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Integrated Vehicle Control via Coordinated Steering and Wheel Torque Inputs

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Abstract

A controller was developed to govern the lateral position of a highway-speed vehicle using frequency-weighted coordination of front steering and torque inputs. The MISO design problem was recast as a SISO approach by using a cascaded design technique: the first step determined the relative contribution of each control input as a function of frequency; secondary design steps utilized classical SISO approaches. For the vehicle control problem, the torque steering inputs were designed to act only as high-frequency inputs, while standard front steering was weighted for DC and low-frequency inputs. This controller was then tested on an experimental vehicle system.

1. Introduction

The independent use of either front steering, rear steering, and differential torque steering to control the yaw rate of a vehicle is a well-studied area of vehicle control. However, two issues are driving the vehicle research toward coordinated use of steering inputs: safety considerations have become a primary selling point with new vehicles; and newer vehicle designs such as electric vehicles are offering increasing opportunities for coordinated steering control.

Historically, the coordinated use of steering inputs is not a new concept. Several authors, notably [1], have established that independent control of several steering inputs can grant the vehicle system certain desirable properties such as complete dynamic separation between side-slip and yaw rate states. Other studies, such as [2], have investigated the wheel usage during coordinated maneuvers using the concept that maximum tire-force usage is a true measure of the system stability margin. Unfortunately, coordinated-steering investigations in general rely on the assumption that the system states such as side-slip angle and wheel forces are readily available, while in practice they are usually only estimated with a large margin of error.

The scope of the vehicle control literature is quite extensive, and the reader is referred to review articles [3] and [4] for appropriate summaries of the field. The intent of this article is to present a methodology for dealing with a steering coordination task, and the control methodology has been intentionally limited to a relatively simple linear model. Other authors have appropriately dealt with the system nonlinearities, most notably in [5], [6], [7], and [8].

Both wheel steering and wheel torque methods of driving are known to exhibit strong non-linearities as the steering inputs saturate, and many authors have dealt with these nonlinearities in the structure of the control algorithm. Rather than deal with the nonlinearities either through robust analysis or adaptation, a better integration of existing control strategies was sought.

The controller methodologies considered are the linear PQ design method developed in Schroeck and Messner [9]. The approach in [9] is considered primarily because it takes advantage of key structural properties of the vehicle dynamics and has demonstrated performance in Dual-Input Single-Output (DISO) systems such as hard-disk head controllers. This controller is examined in this study both in simulation and on an experimental scale vehicle given center-of-gravity lateral position as the sole feedback state.

The remainder of the article is organized as follows: First, an overview is presented of the vehicle dynamics under study as a Multi-Input Single-Output (MISO) controller design problem. The next section introduces the MISO design technique referred to as the PQ controller. The third section examines the specific implementation of the MISO controller on the vehicle system, and the fourth section presents implementation results on an experimental vehicle. A conclusion then summarizes the main points of the paper.

2. Vehicle Dynamics

Modeling of the vehicle dynamics is accomplished by fixing a coordinate system to the center of gravity (CG) of the vehicle and applying Newton's equations. Roll, pitch, bounce, and deceleration dynamics are neglected to simplify the vehicle dynamics to two degrees of freedom: the lateral position states and yaw angle states. The model is further simplified by assuming that each axle shares the same steering angles, and that consequently each wheel produces the same wheel angle steering forces. The resulting linear dynamic model, known as the bicycle model, conceptually matches a bicycle constrained to in-plane motion. The use of this model is explained in detail by Dugoff, Fancher, and Segel [10]. Although the bicycle

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model is relatively simple, it has been verified as a good approximation for full-size vehicle dynamics when accelerations are limited to 0.3 g's [11].

This study restricts the analysis to consider only the linear vehicle model. This study does not investigate tire saturation behavior; in such cases a model-switching method could be used when tire forces saturate as measured by the ABS or similar wheel slip sensor.

Additional assumptions must be made to extend the traditional linear vehicle model to the case where a differential torque command is used to steer the vehicle. The drive torque can be separated into two components: the steady-state driving torque and the transient steering torque. The steady-state driving torque is assumed to not produce a steering moment and not to affect the transient vehicle dynamics. The steering torque inputs are assumed to be "odd" about the longitudinal axis and are transmitted via the front axle only. That is, if the right-hand side of the vehicle is sent a positive steering torque command, the left-hand side of the vehicle is sent a steering torque of equal and opposite sign.

With the above assumptions, a state-space model can be obtained using the following methodology: First, the vehicle dynamics are written in state-space form with the tire forces acting as inputs to the system. We then solve for the tire forces as functions of the vehicle's lateral velocity and yaw rate, and control inputs from the from front steering and wheel torque inputs, thus completing the state-space representation. As a sign convention, the Society of Automotive Engineers standard coordinate system convention is used with the z-axis pointing into the road surface. The wheel torque that tends to spin the vehicle in the positive yaw direction, shown in Fig. 3, will be considered positive.

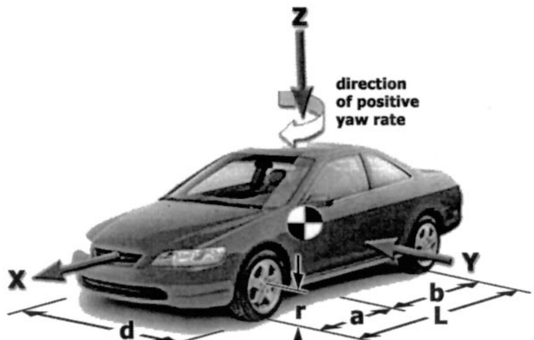

Figure 1: Vehicle notation.

The state equations for the controller design are formed in terms of error states as the vehicle is forced to follow a trajectory:

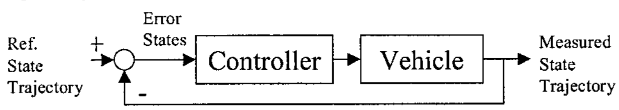

Figure 2: Error states.

The error state equations are as follows:

$$\dot{x} = A \cdot x + B \cdot u \tag{1}$$

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & -\dfrac{C_{\alpha f}+C_{\alpha r}}{m \cdot U} & \dfrac{C_{\alpha f}+C_{\alpha r}}{m} & -\dfrac{a \cdot C_{\alpha f} - b \cdot C_{\alpha r}}{m \cdot U} \\ 0 & 0 & 0 & 1 \\ 0 & -\dfrac{a \cdot C_{\alpha f} - b \cdot C_{\alpha r}}{I_z \cdot U} & \dfrac{a \cdot C_{\alpha f} - b \cdot C_{\alpha r}}{I_z} & -\dfrac{a^2 \cdot C_{\alpha f} + b^2 \cdot C_{\alpha r}}{I_z \cdot U} \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 \\ \dfrac{C_{\alpha f}}{m} & \dfrac{C_{\alpha r}}{m} & 0 \\ 0 & 0 & 0 \\ \dfrac{C_{\alpha f} \cdot a}{I_z} & \dfrac{-C_{\alpha r} \cdot b}{I_z} & \dfrac{d}{I_z \cdot r} \end{bmatrix},$$

$$x = \begin{bmatrix} y & \dot{y} & \Psi & \dot{\Psi} \end{bmatrix}^T \quad u = \begin{bmatrix} \delta_f & \delta_r & \Delta T \end{bmatrix}$$

With states and control inputs:

y = lateral position error [m]

ψ = yaw angle of the vehicle w.r.t. ground [rad]

ΔT , δ_f = steering torque input and front steering angle.

and plant parameters:

m = vehicle mass [kg]	(6.02)
I_z = mom. of inertia, z-axis [kg-m^2]	(0.15)
U = constant longitudinal velocity [m/s]	(4.0)
$C_{\alpha f}$, $C_{\alpha r}$ = f. and r. wheel cornering stiffness,	(39,60)
a, b = length of f, r axle from C.G. [m],	(0.137,0.22)
r = wheel radius [m],	(0.02995)
d = dist. from centerline to wheel [m]	(0.115)

The values in parenthesis represent the numerical values measured for the scale experimental vehicle. If the torque input is ignored, the resulting linear state-space model agrees with published dynamics from [12], among others within a non-dimensional framework [13].

The transfer functions $G_i(s)$ from front, rear, and differential torque steering to lateral position are given from the transformation $G_i(s) = C(sI-A)^{-1}B_i$. The characteristic equation is given as *den*, defined as

$$den = s^2 + a_1 \cdot s + a_0 \tag{2}$$

$$a_1 = \frac{\left(C_{\alpha f} + C_{\alpha r}\right)}{mU} + \frac{\left(C_{\alpha f} a^2 + C_{\alpha r} b^2\right)}{I_z U}$$

$$a_0 = \frac{C_{\alpha f} C_{\alpha r} L^2}{m I_z U^2} - \frac{\left(a C_{\alpha f} - b C_{\alpha r}\right)}{I_z}$$

The transfer functions are:

$$\frac{y(s)}{\delta_f(s)} = \frac{\frac{C_{af}}{m}\cdot s^2 + \frac{C_{af}C_{ar}bL}{mUI_z}\cdot s + \frac{C_{af}C_{ar}L}{mI_z}}{den}, \tag{3}$$

$$\frac{y(s)}{\delta_r(s)} = \frac{\frac{C_{ar}}{m}\cdot s^2 + \frac{C_{af}C_{ar}aL}{mUI_z}\cdot s - \frac{C_{af}C_{ar}L}{mI_z}}{den},$$

$$\frac{y(s)}{\Delta T(s)} = \frac{-\frac{d}{rmUI_z}\left(a\cdot C_{af} - b\cdot C_{ar}\right)\cdot s + \frac{d}{rmI_z}\cdot\left(C_{af} + C_{ar}\right)}{den}.$$

The lateral position error given all three separate inputs is given by:

$$y(s) = G_1(s)\cdot u_1(s) + G_2(s)\cdot u_2(s) + G_3(s)\cdot u_3(s) \tag{4}$$

A clear goal of the MISO vehicle controller problem is to coordinate the inputs to prevent tire force saturation. In particular, coordination between the wheel steering and torque steering control inputs is especially critical; if the front steering inputs are opposing the torque steering inputs, then the tire forces are unnecessarily high. For this reason, a control design methodology was sought that specifically accounted for coordination between input channels.

3. MISO Controller Design Methods

To accommodate a frequency weighting constraint between the steering and torque inputs, a MISO controller design methodology was chosen that was originally presented in Schroeck and Messner [9]. To explain this method, a simple DISO plant is first considered with three controllers as shown in Figure (3):

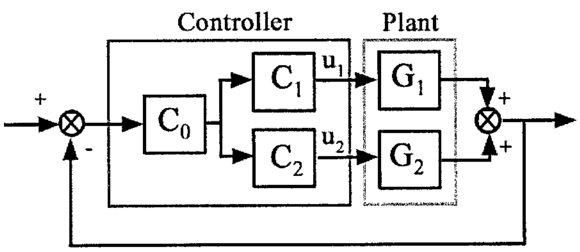

Figure 3: DISO plant.

To design the controller, first the ratio of the transfer functions G_1 and G_2 are taken as a new plant P as shown in Figure 4. A controller Q is then designed to shape P such that a desired frequency roll-off is achieved. High values of the loop gain PQ(jω) indicate that input 2 dominates at this frequency while low values indicate that input 1 dominates. Thus, if G_1 is a fine, high-frequency actuator and G_2 is a coarse, low-frequency actuator, a large PQ loop gain is desired at low frequencies and low loop gain is desired at high frequencies. Thus the frequency weighting problem is simply re-cast as a controller loop-shaping problem. Selection of Q decides the relative frequency separation of the parallel subsystems formed by C_1G_1 and C_2G_2.

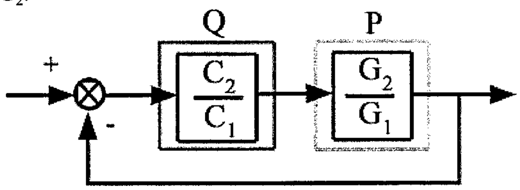

Figure 4: DISO plant.

After Q is designed, this controller transfer function is then broken up between C_2 and C_1 such that each controller is realizable. If Q is realizable, then a natural choice is $C_1 = 1$, $C_2 = Q$.

With C_1 and C_2 fixed, the controller design problem becomes a SISO design problem in terms of selecting C_0 to stabilize G_{SISO} as shown in Figure 5. Again, any design technique can be used to design C_0, but loop-shaping procedures are quite easy to implement.

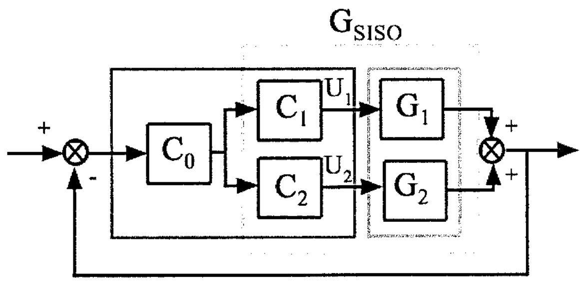

Figure 5: G_{SISO} plant and C_0 controller.

The extension of the PQ design technique is easily extended to higher order models, however for the vehicle steering coordination task, a two-input coordination task for wheel steering and torque steering for a front-wheel drive vehicle serves as a good example for this technique.

4. Vehicle Controller Design

To demonstrate the design of a coordinated steering PQ controller, a vehicle plant is considered where a front-wheel-drive vehicle's lateral position on the road is controlled. The feedback is measured lateral position error as measured from a sensor mounted at the C.G. of the vehicle. Both wheel steering and torque steering are available as steering inputs, but both must act through the same tires, thus coordination is necessary for best performance.

The vehicle model is chosen as the measured vehicle parameters substituted into the state space equations of equation (1). The Bode plots of the system response is shown in Figure 6.

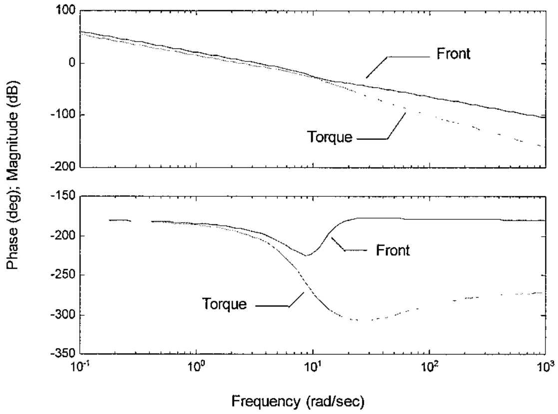

Figure 6: Bode plots from front and torque steering inputs to lateral position.

To coordinate the two inputs, the front steering input is chosen as the course input (G_2), while the torque steering input is chosen as the fine input (G_1). Therefore, for low frequency inputs the front wheel steering will be primarily be active, while for high-speed inputs the torque steering inputs will become active. The Bode plot of P(s) = $C_2(s)/C_1(s)$ is given in Figure 7.

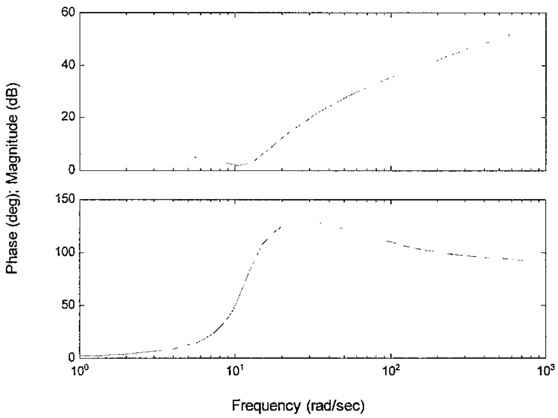

Figure 7: Bode plot of P(s).

Note that the poles cancel between $G_1(s)$ and $G_2(s)$ when P(s) is formed. Also note that front steering inputs dominate at very high frequencies due to the difference in the relative order of the input transfer functions in Equation (3).

The compensator Q(s) must be chosen with some caution. Since torque inputs are only desired to assist during rapid maneuvers, the loop gain P(s)Q(s) must be small at high frequencies and large at low frequencies. One method of achieving this would be to introduce a single very low frequency pole into P(s). However, this additional pole would introduce both a low frequency pole and zero into $G_{SISO}(s)$. Thus, a higher-order controller Q(s) is required.

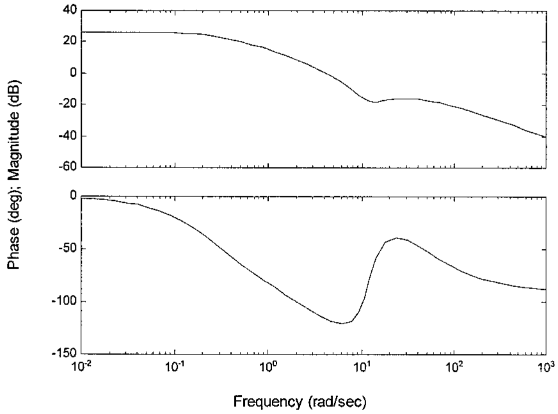

Figure 8: Bode plot of P(s)Q(s).

For this example, Q(s) was chosen as:

$$Q(s) = \frac{10}{0.667 \cdot s^2 + 3.5333 \cdot s + 1} \tag{5}$$

Figure 8 shows the Bode plot cascaded system P(s)Q(s) revealing the desired weighting. Since Q(s) is realizable, the controller Q was separated as $C_1(s) = 1$ and $C_2(s) = Q(s)$. Using these values for $C_1(s)$ and $C_2(s)$, $G_{SISO}(s)$ is formed and the Bode plot of the uncompensated system is shown in Figure 9.

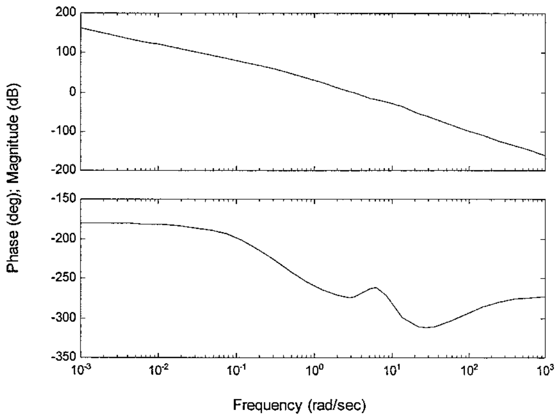

Figure 9: Bode plot of $G_{SISO}(s)$.

A controller $C_0(s)$ is then designed using classical controller design techniques to achieve sufficient phase via a double-lead compensator.

$$C_0(s) = \frac{1014 \cdot s^2 + 1115 \cdot s + 424}{s^2 + 72 \cdot s + 3232} \tag{6}$$

The resulting $C_0(s)G_{SISO}(s)$ is shown in Figure 10.

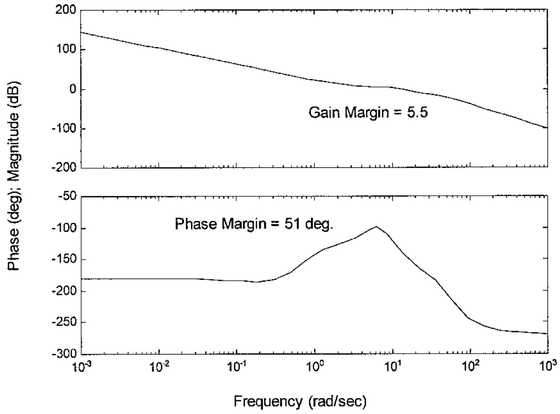

Figure 10: Bode plot of $C_0(s)G_{SISO}(s)$.

5. Implementation on the Illinois Roadway Simulator

To test the PQ controller of Section 4, both simulation and experimental tests were performed using the Illinois Roadway Simulator (IRS) and a scale vehicle with front, rear, and torque-steering capability. The IRS is an experimental test bed consisting of approximately 1/8 scale vehicles running on a simulated road surface, where the vehicles are held fixed with respect to inertial space and the road surface moves relative to the vehicle. An analogy would be wind tunnel testing of aerospace systems. There are many advantages to using scale vehicles for testing including cost, durability, repeatability, safety, and flexibility of the experiment. Additionally, extensive testing has established a very high degree of dynamic similitude between the scale test vehicles and full-sized vehicles [13, 14].

The vehicle's velocity of 4.0 m/s was chosen to represent an average full-sized passenger vehicle operating at highway speeds of 65-70 mph. Further information about the treadmill system, vehicle setup, and dynamic similitude can be found in [14].

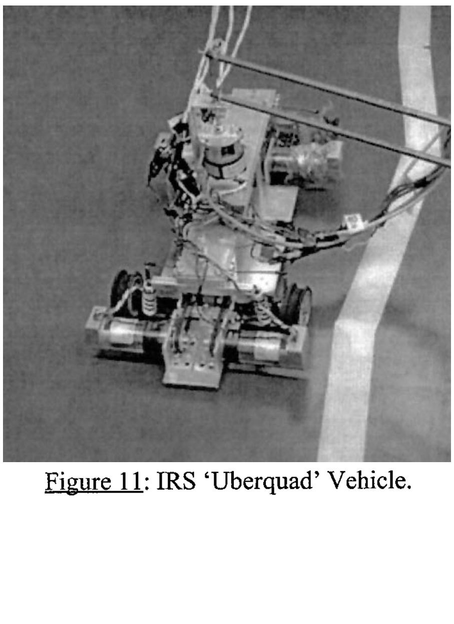

Figure 11: IRS 'Uberquad' Vehicle.

The vehicle steering and torque inputs are actuated via permanent magnet DC motors. Because the motor torque is proportional to input current for DC motors, the wheel and steering torque can be commanded directly. Although this direct method torque input is not currently practiced on conventional vehicles, future vehicle designs using hybrid electric/internal-combustion engine drive systems will likely have this capability. Further, the high-bandwidth drive motors on IRS scale vehicles can be made to emulate a conventional drive/braking system by modifying the control algorithm to exhibit powertrain and braking dynamics.

For this study, actuator dynamics were included in the test vehicle but not in the design steps. Using published values for full-sized vehicles as a guide, full-sized steering actuators can be approximated as second order systems ranging in bandwidth between 3 and 6 Hz. All steering actuators on the vehicle were chosen to represent linear 5 Hz second-order actuators with critical damping. Since the dynamics are not included in the controller design, experimental testing on the vehicle containing such dynamics provides a measure of robustness of the resulting controller.

The position of the center of gravity of vehicle was monitored via a light-weight arm mounted to the vehicle from a fixed ground position. Monitoring the angles of the arm via integrated encoders allowed the vehicle states to be measured [14].

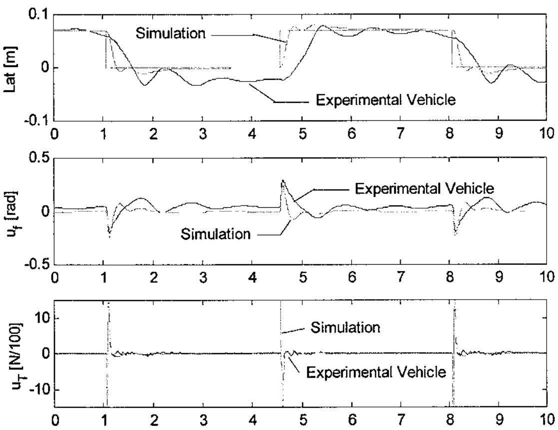

Figure 12: Experimental results

Shown in Figure 12 are the results of implementing the coordinated controller on the IRS vehicle system. Clear frequency separation of the inputs is achieved while maintaining a similar phase on the control inputs. Note that the vehicle response contains biases due to the lack of an integrator in the system and the fact that the wheels are not exactly aligned. Additionally, it should be noted that the response is less damped than predicted primarily due to the presence of high-frequency actuator dynamics that introduce a control input delay. This delay is especially noticeable on the front steering input. To ensure that the high-frequency torque input was actually affecting the vehicle, the torque inputs were shut off in simulation with the result being vehicle instability. This clearly justified the use of torque inputs.

Lateral position control at high speeds is known to be a difficult problem using only the lateral position state for feedback [15]. Our difficulty in achieving a high system damping is consistent with this previous published work. With this in mind, the tracking results of Figure 12 are fairly good.

Efforts were made to increase the system damping, but such efforts required higher-order controllers containing derivative approximations that were unrealizable. In the IRS vehicle system with encoder-quantized feedback, derivatives of higher order than 2 contain too much noise for useful control purposes.

Additionally, it was found that the pole assignment in designing Q(s) to perform frequency-based control input weighting introduced very lightly damped zeros close to the origin. These zeros effectively limited the closed loop phase margin to 55 degrees. Usual implementations of the PQ controller on systems that naturally contain larger frequency separations usually involve systems whose subsystem pole locations do not cancel in the PQ design step. In such a case, the uncancelled poles assist in the frequency shaping of the control input weights. In the vehicle implementation, each branch of the MISO vehicle system share the same pole locations, therefore artificial poles must be introduced that in later design steps become lightly damped zeros. Clearly, a more computer-orientated approach such as an H-infinity method could be used for the choice of Q(s). However, here we present the basic concept and leave it for future work to explore the optimal design of the controllers

6. Conclusions

A coordinated MISO controller was introduced to achieve the high-speed lateral position-tracking task. The design method imposed both frequency separation constraints on the control inputs in addition to addressing traditional stability concerns. Experimental results on a scaled test vehicle confirmed the design.

The results of both the controller design and the experimental implementation indicate a distinct benefit associated with the separation of input frequencies. This is not unlike the use of frequency weighting in other MISO control schemes such as LQ regulation. The main difference would be in the output feedback representation of the PQ versus a state feedback frequency-shaped LQR representation. However, the conceptual similarities would be interesting topics for future investigations.

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