IMECE2003-41609

DIMENSIONLESS SENSITIVITY METHODS TO IDENTIFY VEHICLE CORNERING STIFFNESS FROM YAW RATE MEASUREMENTS

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ABSTRACT

A significant amount of research has focused on modelbased identification of vehicle behavior using Kalman-filter or similar approaches with sometimes complex, high-order or nonlinear vehicle models to achieve estimation accuracy. This work examines the model complexity versus accuracy tradeoff with a bias toward greatly reducing the complexity of the identification model even if this allows some identification inaccuracy. By using the simplest model possible, but no simpler, the goal is to achieve fast convergence. Model simplification is obtained using a novel dimensionless method that exposes explicit and implicit coupling between Bode parameter sensitivities, a coupling that constrains the possible parameter variations. To demonstrate this method, vehicle yaw rate data is used to attempt to identify the cornering stiffness parameter governing the tire-road interaction. Simulation results and experimental implementation on a research vehicle under changing road conditions are presented.

INTRODUCTION

On-board vehicle electronic systems are increasingly using dynamic models of vehicle behavior rather than heuristic or look-up-table methods. The growing list of model-based systems is significant even considering just the area of chassis control: anti-lock brake systems (ABS), traction control systems (TCS), directional stability enhancements such as direct yaw control systems (DYC) and electronic stability programs (ESP) [1, 2], integrated roll-stability programs, and fault-detection subsystems. These model-based controllers require knowledge or approximation of dynamic models, which in turn depend on a number of physical parameters whose values must be measured off-line or determined during operation. Because off-line identification may be difficult or impractical to predict for every driving situation, on-line parameter estimation is attractive. Consequently, a growing amount of research has focused on on-line, and often real-time, estimation of chassis parameters.

The task of identification is particularly difficult in chassis dynamics because of the nature of the vehicle system. Even with a linear model, it can be difficult or dangerous to provide sufficiently exciting inputs. Most drivers would not intentionally swerve - or tolerate excitation inputs - on an icy or frictioncompromised road solely to establish the exact value of a friction parameter. Therefore, the identification algorithm must wait for persistence of excitation (PE)conditions to be satisfied. The topic of PE is beyond the scope of this discussion, and it will be assumed in later testing that such conditions exist. However, it should be clear that, given the potentially short duration of driving situations that satisfy PE, the convergence time of any identification algorithm must be minimized.

The ability to achieve fast estimation is related to the assumed complexity of the model. In some research there is a subtle assumption that if the identification model is more complete and complex, then the estimated values will be

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closer to the true values [3], while others have questioned this assumption [4, 5]. Considering the number of publications that seek to simultaneously estimate a large number of parameters using linear or nonlinear Kalman filter approaches [3, 6, 7], the academic opinion may appear to be biased toward complexity. Some identification algorithms applied to linearized models have reported computational demands on the order of 10^4 to 10^6 computations per sample period [6]. Nonlinear models requiring gradient methods suggest even slower convergence and higher computational demand [3].

The problems of computation, system excitation, and convergence time can be alleviated if only a very small number of parameters are estimated. The identification approach presented in this work is a much-simplified approach based on using sensitivity invariants to extract the maximum amount of available system information prior to identification. The result is an identification algorithm whose computational demands are simplified to the order of 10^1 to 10^2 floating-point calculations per time step.

The remaining sections are summarized as follows: Section 2 presents the governing equations of motion for the vehicle dynamics considered in this study. Section 3 introduces concepts of sensitivity invariance that show explicit and implicit coupling of Bode parameter sensitivities within the system model. Section 4 applies a temporal and spatial parametrization to the vehicle equations that eliminates sensitivity invariants and produces a dimensionless model representation. Section 5 develops an identification model to specifically estimate tire properties from the reduced parameter model based on measurement of the vehicle's yaw rate measurement. Section 6 presents implementation results from simulations and from testing on an experimental vehicle. Results of this testing are then discussed, and conclusions then summarize the main points of this study.

VEHICLE DYNAMICS

The vehicle dynamic model used in this study is known as the bicycle model because the dynamics conceptually model a bicycle whose motion is constrained to planar maneuvers [8,9]. The dynamic equations are obtained by fixing a coordinate system to the center of gravity (CG) of the vehicle and solving for the equations of motion. Roll, pitch, bounce, aerodynamics, and deceleration dynamics are neglected to simplify the vehicle motion to two degrees of freedom: the lateral position and yaw angle. The model is further simplified by assuming that each tire on an axle produces the same lateral forces. As a coordinate system convention, the Society of Automotive Engineers standard is used with the z-axis pointing into the road surface as shown in Fig. 1.

Traditionally, the bicycle model is formulated in transferfunction or state-space form using the front wheels as steering inputs. Equations of motion are derived directly from Newtonian



Figure 1. VEHICLE COORDINATES.

dynamics, and all states are measured from the center-ofgravity. The resulting transfer function from the bicycle model correlating the vehicle yaw rate, $\dot{\psi}$, to the front steering angle, δ_f , is given by:

$$\frac{\dot{\Psi}(s)}{\delta_{f}(s)} = \tag{1}$$

$$\frac{\frac{C_{\alpha f} \cdot a}{I_{z}} \cdot s + \frac{C_{\alpha f} \cdot C_{\alpha r} \cdot L}{m \cdot I_{z} \cdot U}}{s^{2} + \left(\frac{C_{\alpha f} + C_{\alpha r}}{m \cdot U} + \frac{C_{\alpha f} \cdot a^{2} + C_{\alpha r} \cdot b^{2}}{I_{z} \cdot U}\right) \cdot s + \frac{C_{\alpha f} \cdot C_{\alpha r} \cdot L^{2}}{m \cdot I_{z} \cdot U^{2}} - \frac{a \cdot C_{\alpha f} - b \cdot C_{\alpha r}}{I_{z}}}{I_{z}}$$

With the parameters given by:

т	= vehicle mass	(5.451 kg)	
I_z	= vehicle moment of inertia	$(0.1615 kg \cdot m^2)$	(2)
U	= vehicle longitudinal velocity	(4.0 m/s)	
a	= distance from C.G. to front axle	(0.1461 m)	
b	= distance from C.G. to rear axle	(0.2191 m)	
L	= vehicle length, $a + b$	(0.3652 m)	
$C_{\alpha f}$	= cornering stiffness, front 2 tires	(65 N/rad)	
$C_{\alpha r}$	= cornering stiffness, rear 2 tires	(110N/rad)	

The values in parenthesis correspond to the measured values for the experimental scale vehicle used later to validate the identification approach.

With regard to the underlying model, the only knowledge of the tire-road interface is represented by the front and rear cornering stiffness parameters, $C_{\alpha f}$ and $C_{\alpha r}$. The bicycle model does not account for nonlinear tire dynamics. The cornering stiffness represents the slope at the origin of the curve representing the lateral tire forces as a function of the sliding angle of the tire with respect to the road. Although the bicycle model is relatively simple, many investigations have verified that it remains a good approximation for full-size vehicle dynamics as long as accelerations are limited to 0.3 g's [10]. In the presence of changing road conditions at a known velocity, the principle unknown variables in the bicycle model are the cornering stiffness values. Estimation of these variables is therefore a primary goal of this study.

The yaw-rate transfer function is presented because control of yaw-rate offers a very direct way to assist the human driver. On-board vehicle controllers often act under very restricted preview - if any - of the road error. Under limited preview conditions, human drivers appear to most correlate their steering inputs to the yaw rate of the vehicle [11]. Therefore, a primary task for many driver-assist programs is to maintain proper correlation between human steering input and vehicle yawrate despite disturbances or changing road conditions [1, 12]. While other states or states combinations could be used in this identification study (with possibly better results), a yaw-rate sensor is already packaged with many vehicle chassis control systems.

SENSITIVITY INVARIANTS

The concepts of sensitivity invariance used in this work are based on the results of two theorems. The first was developed by Euler in the late 1700's and is known as Euler's Homogenous Function Theorem (EHFT) [13]. It states that, given an arbitrary function of the form:

$$y = f(x_1, x_2, \dots, x_n) \tag{3}$$

that is made homogenous to the constants, A,B,C, when the function is written as:

$$K \cdot y = f(K_1 \cdot x_1, K_2 \cdot x_2, \dots, K_n \cdot x_n)$$
(4)

where the constants K and K_i are constrained by:

$$K = A^{a} \cdot B^{b} \cdot C^{c} \dots$$

$$K_{i} = A^{a_{i}} \cdot B^{b_{i}} \cdot C^{c_{i}} \dots$$
(5)

Then the function *y* is also a solution to the following set of equations:

$$a \cdot y = a_1 \cdot x_1 \cdot \frac{\partial y}{\partial x_1} + a_2 \cdot x_2 \cdot \frac{\partial y}{\partial x_2} + \dots + a_n \cdot x_n \cdot \frac{\partial y}{\partial x_n}$$
(6)

$$b \cdot y = b_1 \cdot x_1 \cdot \frac{\partial y}{\partial x_1} + b_2 \cdot x_2 \cdot \frac{\partial y}{\partial x_2} + \dots + b_n \cdot x_n \cdot \frac{\partial y}{\partial x_n}$$
(6)

$$c \cdot y = c_1 \cdot x_1 \cdot \frac{\partial y}{\partial x_1} + c_2 \cdot x_2 \cdot \frac{\partial y}{\partial x_2} + \dots + c_n \cdot x_n \cdot \frac{\partial y}{\partial x_n}$$

$$\vdots$$

While the above expression generally concludes the mathematical presentation of the EHFT, it is better understood in a modern systems context by rewriting it as:

$$\begin{bmatrix} a \\ b \\ c \\ \vdots \end{bmatrix} = \begin{bmatrix} a_1 \ a_2 \ \dots \ a_n \\ b_1 \ b_2 \ \dots \ b_n \\ c_1 \ c_2 \ \dots \ c_n \\ \vdots \ \vdots \ \vdots \ \vdots \ \vdots \end{bmatrix} \cdot \begin{bmatrix} S_{x_1}^{y} \\ S_{x_2}^{y} \\ \vdots \\ S_{x_n}^{y} \end{bmatrix}$$
(7)

where each $S_{x_i}^y$ represents the sensitivity of the output with respect to the parameter x_i , i.e. the sensitivity function of Bode [14]. Each of the above rows represents an equation for a sensitivity invariant, i.e. a coupling between parameter gradients with regard to the system output.

The second theorem used in this work is known as the Pi Theorem [15, 16]. It is based on the observation that all true mathematical descriptions of physical systems are mathematically homogenous when changes in the units of length, mass, time, charge, temperature, or any consistently applied unit system are applied. The Pi Theorem (not proven here) states that: only systems descriptions whose parameters and outputs are all dimensionless will minimize the parameter sensitivities predicted by the EHFT. The pi-theorem further suggests a very simple methodology of parameterizing system models to form dimensionless system descriptions that eliminate any sensitivity invariance predicted by the EHFT. Because the application of the Pi Theorem is relatively novel in a system identification context, it is presented in detail below for the system identification task.

MODEL REPARAMETRIZATION

The author's original use of the dimensionless approach was motivated by the use of a scale-sized vehicle testbed and the corresponding need to address size scaling in order to represent the most average vehicle system possible. The method of the Pi Theorem is summarized as follows: First, one chooses all the parameters the may enter a mathematical representation (i.e. system model). In the case of the vehicle dynamics the 7 parameters of the bicycle model of Eqn. (2) are the most obvious choice. From these parameters, the dimensional span of the parameters are then inferred by the units on the parameters (i.e. a span of Length-Mass-Time for Newtonian systems, or Length-Time for Kinematics, or Voltage-Current-Time for Circuits, etc). Next, a small subset of the parameter set is chosen as a new unit basis, and the remaining parameters are 'measured' with respect to these new 'units' to form a new dimensionless parameterization. Under changes in the new unit bases, the original model can always be rewritten in the dimensionless form and will contain only pi parameters.

An example illustration of the pi-theorem applied to the vehicle system is now given. For vehicle dynamics, the unit system spans the Mass-Length-Time dimensions. A natural parameter-unit to measure mass is the vehicle's mass, m, for length would be the vehicle's length, L, and for time would be the time to traverse the vehicle length at a constant forward velocity, U/L. Note that this time unit clocks at a constant ratio to the wheel pulses from the anti-lock brake system. The five remaining parameters in the bicycle model, namely $a, b, C_{\alpha f}, C_{\alpha r}$ and I_z , are then 'measured' in this new parameterunit system. This method re-measuring, while not intuitive in an engineering sense, is very simple. For instance, an object p weighing 1 kg remeasured in the vehicle system with a vehicle weighing m = 1000 kg would then weigh, p/m = 0.001. Note that the remeasured object now appears unitless in our old system (insights from this observation by Buckingham and others lead to the Pi Theorem). Symbolically, we can remeasure any parameter by solving a dimensional equation that seeks to produce dimensionless numbers. For instance, the cornering stiffness has units of $[M \cdot Le \cdot T^{-2}]$ where M, Le, T represent mass, length, and time scaling units. Brackets are used here as an operator that extracts the dimension of the variables within the brackets (in the remainder of this paper, it should be clear by the context when brackets are meant to refer to dimensions of variables or to matrix notation). To find how cornering stiffness should be re-measured in the new vehicle unit system, the dimensional equation becomes:

$$\begin{bmatrix} C_{\alpha f} \cdot m^{i} \cdot U^{j} \cdot L^{k} \end{bmatrix} =$$

$$\begin{bmatrix} \left(M \cdot Le \cdot T^{-2} \right) \cdot \left(M \right)^{i} \cdot \left(Le \cdot T^{-1} \right)^{j} \cdot \left(Le \right)^{k} \end{bmatrix} = \begin{bmatrix} M \cdot Le \cdot T \end{bmatrix}^{0}$$
(8)

The variables i, j, k denote integers. Equating powers to solve for i, j, k, three equations are obtained:

$$1+i = 0$$
 (9)
 $-2-j = 0$
 $1+j+k = 0$

Solving the equations gives i = -1, j = -2, and k = 1. The corresponding equation for the new parameter in the vehicle unit system is $\frac{C_{\alpha f'}L}{m \cdot U^2}$. In the early 1900's, Buckingham denoted such new parameters with the symbol π_i , where the subscript *i* was usually a number [13]. The name "pi parameter" eventually became common to denote these types of parameters.

Repeating the previous calculations for the remaining four parameters produces a total of five pi parameters:

$$\pi_1 = \frac{a}{L}, \pi_2 = \frac{b}{L}, \pi_3 = \frac{C_{\alpha f} \cdot L}{m \cdot U^2}, \pi_4 = \frac{C_{\alpha r} \cdot L}{m \cdot U^2}, \pi_5 = \frac{I_z}{m \cdot L^2}.$$
 (10)



Figure 2. RELATIVE DISTRIBUTION OF π_5 .

Comparing the standard form to this new dimensionless form, the standard model consists of 8 parameters while the dimensionless model contains only 5 parameters. This fact already improves excitation conditions and convergence rates considerably with regard to identification. In the state-space form of the model, a pi-parameterization is equivalent to a careful, parameter-based choice in similarity transform and temporal scaling. A more compelete discussion of this equivalence can be found in [17].

There are several advantages to the use of dimensionless pi-parameters over traditional parameterizations. Generally, and with the vehicle dynamics considered here, pi-parameters are well-scaled, always positive, and should be well-scaled, i.e. have magnitudes approximately equal to 1. While distributions of standard vehicle parameters (mass, inertia, etc.) are not shown due to space constraints, such attributes are not exhibited by dimensioned vehicle parameters. Indeed, dimensioned parameters may vary vehicle-to-vehicle by nearly an order of magnitude. Pi parameters are very tightly grouped in pi space, and so their frequency distributions are localized and generally well-defined (see Fig. 2). Finally, the pi-parameters, because they are dimensionless, are independent of the unit system used (British Standard, SI, MKS, etc.) and thus are universal.

The true distribution and cause of the Gaussian-like fit of the pi-parameters is unknown, but the assumption of a normal distribution allows a numerical definition of an average parameter set. Using pi-values from over 700 vehicles, the average pi-values are calculated and shown in Eqn. (11). If these are compared to previous reported values calculated when the database was only 30 vehicles [18], the averages are seen to be nearly identical:

$$\overline{\pi_{1}} = 0.4431$$
(11)
$$\overline{\pi_{2}} = 1 - \overline{\pi_{1}}$$

$$\overline{\pi_{3}} = 145.68/U^{2}$$

$$\overline{\pi_{4}} = 1.0977 \cdot \overline{\pi_{3}}$$

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$$\overline{\pi_5} = 0.2510$$

While the pi-theorem method of parameterizing the system model explicitly eliminates sensitivity invariance due to dimensional scaling, other invariances may be present. Such invariances may arise because similar systems (like passenger vehicles) are often designed or evolved under very similar constraints. Intuitively, their designs may tend to converge to an optimal manifold in the pi space. Such sensitivity relationships are hereafter referred to as *implicit* sensitivity invariances to distinguish them from the explicit sensitivity invariances predicted by the EHFT.

Implicit sensitivity relationships can be found by numerical pattern recognition, which in practice involves simply plotting parameters against each other looking for relationships. In many cases, very well-defined curves can be observed in the form of simple lines or polynomials [17]. What appear to be simple pi-relationships in the dimensionless parameter domain often map to nonlinear and/or power-law relationships in the standard domain, and such relationships are easily overlooked when there is a nontrivial amount of scatter or measurement error present. In the vehicle dynamics case, the relationship between the third and fourth pi parameter is scattered normally about a line, $\pi_4 \cong 1.0977 \cdot \pi_3$ [17]. Also, the first and second pi parameters are related exactly by a line, $\pi_2 = 1 - \pi_1$. The first implicit invariance relationship is a mathematical statement of the intuitive observation that the tire/road interaction the back tires will generally be a constant multiple of the interaction of the front tires. The second relationship represents the geometric constraint (by definition) that the vehicle length is the sum of the a and b parameters. While implicit sensitivity invariances require analysis by the engineer developing the model, their discovery can greatly simplify model representation. In the case of chassis dynamics, they will be used to greatly simplify the identification model.

DEVELOPMENT OF THE IDENTIFICATION MODEL

The transfer function representation of Eqn. (1) is easily parameterized to a dimensionless form either by direct substitution of the pi values or by simple remeasure of the time and spatial units in the new unit system. One must exercise caution with the time scaling, as the Laplace variable carries units of inverse-time and must be scaled as well. The scaled Laplace variable will be denoted hereafter by the symbol, \bar{s} . The yaw-rate transfer function, with bars representing time-units that are scaled to dimensionless time, is as follows:

$$\frac{\overline{\dot{\psi}}(\overline{s})}{\delta_f(\overline{s})} = \tag{12}$$

$$\frac{\frac{\pi_1 \cdot \pi_3}{\pi_5} \cdot \overline{s} + \frac{\pi_3 \cdot \pi_4}{\pi_5}}{\overline{s}^2 + \left(\pi_3 + \pi_4 + \frac{\pi_1^2 \cdot \pi_3 + \pi_2^2 \cdot \pi_4}{\pi_5}\right) \cdot \overline{s} + \frac{\pi_3 \cdot \pi_4 - \pi_1 \cdot \pi_3 + \pi_2 \cdot \pi_4}{\pi_5}}$$

Examining this dimensionless model, we note that nearly all the pi-parameters are constant and approximately given by the average measurements of Eqn. (11), with the sole exception being the π_3 parameter. This parameter remains the only varying, unknown parameter with respect to either vehicle velocity or road-tire interface (tire force). Explicitly extracting the π_3 variable from the transfer function, we obtain:

$$\overline{\overline{\psi}(\overline{s})} = (13)$$

$$\frac{p_1}{p_5} \cdot \pi_3 \cdot \overline{s} + \frac{p_4}{p_5} \cdot \pi_3^2$$

$$\frac{p_1}{\overline{s}^2 + \left(1 + p_4 + \frac{p_1^2 + (1 - p_1)^2 \cdot p_4}{p_5}\right) \cdot \pi_3 \cdot \overline{s} + \frac{\pi_3^2 \cdot p_4 - p_1 \cdot \pi_3 + (1 - p_1) \cdot p_4 \cdot \pi_3}{p_5}}$$

with p_1 and p_5 equal to 0.4431, 0.2510, i.e. the average values of π_1 and π_5 . The value of p_4 is set to 1.0977, i.e. the slope of the implicit π_3 - π_4 relationship (not plotted in this work, see [18]). It should be clear that plots of transfer function coefficients will be nonlinear with respect to changes in the π_3 parameter.

If a discrete model of the system of Eqn. 13 is formed, and the coefficients are plotted as a function of π_3 , the coefficients are found to be almost perfectly linear in the parameters under π_3 variations. The discrete z-transform model was obtained from a zero-order hold with a sample time (0.001 seconds).

$$\frac{\dot{\Psi}(z)}{\delta_f(z)} = \frac{b_1 \cdot z^{-1} + b_2 \cdot z^{-2}}{1 + a_1 \cdot z^{-1} + a_2 \cdot z^{-2}} \tag{14}$$

The linear dependence of the discrete coefficients b_1, b_2, a_1 and a_2 on the pi-parameter was unexpected, so the algebraic conversion from the Laplace to the z-domain via a zero-orderhold [19,20] was solved. Under the unrestrictive assumption that $T \ll \pi_i$, one obtains the following algebraic relationships for the z-transfer function coefficients of Eqn. (14):

$$b_{1} \cong m_{1} \cdot \pi_{3}$$

$$b_{2} \cong -b_{1}$$

$$a_{1} \cong (-2 + m_{2} \cdot T) \cdot \pi_{3}$$

$$a_{2} \cong (1 - m_{2} \cdot T) \cdot \pi_{3}$$

$$m_{1} = \frac{p_{1}}{p_{5}} \cdot T$$

$$m_{2} = 1 + p_{4} + \frac{p_{1}^{2} + (1 - p_{1})^{2} \cdot p_{4}}{p_{5}}$$
(15)

The pi-parameterized system is well scaled numerically compared to the standard representation, whose coefficients

may vary over many orders of magnitude. This fact is utilized along with the assumption that $T \ll \pi_i$ to simplify the discrete representation to the point of a linear coefficient dependence. Thus, the approximations of Eqn. (15) appear to be suitable over a wider range of road variations for vehicle dynamics using the dimensionless parameterization than for standard representations.

Substitution of the above relationships into the z-transfer function representation yields the following relationship:

$$\frac{\dot{\psi}(z)}{\delta_f(z)} = \frac{(m_1 \cdot \pi_3) \cdot z^{-1} - (m_1 \cdot \pi_3) \cdot z^{-2}}{1 + (-2 + m_2 \cdot \pi_3) \cdot z^{-1} + (1 - m_2 \cdot pi_3) \cdot z^{-2}}$$
(16)

Note that the model is now parameterized explicitly in one parameter, and the coefficients are all linear in this parameter. To identify this function using standard identification algorithms, the model is rewritten in difference-equation form:

$$\dot{\psi} - 2 \cdot \dot{\psi}(z-1) + \dot{\psi}(z-2) =$$

$$-m_2 \cdot \pi_3 \cdot \dot{\psi}(z-1) + m_2 \cdot \pi_3 \cdot \dot{\psi}(z-2) +$$

$$+m_1 \cdot \pi_3 \cdot \delta_f(z-1) - m_1 \cdot \pi_3 \cdot \delta_f(z-2)$$
(17)

Using the delay-operator notation, define the following:

$$y(q) = \dot{\psi} - 2 \cdot \dot{\psi}(z-1) + \dot{\psi}(z-2)$$
(18)

$$\theta^{T} = \pi_{3}$$

$$\phi^{T}(q-1) = -m_{2} \cdot \pi_{3} \cdot \dot{\psi}(z-1) + m_{2} \cdot \pi_{3} \cdot \dot{\psi}(z-2) + m_{1} \cdot \pi_{3} \cdot \delta_{f}(z-1) - m_{1} \cdot \pi_{3} \cdot \delta_{f}(z-2)$$

These definitions then produce a one-parameter regression model:

$$y(q) = \mathbf{\phi}^T (q-1) \cdot \mathbf{\theta} \tag{19}$$

This linear, one-parameter form of the yaw-rate equation of very well suited for recursive least-squares estimation (RLSE). A standard RLSE algorithm with a forgetting factor is given by Astrom and Wittenmark [21] as:

$$K(q) = P(q-1)\varphi(q-1) \cdot$$

$$\left(\lambda + \varphi^{T}(q-1)P(q-1)\varphi(q-1)\right)^{-1}$$

$$\hat{\theta}(q) = \hat{\theta}(q-1) + K(q) \cdot \left(y(q) - \varphi^{T}(q-1) \cdot \hat{\theta}(q-1)\right)$$

$$P(q) = \left(I - K(q)\varphi^{T}(q-1)\right)P(q-1)/\lambda$$
(20)

with the condition that the initial values of the parameter estimate, $\hat{\theta}(q_0)$, and covariance matrix, $P(q_0)$, are specified by

the user. In the derivation of this algorithm, it is assumed that the term $\Phi^T(q-1) \cdot \Phi(q-1)$ is nonsingular for all $q > q_0$, with $\Phi(q-1)$ defined as:

$$\Phi(q-1) = \begin{bmatrix} \varphi^T(1) \\ \vdots \\ \varphi^T(q-1) \end{bmatrix}$$
(21)

The forgetting factor is given by the term λ . A step-by-step derivation and explanation of this procedure is presented by Astrom and Wittenmark [21].

The computational overhead of this simplified algorithm is especially low, on the order of 10 floating point computations per sample cycle. This simplicity allows the algorithm to be implemented even on very basic and low-cost embedded microprocessors. This computational simplicity, due again to sensitivity invariance, stands in very sharp contrast to the Kalman-filter and nonlinear gradient-based approaches discussed earlier that are report per-time-step computations at least three orders of magnitude higher. However, there is a tradeoff in accuracy.

Because the discrete function to be identified is now an approximation to an average continuous-time system representation, which itself is an average approximation to the specific vehicle, one should not expect *exact* matching of the identified parameter to the the true parameter. The possible mismatch is a tradeoff of accuracy for faster estimation. For vehicle control, one might desire access to fast (realtime) updates of the parameter estimate, very fast parameter convergence, and very lax excitation conditions rather than very accurate but very slow estimation of true parameters. A quickly-updating but biased estimate may be very useful as an indication of parameter variation and may be more useful to crash prevention and controller scheduling than a very accurate but slowly converging algorithm.

The procedure for identification can now be summarized in the following steps:

- 1. Choose a sample time, T_s , to sample the yaw rate and front steering inputs.
- 2. Calculate the corresponding dimensionless sampling time, T, by multiplying T_s by the vehicle velocity and dividing by the vehicle length.
- 3. Calculate m_1 and m_2 from Eqn. (15) using the average pi values from Eqn.(11)
- 4. At each time instant T_s , sample the yaw-rate and control input, filter both using the appropriate filters and create a dimensionless yaw rate by multiplying the dimensioned measurement by the vehicle length and dividing by the vehicle velocity (note that the control input is already dimensionless, as it is in radians).

- 5. Using the dimensionless yaw rate, calculate the values y(q) and $\varphi^T(q-1)$ in Eqn. (18)
- 6. Update the parameter estimates using the desired algorithm, in this case the RLSE of Eqn. (20).

TESTING AND IMPLEMENTATION OF THE IDENTIFICATION ALGORITHM

Before discussion of implementation, it should be mentioned that the discovery of sensitivity invariants by the authors was serendipitous and the result of a novel and experimental approach to vehicle studies [17]. In order to study dangerous vehicle controllers in a safe and affordable manner, a scale vehicle roadway simulator was developed where autonomous and remotely-human-driven scale-sized vehicles are driven on a large treadmill 'roadway' to simulate highway driving [18, 22]. An analogy to this vehicle/treadmill system would be the well-known aircraft/wind-tunnel testing system in aerodynamic studies. The Pi Theorem method is commonly used in the area of fluids and heat-transfer problems in order to guarantee that research results will scale and compare correctly with respect to changes in experimental setup. The the bicycle model parameters presented in Eqn. (2) represent measurements taken from a scale vehicle that is tested at a velocity, U, of 4.0 m/s. This corresponds approximately to a full-size vehicle at 63 m.p.h. These parameters were used for development of a simulation study.

The identification method was tested in simulation as the vehicle was made to follow a reference square wave of amplitude 0.06 meters to represent repeated, aggressive lane-changing maneuvers in a full-size vehicle. The steering output of the controller design was filtered with a 5 Hz, second-order filter of unity gain and damping ratio of 0.707 to simulate a steering actuator that will be present on the test vehicle. Every 10 seconds, the front and rear cornering stiffness are changed from the nominal values of Eqn. (2) to a value of 50% nominal to simulate a reduced road-friction situation. The instances of friction change are indicated in Fig. 5 with vertical dashed lines in each of the plots. The plant input (wheel angle in radians) and plant output (simulated yaw rate in rad/sec) were sampled every 0.001 seconds. The sampled signals were then filtered digitally with a 4th-order Bessell filter with lower and upper passband edge frequencies of 40 and 150 rad/sec respectively. The purpose of the lowpass effect is to remove high-frequency aliasing effects due to sampling, while the purpose of the highpass effect is to remove signal biases due to constant disturbances (such as slight wheel misalignment). The forgetting factor was set to 0.99995 for both the simulation and experimental studies. This value was chosen by manual tuning of the simulation until a 'good' tradeoff between convergence rate and forgetting was achieved.

The top plot of Fig. 3 shows the lateral position of the vehicle as it attempts to track the reference position (shown in



Figure 3. IDENTIFICATION ALGORITHM APPLIED TO THE SIMULATED VEHICLE

dotted lines). The middle plot is the corresponding yaw-rate measurement. The bottom plot is the estimated π_3 parameter using the identification algorithm previously described plotted alongside measured values of the π_3 parameter.

Based on the good convergence exhibited by the simulation study, an experimental investigation was implemented on the actual test vehicle. While the bicycle model parameters of the vehicle were measured off-line and are the same values as reported in Eqn. (2), these values were recorded with a very large uncertainty, especially in the inertia and cornering stiffness measurements. The experimental vehicle is used to introduce a real-world plant that exhibits nonlinearities, unmodeled dynamics, and disturbances that are also present in passenger vehicles that are otherwise ignored in a simulation study. For the experimental vehicle, the defining length, mass, and velocity correlate well to an 'average' full-sized vehicle at a speed of 63 mph [17, 18].

To test the vehicle under a driving situation that exhibited a severe change in road friction, the vehicle was driven on a treadmill where one-half of the treadmill was dry and one half was wetted. In off-line testing, the cornering stiffnesses appeared to be reduced by a factor of approximately a half of the dry-road value [22], a friction change more resembling an icy road for a full-size test situation. The vehicle was made to follow a period 10 square-wave of amplitude 15 cm on the dry portion of the treadmill. After 60 seconds, a reference change represented by a steep ramp input up to an offset of 45 cm forced the vehicle onto the wetted partition of the road for 20 seconds, after which the vehicle was ramped back to the dry portion. The partition between dry and wet road is shown in Fig. 4 top plot by the dotted line at zero.



Figure 4. IDENTIFICATION ALGORITHM APPLIED TO THE EXPERIMENTAL VEHICLE

The implementation results from the experimental vehicle testing are shown in Fig. 4. Again, the top plot shows the lateral position of the vehicle on the road, the middle plot shows the yaw-rate measurement, and the bottom plot shows the recursive estimate of the π_3 parameter as a function of time, as well as the dry-road value that was measured off-line beforehand.

The vehicle is seen to clearly have difficultly maintaining tracking performance on the wetted road surface. Most untrained human drivers would crash their vehicle in this situation, and the autonomous steering controller was only barely able to maintain vehicle directional control.

DISCUSSION

While the simulation results show correct convergence to true parameters, it is difficult to evaluate the estimation accuracy of experimental results because the definition of 'true' cornering stiffness unknown. Cornering stiffness is very difficult to define in practice as the strict definition of cornering stiffness is the slope at the origin of the nonlinear curve relating lateral force versus slip. The force is measured in the side (lateral force) direction when the tire is sliding at different angles (slip) with respect to the underlying surface. The cornering stiffness as estimated in this work is a lumped, linear estimate of tire-road interaction that best fits one output measure of chassis dynamics. The tire may be operating at any location on the force-slip curve. This experimental implementation was conducted on wet treadmill similar to an icy roadway, which for short durations will cause especially nonlinear tire behavior. Therefore, the estimation algorithm at these times is not providing the correct estimate of the actual tire cornering stiffness, but rather is identifying the cornering stiffness that best matches the observed vehicle behavior.

However, the tire-force curve is never *exactly* linear at any region, and for purposes of indicating changes in vehicle chassis behavior a best-approximation can actually be very useful. Both the simulation and experimental testing showed definite *changes* in the pi-parameter estimate when the cornering stiffness and road surface were suddenly varied. The parameter estimate lagged the parameter change slightly, but repeated testing seemed to show that this lag is related to primarily to lack of excitation of the system and the choice of a relatively large amount of memory in past estimates (i.e. a forgetting factor was chosen very close to 1).

For both simulation and experimental studies, the steering inputs did not appear to provide enough excitation for the parameters to converge during straight-line driving. Clearly, the parameters are updated only during maneuvers that produced relatively large yaw rates. In the experimental case, some parameter drift is very obvious in the estimate, and this is likely due to the large amount of correlated noise in the yawrate measurement. For this test, yaw rate was obtained an encoder attached to the top of the vehicle [22] and this likely increases correlation between vehicle bounce and yaw angle due to bounce of the sensing arm. In full-sized vehicles using inertial units, such correlation would not be as likely or could be better mitigated in the sensor design and placement.

CONCLUSIONS AND FUTURE WORK

This study presented a concept of sensitivity invariance within a vehicle identification framework focusing on roadtire interaction at highway speeds. By using sensitivity invariance, the vehicle model was reduced from a nonlinear, seven-parameter estimation problem to a linear, one-parameter representation suitable for identification. Both simulation and experimental studies demonstrated the method.

Obvious improvements to the algorithm would be to address methods to modify the parameter converge. Well-known techniques include conditional parameter updating, covariance resetting, parallel estimation, leakage approaches, directional forgetting, or robust estimation to name a few [6, 21, 23]. Such approaches were intentionally neglected in this study to maintain simplicity in the presentation of the sensitivity invariance approach and the ability of this approach to identify the model.

The extension of dimensional analysis into identification and systems theory arose from the author's attempt to understand the underlying assumptions of the Pi Theorem. Similar work, particularly circuit network analysis and biological modeling in the period of the 1970's [14, 24, 25], suggests a generality to the identification approach beyond vehicle systems. Extensive work remains in developing automated methods of invariance discovery and automated tools for finding the best parameterdomain representation.

ACKNOWLEDGMENT

The first author gratefully acknowledges the support of the National Science Foundation Graduate Fellowship in completing this work.

REFERENCES

- [1] Ackermann, J., and Sienel, W., 1993. "Robust yaw rate control". *IEEE Transactions on Control and System Technology*, **1**, pp. 15–20.
- [2] Furukawa, Y., and Abe, M., 1997. "Advanced chassis control systems for vehicle handling and active safety". *Vehicle System Dynamics*, 28, pp. 59–86.
- [3] Russo, M., Russo, R., and Volpe, A., 2000. "Car parameters identification by handling maneuvers". *Vehicle System Dynamics*, 34, pp. 423–436.
- [4] Smith, D. E., and Starkey, J. M., 1994. "Effects of model complexity on the performance of automated vehicle steering controllers: Controller development and evaluation". *Vehicle System Dynamics*, 23, pp. 627–645.
- [5] Smith, D. E., and Starkey, J. M., 1995. "Effects of model complexity on the performance of automated vehicle steering controllers: Model development, validation and comparison". *Vehicle System Dynamics*, 24, pp. 163– 181.
- [6] Samadi, B., Kazemi, R., Nikravesh, K. Y., and Kabganian, M., 2001. "Real-time estimation of vehicle state and tireroad friction forces". In Proceedings of the 2001 American Control Conference, Washington D.C., pp. 3318–3323.
- [7] Venhovens, P. J. T., and Naab, K., 1999. "Vehicle dynamics estimation using kalman filters". *Vehicle System Dynamics*, 32, pp. 171–184.
- [8] Alleyne, A., 1997. "A comparison of alternative intervention strategies for unintended roadway departure (urd) control". *Vehicle System Dynamics*, 27, pp. 157– 186.
- [9] Dugoff, H., Fancher, P. S., and Segel, L., 1970. "An analysis of tire traction properties and their influence on vehicle dynamic performance". *SAE Transactions*, **79**, pp. 341–366.
- [10] LeBlanc, D. J., Johnson, G. E., Venhovens, P. J. T., Gerber, G., DeSonia, R., Ervin, R. D., Lin, C.-F., Ulsoy, A. G., and Pilutti, T. E., 1996. "Capc: A road-departure prevention system". *IEEE Control Systems Magazine* [December], pp. 61–71.
- [11] McLean, J. R., and Hoffman, E. R., 1973. "The effects of restricted preview on driver steering control and performance". *Human Factors*, **15**, pp. 421–430.
- [12] Peng, H., and Tomizuka, M., 1993. "Preview control for

vehicle lateral guidance in highway automation". *ASME Journal of Dynamic Systems, Measurement and Control*, **115**, pp. 679–686.

- [13] Buckingham, E., 1914. "On physically similar systems; illustrations of the use of dimensional equations". Physical Review, 4 (2nd series), pp. 345–376.
- [14] Frank, P. M., 1978. *Introduction to System Sensitivity Theory*. Academic Press, New York.
- [15] Taylor, E. S., 1974. *Dimensional Analysis for Engineers*. Clarendon Press, Oxford.
- [16] Bridgman, P. W., 1943. *Dimensional Analysis*. Yale University Press, New Haven, Conneticut.
- [17] Brennan, S., 2002. On Size and Control: The Use of Dimensional Analysis in Controller Design. PhD Thesis, Dept. of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, December.
- [18] Brennan, S., and Alleyne, A., 2001. "Using a scale testbed: Controller design and evaluation". IEEE Control Systems Magazine, **21** (3) [June], pp. 15–26.
- [19] Franklin, G. F., Powell, J., and Emami-Naeini, A., 2002. *Feedback Control of Dynamic Systems*. Prentice Hall, Upper Saddle River, New Jersey.
- [20] Astrom, K. J., and Wittenmark, B., 1997. Computer-Controlled Systems: Theory and Design. Prentice Hall, Upper Saddle River, New Jersey.
- [21] Astrom, K. J., and Wittenmark, B., 1995. *Adaptive Control*. Addison-Wesley, Reading, Massachusetts.
- [22] Brennan, S., 1999. Modeling and control issues associated with scaled vehicles. MS Thesis, Dept. of Mechanical and Industrial Engineering, University of Illinois at Urbana-Champaign, Urbana, IL, August.
- [23] Ljung, L., 1999. *System Identification: Theory for the User*. Prentice Hall, Upper Saddle River, New Jersey.
- [24] Eslami, M., 1994. Theory of Sensitivity in Dynamic Systems, An Introduction. Springer-Verlag, New York.
- [25] McMahon, T. A., and Bonner, J. T., 1983. *On Size and Life*. Scientific American Books, Inc., New York.