloc_ok
dglo_ok

% {global} = {r} + [A] * {local}
%
% homogeneous coordinates
% { 1   }   [ 1   0    0    0   ]   { 1   }
% {glo_x} = [ rx  a11  a12  a13 ] * {loc_x}
% {glo_y}   [ ry  a21  a22  a23 ]   {loc_y}
% {glo_z}   [ rz  a31  a32  a33 ]   {loc_Z}
%
% {glo_hom} = [T] * {loc_hom]
loc_hom = [ 1 1 1 1 ; loc_ok ];
glo_hom = [ 1 1 1 1 ; glo_ok ];
T = glo_hom * inv( loc_hom );
r = T(2:4,1)
A = T(2:4,2:4)
det_a = det(A)

% predict global for all
glo_pred = r*ones(1,5) + A*loc
glo_sort

% bottom of h04_ec
```

ME 581 – H04 **Name** _____

```
>> h04_ec

dloc =
         0    3.0000    2.0000    1.0000    4.0000
    3.0000         0    3.6056    3.1623    7.0000
    2.0000    3.6056         0    2.2361    4.4721
    1.0000    3.1623    2.2361         0    4.1231
    4.0000    7.0000    4.4721    4.1231         0

dglo =
         0    1.9998    2.2361    4.4722    2.3043
    1.9998         0    1.0002    4.0000    4.0180
    2.2361    1.0002         0    4.1233    4.0772
    4.4722    4.0000    4.1233         0    6.6585
    2.3043    4.0180    4.0772    6.6585         0

dloc_ok =
         0    2.0000    1.0000    4.0000
    2.0000         0    2.2361    4.4721
    1.0000    2.2361         0    4.1231
    4.0000    4.4721    4.1231         0

dglo_ok =
         0    1.9998    1.0002    4.0000
    1.9998         0    2.2361    4.4722
    1.0002    2.2361         0    4.1233
    4.0000    4.4722    4.1233         0

r =
     2
    -3
     4

A =
    0.2870    0.5100    0.8110
   -0.2410    0.8580   -0.4540
   -0.9270   -0.0650    0.3690

glo_pred =
    2.0000    4.4330    2.5740    2.5100   -1.2440
   -3.0000   -4.3620   -3.4820   -2.1420   -1.1840
    4.0000    5.1070    2.1460    3.9350    2.5240

glo_sort =

    2.0000    4.4330    2.5740    2.5100   -1.2440
   -3.0000   -4.3620   -3.4820   -2.1420   -1.1840
    4.0000    1.1070    2.1460    3.9350    2.5240
```

ME 581 – H04 **Name** _____

```
% h04_ec02.m - ME 581 H04 extra credit
%   v02 exhaustive search with affine [A]
% HJSIII, 19.04.05

clear

% constants
d2r = pi /180;

% local and global coordinates
loc = [  0   0   0 ;
         0   0   3 ;
         2   0   0 ;
         0   1   0 ;
         0   0  -4 ]';

glo = [  2.574  -3.482  2.146 ;
         2.000  -3.000  4.000 ;
         2.510  -2.142  3.935 ;
        -1.244  -1.184  2.524 ;
         4.433  -4.362  1.107 ]';

% zap one local at a time
list_local_all = [ 1 2 3 4 ;
                   1 2 3 5 ;
                   1 2 4 5 ;
                   1 3 4 5 ;
                   2 3 4 5 ];
for i_list = 1 : 5,
  list_loc = list_local_all( i_list,: );
  loc_ok = loc(:,list_loc);

% try all global
  for j1 = 1 : 5,
    for j2 = 1 : 5,
    if j2~=j1,
      for j3 = 1 : 5,
      if j3~=j2 & j3~=j1,
        for j4 = 1 : 5;
        if j4~=j3 & j4~=j2 & j4~=j1,
          list_glo = [ j1  j2  j3  j4 ];
          glo_ok = glo(:,list_glo);

% homogeneous coordinates for affine [A]
          loc_hom = [ 1 1 1 1 ; loc_ok ];
          glo_hom = [ 1 1 1 1 ; glo_ok ];
          T = glo_hom * inv( loc_hom );
          r = T(2:4,1);
          A = T(2:4,2:4);
          if abs( det(A)-1 ) < 0.0002,
            if max( max( abs(A) ) ) < 1.001,
              list_loc
              list_glo
              r
              A
            end  % bottom of if max
          end  % bottom of if abs

        end, end  % bottom of for/if j4
      end, end  % bottom of for/if j3
    end, end  % bottom of for/if j2
  end  % bottom of for j1
end  % bottom of for i_list

% bottom of h04_ec02
```

ME 581 – H04 **Name** _____

```
% h04_ec03.m - ME 581 H04 extra credit
%   v03 exhaustive search with orthonormal [A]
% HJSIII, 19.04.05

clear

% constants
d2r = pi /180;

% local and global coordinates
loc = [  0   0   0 ;
         0   0   3 ;
         2   0   0 ;
         0   1   0 ;
         0   0  -4 ]';

glo = [  2.574  -3.482  2.146 ;
         2.000  -3.000  4.000 ;
         2.510  -2.142  3.935 ;
        -1.244  -1.184  2.524 ;
         4.433  -4.362  1.107 ]';

% zap one local at a time
list_local_all = [ 1 2 3 4 ;
                   1 2 3 5 ;
                   1 2 4 5 ;
                   1 3 4 5 ;
                   2 3 4 5 ];
for i_list = 1 : 5,
  list_loc = list_local_all( i_list,: );
  loc_ok = loc(:,list_loc);

% try all global
  for j1 = 1 : 5,
    for j2 = 1 : 5,
    if j2~=j1,
      for j3 = 1 : 5,
      if j3~=j2 & j3~=j1,
        for j4 = 1 : 5;
        if j4~=j3 & j4~=j2 & j4~=j1,
          list_glo = [ j1  j2  j3  j4 ];
          glo_ok = glo(:,list_glo);

% orthonormal [A]
          fi = [ 1 1 1 1 ];
          [ A, r ] = ab2rt( loc_ok, glo_ok, fi );
          glo_pred = r*ones(1,4) + A*loc_ok;
          if max( max( abs(glo_ok - glo_pred) ) ) < 0.0002,
            list_loc
            list_glo
            r
            A
          end  % bottom of if max
        end, end  % bottom of for/if j4
      end, end  % bottom of for/if j3
    end, end  % bottom of for/if j2
  end  % bottom of for j1
end  % bottom of for i_list

% bottom of h04_ec03
```