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Driver Assisted Yaw Rate Control

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Abstract

A yaw rate controller was developed that utilizes the rear wheels of a vehicle to improve steering performance. The driver retains control over the front wheels, while a Model Reference Controller (MRC) commands the rear wheels. Experimental results were obtained by implementing the MRC on a scale vehicle. These results indicate significant performance improvement over proportional yaw-rate controller methods often used for rear-wheel control. Specifically, the MRC method was found to be less sensitive to model non-linearities such as actuator dynamics and steering linkage kinematics.

1. Introduction

To satisfy increasing efficiency, performance, and safety requirements demanded of new vehicles, many vehicle subsystems have been augmented with automated controllers. In terms of vehicle performance and handling, the assistance these controls provide in the braking, traction control, and steering of a vehicle has led to significant improvements in performance. Driver Assisted Control (DAC) can be used to provide stability and performance, while still allowing the driver to dictate the path of the vehicle, by assisting the driver in her/his directional control of the vehicle. Parameters such as yaw angle, yaw rate, lateral velocity, and lateral acceleration would be sensed or estimated to provide the control with the necessary information to achieve desired transient and steady state performance based on the driver's input to the vehicle.

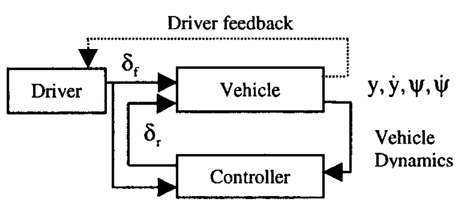

Figure 1: DAC Schematic

The motivations for the DAC controller are twofold: (i) to give the driver a "tunable" vehicle in terms of its handling performance and (ii) to give the vehicle a more stable and predictable response at higher speeds. For this investigation, the DAC was implemented on a vehicle by using the rear wheels as the control input. As illustrated in Fig. 1, this still allows the driver to steer the vehicle's front wheels in a customary open loop fashion. A failure detection method can also be used to lock the rear wheels at zero position if the control becomes unstable for some reason. All of these features are advantages because they utilize the rear wheels as input to alter the dynamic response of the vehicle while still allowing the driver to dictate the vehicle path.

There is a wealth of literature on lateral vehicle dynamics modeling and automatic 2WS and 4WS control designs as detailed in the survey article by Tomizuka & Hedrick (1995). Additionally, there have been many studies done on 4WS approaches to change the vehicle's dynamics (Yamamoto, 1991, Sato, et al 1991, Lin, 1992, Inoue & Sugasawa, 1993, Furukuwa & Abe, 1997). Typically, the strategies associated with 4WS can be categorized into feedforward and feedback approaches (Inoue & Sugasawa, 1993). The feedforward approaches usually set the rear steer angles to be a function of the front steer angles: for example, a pure gain.

$$\delta_r = K_{steer}(V) \cdot \delta_f \tag{1}$$

In this case the goal of the controller is simply to minimize the steady state sideslip angle of the vehicle using a predefined gain as a function of the vehicle speed. Other feedforward approaches use a filtered or delayed value:

$$\delta_r = \frac{K_1 s + K_0}{\tau s + 1} \cdot \delta_f \tag{2}$$

where the values of the constants may be determined via Linear Optimal Control (Cho & Kim, 1995). The goal of the filtering is usually to alleviate the reverse vehicle sideslip during transient responses. There have been several feedback approaches to 4WS as well. The simplest ones have been straightforward extensions of the feedforward strategies where the vehicle yaw rate was fed back to the rear wheels.

$$\delta_r = K_{steer} \cdot \delta_f + K_{yaw} \cdot \dot{\psi} \tag{3}$$

The goal of the yaw rate feedback is to utilize a measurable variable to provide additional sideslip minimization at higher vehicle speeds. Some investigators considered full state feedback but lateral velocity states are inherently difficult to determine.

Relatively few of the previous investigations attempt to actually make the vehicle behave as if it had a different set of dynamics through feedback. Lee (1990) details work completed at General Motors Research Labs on several different strategies: rear wheel assist, front wheel and rear wheel assist, driver open loop (DOL), and driver in the loop (DIL). Several different combinations of feedback parameters were tried as well: yaw rate, sideslip, and lateral velocity. The potential drawback to several of these

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controllers is the use of the front wheels as the control augmenting the driver's commands. A failure in the controller hardware would not allow the driver to stabilize the vehicle because his/her input is the front wheels as well. For this very reason, the DAC strategies completed in the present work use the rear wheels as the control input.

The rest of the paper is organized as follows. In Section 2, an introduction to Model Reference Control is presented. An experimental validation of this control and a comparison to a proportional controller is presented in Section 3. Discussion is provided in Section 4 explaining various methods to overcome inherent model sensitivity to the velocity parameter. A conclusion then summarizes the main points of the paper

2. Model Reference Control

Drivers often describe the feel of the vehicle based on the dynamic model of the vehicle rather than time domain (rise time, steady state error) or frequency domain (bandwidth, phase lag) criteria. It is assumed that the driver has specified the desired performance characteristics in terms of a reference model. The purpose of the controller presented here is to ensure that the vehicle tracks the reference model. The basic Model Reference Control (MRC) (Astrom & Wittenmark, 1997) control strategy will be used along with a modification based on rejection of known disturbance dynamics. The driver front steer input will act as a reference command to the MRC of the rear steer model while simultaneously acting as a known output disturbance to the vehicle's yaw rate. For the standard MRC we assume the plant can be modeled as a ratio of two linear polynomials,

$$\frac{Y(s)}{U(s)} = \frac{B(s)}{A(s)}. \tag{4}$$

The polynomial A is assumed to be monic and of degree n. B can be non-monic and of degree less than or equal to n. It is also assumed that the polynomials are relatively prime, i.e. they have no common factors. The desired closed loop performance is:

$$\frac{Y(s)}{U_c(s)} = \frac{B_m(s)}{A_m(s)} \tag{5}$$

and the control law is given by:

$$Ru(t) = Tu_c(t) - Sy(t) \tag{6}$$

where R, S, T are polynomials in the Laplace operator s. The controller consists of a feedforward term (T/R) and a feedback term (S/R). The idea behind the controller is to cancel out the unwanted plant dynamics and replace them with the designer's own desired dynamics.

For the DAC case, we assume the same reference model from Equation (5) but the plant dynamics of Equation (4) are now given as

$$A(s)y = B(s)u + \underbrace{\Delta}_{disturbance} \tag{7}$$

Assume that the disturbance, Δ, can be separated into two parts: known dynamics and unknown disturbances. Assuming a rational, causal transfer function representation for the disturbance gives:

$$\Delta(s) = \underbrace{\frac{B_d(s)}{A_d(s)}}_{\substack{known\\dynamics}}\gamma(s) + \underbrace{v(s)}_{\substack{unknown\\disturbance}} \tag{8}$$

The term, γ, is defined as the disturbance generator for the known disturbance. The unknown disturbance term, v, contains both unmodeled dynamics of the system as well as external disturbances (e.g. wind) that are not known. Suppressing the Laplace operator for convenience, Equation (7) can be rewritten as

$$A_d A y = A_d B u + B_d \gamma + A_d v \tag{9}$$

The control signal has the same structure as (6), where R, S, T are controller design polynomials. Substituting (6) into (9) gives:

$$\frac{A_d(AR+BS)}{R} y = \frac{A_d BT}{R} u_c + B_d \gamma + A_d v \tag{10}$$

$$\Rightarrow y = \frac{BT}{AR+BS} u_c + \frac{RB_d}{A_d(AR+BS)}\gamma + \frac{R}{(AR+BS)} v \tag{11}$$

Considering (6) again and simplifying for the input u:

$$Ru = Tu_c - S\left(\frac{BT}{AR+BS}u_c + \frac{RB_d}{A_d(AR+BS)}\gamma + \frac{R}{(AR+BS)}v\right) \tag{12}$$

$$\Rightarrow u = \frac{AT}{AR+BS}u_c - \frac{B_d S}{A_d(AR+BS)}\gamma - \frac{S}{(AR+BS)}v \tag{13}$$

For the DAC controller, let G_m be a reference model that obtains the desired response from the driver's front steer input to the vehicle's yaw rate.

$$y_{des} = \dot{\psi}_{des} = G_m \delta_f = \frac{B_m}{A_m}\delta_f \tag{14}$$

Therefore, the command input u_c is actually the driver steer command δ_f.

$$u = \frac{AT}{AR+BS}\delta_f - \frac{B_d S}{A_d(AR+BS)}\gamma - \frac{S}{(AR+BS)}v \tag{15}$$

However, the disturbance generator is also the front steer angle command, $\gamma = \delta_f$, since it is the driver's steer input that causes uncontrolled changes in the yaw rate. Consequently, the input can be simplified as:

$$u = \underbrace{\frac{A_d AT - B_d S}{A_d(AR+BS)}\delta_f}_{feedforward} - \underbrace{\frac{S}{(AR+BS)}v}_{feedback} \tag{16}$$

where the disturbance polynomials A_d and B_d are the denominator and numerator polynomials for the transfer function from front steer command to yaw rate. As is evident, Equation (16) represents a 2 DOF controller with both a feedforward and feedback component. Ideally, the better the models for the front and rear steer transfer functions, the larger the relative activity of the feedforward portion of the controller.

The uniqueness of this model reference approach is that the effect of the driver's front steer input on the vehicle yaw rate is formulated as an output disturbance which should be rejected by the controller governing the rear wheels. Obviously, the front steer will be aiding in the output yaw rate tracking of the desired reference model for much of the time. Therefore, the controller's rear wheel steering will add only that incremental steer effort which is deemed necessary. Since the steady state gain of the yaw rate from front steer input is a function of the vehicle's understeer gradient (Genta, 1997), the choice of both transient and steady state reference model performance completely dictates the vehicle's understeer gradient.

3. Experimental Results

The experimental results of the control approach were investigated using the Illinois Roadway Simulator (IRS). Scale vehicles in sizes ranging from 1/10 to 1/8 scale are driven on a treadmill surface that moves the road surface beneath the vehicle. Measurements of the vehicle's position, yaw angle, and yaw rate are obtained by a linkage system attached to the vehicle that uses encoders and the linkage kinematic equations. Previous comparisons between the scale vehicle used in this study and various full sized vehicles show similar pole-zero locations and trends. Further details of the IRS system may be found in Brennan et al. (1998).

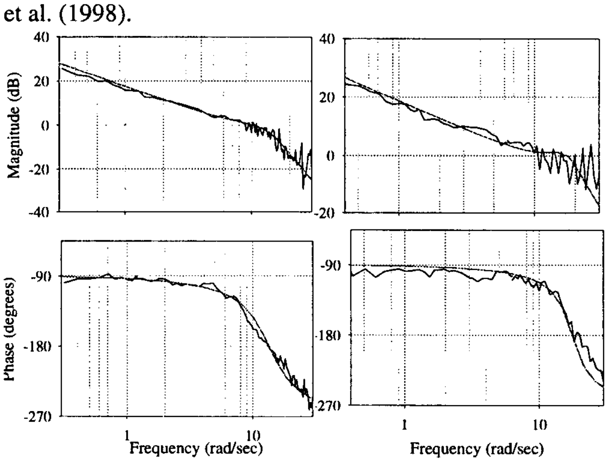

Figure 2: Frequency responses from front (left) and rear (right) steer command to yaw angle

For the experimental testing conducted in this paper, a 4 WD 1/10 scale vehicle was driven at 1.2 m/s (30 mph scale speed). The vehicle position, yaw angle, and yaw rate were measured from the center of gravity of the vehicle. The front steer commands to control the vehicle's lateral position on the IRS are controlled via a person operating a driving console near the vehicle, or from a digital signal analyzer when obtaining the frequency responses. The frequency responses from front and rear steer angles to vehicle yaw rate are shown in Figure 2. The corresponding transfer functions are

$$G_f = \frac{13480}{s(s^2+10.3s+180)} \quad \frac{\text{deg}}{\text{Volt}} \qquad (17)$$

$$G_r = \frac{26500}{s(s^2+8.5s+310)} \quad \frac{\text{deg}}{\text{Volt}} \qquad (18)$$

The transfer functions in Equations (17) and (18) are from steer command to yaw angle; hence the free integrator. To get the yaw rate transfer functions, a simple elimination of the free integrator would suffice. Using these models, and the reference model:

$$G_m(s) = \frac{34370}{s^2+30\cdot s+306} \qquad (19)$$

This reference model corresponds to increasing the DC gain and the bandwidth by approximately 1.5 while adding damping to the open-loop system. Using this reference model, the designed MRC polynomials are given as:

$$R(s) = s + 41.5 \qquad (20)$$

$$S(s) = 0.0092s - 0.2545 \qquad (21)$$

$$T(s) = -1.2971 - 25.9426 \qquad (22)$$

$$A_o(s) = s + 20 \qquad (23)$$

The reference model in (19) was chosen to give a fast, stable, transient response.

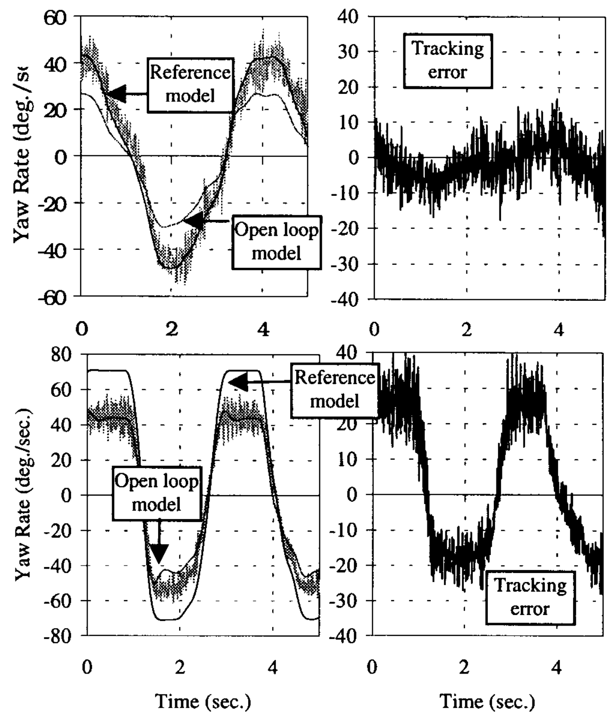

Figure 3: Reference model vs. experimental yaw rate. Both MRC in use (top) and disabled (bottom) are shown with tracking errors (right).

Figure 3 gives a comparison between the reference model and the actual vehicle yaw rate for a vehicle performing a double lane change under manual driver control. Also shown are the output of the reference model and the response of the vehicle without DAC for the same double lane change as before. Clearly, with the DAC, the vehicle's yaw degree of freedom behaves much more like the desired reference model.

A particularly interesting aspect of the tracking error is that the error is largest when the actuator slew rate is high. An analysis of the dynamic response of the steering actuators demonstrated that they are rate limited (Brennan et al., 1998). The inability of the actuator to achieve the reference command imposes an upper limit on the controller bandwidth. Since full-scale vehicles using hydraulic steer inputs are also rate limited, this bandwidth limitation on the use of linear controller methods is an important consideration.

In the above open-loop dynamics, it is clear that there is an overshoot and slight non-linearity that is particularly influential in one direction of motion. An analysis of the linkage kinematics explains this error. Figure 4 shows the linkage kinematics.

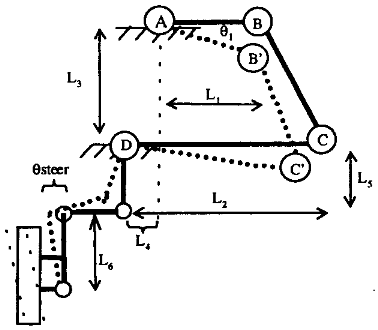

Figure 4: Linkage kinematics for front and rear steer input.

The lengths of the links in the above steering linkage are as follows:

$$\begin{aligned} L_1&=8\text{ mm}\\ L_2&=24\text{ mm}\\ L_3&=52\text{ mm}\\ L_4&=3\text{ mm}\\ L_5 = L_6 &=30\text{ mm} \end{aligned} \tag{24}$$

Examination of the geometric arrangement of the linkages reveals that the steering angle θ_{steer} is equal to the angle ∠CDC'. To solve for ∠CDC', we can examine the system as a four-bar linkage. In standard vector notation,

$$r_1e^{i\theta_1} + r_2e^{i\theta_2} + r_3e^{i\theta_3} + r_4e^{i\theta_4} = 0 \tag{25}$$

We can see that

$$\begin{aligned} r_1 &= L_1\\ r_2 &= \sqrt{L_3^2 + (L_2 - L_1 - L_4)^2}\\ r_3 &= L_2\\ r_4 &= \sqrt{L_4^2 + L_3^2} \end{aligned} \tag{26}$$

If we solve numerically for r_2 and r_4, we find r_2=55.02 mm and r_4 = 52.09 mm. We can also note that θ_4 is the angle of the ground link, and does not change. Solving for θ_4:

$$\theta_4 = \tan^{-1}\left(\frac{L_3}{L_4}\right) \tag{27}$$

we find θ_4 = 176.7 degrees numerically. If we continue the vector equation by expanding the imaginary vectors, we can then collect the real and imaginary portions of the vectors to obtain:

$$\begin{aligned} r_2\sin(\theta_2) &= -r_1\sin(\theta_1) - r_3\sin(\theta_3) - r_4\sin(\theta_4)\\ r_2\cos(\theta_2) &= -r_1\cos(\theta_1) + r_3\cos(\theta_3) - r_4\cos(\theta_4) \end{aligned} \tag{28}$$

If we square both equations and add them, we obtain:

$$\begin{aligned} r_2^2 = {} & r_1^2 + r_3^2 + r_4^2\\ & - 2r_1r_3[\sin(\theta_1)\sin(\theta_3) + \cos(\theta_1)\cos(\theta_3)]\\ & + 2r_1r_4[\sin(\theta_1)\sin(\theta_4) + \cos(\theta_1)\cos(\theta_4)]\\ & - 2r_3r_4[\sin(\theta_3)\sin(\theta_4) + \cos(\theta_3)\cos(\theta_4)] \end{aligned} \tag{29}$$

Collecting terms relating θ_3:

$$\begin{aligned} & r_2^2 - r_1^2 - r_3^2 - r_4^2 - 2r_1r_4\cos(\theta_1 - \theta_4)\\ & = -[2r_1r_3\sin(\theta_1) + 2r_3r_4\sin(\theta_4)]\sin(\theta_3)\\ & \quad - [2r_1r_3\cos(\theta_1) + 2r_3r_4\cos(\theta_4)]\cos(\theta_3) \end{aligned} \tag{30}$$

It is easier to solve this if the following substitution is made

$$\begin{aligned} A &= r_2^2 - r_1^2 - r_3^2 - r_4^2 - 2r_1r_4\cos(\theta_1 - \theta_4)\\ B &= -[2r_1r_3\sin(\theta_1) + 2r_3r_4\sin(\theta_4)]\\ C &= -[2r_1r_3\cos(\theta_1) + 2r_3r_4\cos(\theta_4)] \end{aligned} \tag{31}$$

The equation then becomes

$$A = B\sin(\theta_3) + C\cos(\theta_3) \tag{32}$$

We can solve this if we use the trigonometric relationships:

$$\begin{aligned} \sin(\theta) &= \frac{2\tan(\theta)}{1+\tan^2(\theta)}\\ \cos(\theta) &= \frac{1-\tan^2(\theta)}{1+\tan^2(\theta)} \end{aligned} \tag{33}$$

Substituting, the equation becomes

$$(1 + \tan^2(\theta_3))A = (2\tan(\theta_3))B + (1 - \tan^2(\theta_3))C \tag{34}$$

Which is quadratic in $\tan(\theta_3)$,

$$(A + C)\tan^2(\theta_3) - 2B\tan(\theta_3) + (A - C) = 0 \tag{35}$$

Solving

$$\theta_3 = \operatorname{a\,tan}\left(\frac{B \pm \sqrt{B^2 - A^2 + C^2}}{(A + C)}\right) \tag{36}$$

By plotting the servo angle with respect to the wheel angle, the non-linearity can be seen.

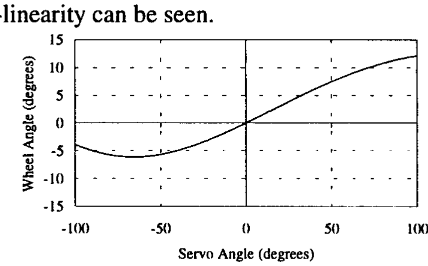

Figure 5: The variation from a linear response due to linkage kinematics.

The system is nominally operated at 0 degrees. However, for large maneuvers as seen in the above open-loop responses the linkage non-linearity becomes evident. This asymmetry is masked with use of the controller due to feedback. In a full sized vehicle, the linkage kinematics would likely be more linear, and so this limitation would likely not be as influential.

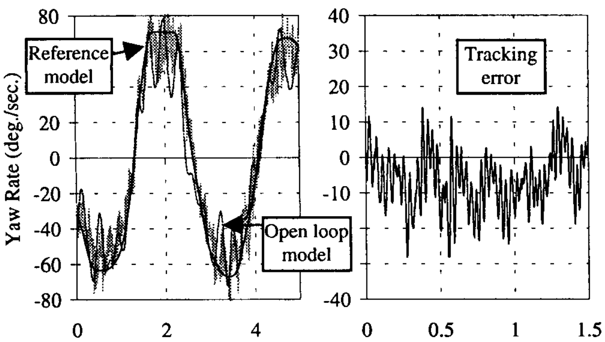

Figure 6: The proportional controller responses. A close-up of the experimental tracking error (right) shows a persistent oscillation.

For comparative purposes, a proportional controller response is shown in Figure 6, where:

$$\delta_r = K_p \cdot (\dot{\psi}_{des} - \dot{\psi}) = K_p \cdot \left(\frac{B_m}{A_m} \delta_f - \dot{\psi} \right) \qquad (37)$$

Although the tracking error appears reasonable in magnitude, the system non-linearities introduce a limit cycle that is highly undesirable. If enough derivative gain is added to where these oscillations are not noticeable, the additional phase lag degrades performance significantly. The best performance was found using the MRC controller utilizing both a feedforward and feedback component.

4. Discussion

Since its basis is an MRC approach, the DAC relies on a fairly accurate model of the vehicle. However the vehicle's dynamics may change significantly with velocity or with road conditions. In an effort to make the control approach feasible across a range of parameter variations, a continuous time RLS adaptive estimator was combined with the original DAC in DePoorter et al, (1998). The estimator operated on a reduced order model of the vehicle. The approach was shown to be successful for the experimental IRS system and was able to account for vehicle parameter changes due to increased speed. However, the proper parameter convergence for the front and rear steer transfer functions required more Persistence of Excitation than would probably be available in an actual vehicle. In a nominal driving condition, which consists of relatively small driver input, the identification scheme may not obtain enough PE to get convergence to the actual parameters. Moreover, the reduced order model may not be accurate at higher speeds. Consequently, a velocity scheduled control approach based on off-line system identification may be more realistic in practice and is the avenue of ongoing IRS research efforts.

5. Conclusions

The experimental results indicate that the MRC design approach increases vehicle performance in a driver-selectable manner without the sensitivity to non-linearities seen in P or PD control. The modeling of the driver input as a known disturbance into the system allows the driver to remain in the control loop and provides a fail-safe method of increasing vehicle performance. The nature of the MRC design approach makes the control sensitive to changes in the plant model. However, these changes are generally in the form of measurable parameter changes, such as changes in vehicle velocity. Future work will focus on control methods that compensate for these model changes.

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